BHASKARACHARYA

The period between 500 and 1200 AD was the golden age of Indian Astronomy. In this long span of time Indian Astronomy flourished mainly due to eminent astronomers like Aryabhat, Lallacharya, Varahamihir, Brahmagupta, Bhaskaracharya and others. Bhaskaracharya's Siddhanta Shiromani is considered as the pinnacle of all the astronomical works of those 700 hundred years. It can be aptly called the "essence" of ancient Indian Astronomy and mathematics. In the ninth century Brahmagupta's Brahmasphutasiddhanta was translated in Arabic. The title of the translation was 'Sind Hind'. This translation proved to be a watershed event in the history of numbers. The Arabs quickly grasped the importance of the Indian decimal system of numbers. They played a key role in transmitting this system of numbers to Europeans. For a long time Europeans were using Roman Numerals, which were very tedious to handle. After accepting the decimal system of numbers, European mathematicians made a remarkable progress in mathematics, but that was about 500 years after Bhaskaracharya.

From 750 AD Onwards India was engulfed in waves of foreign attacks. In 1205 AD Bakhtiyar Khilji destroyed the magnificent Nalanda University, which was a renowned center of knowledge for about 800 years. India was in utter chaotic state till the country was colonized by British. All universities and learning centers in India were destroyed, knowledge was lost and hardly any progress was made in mathematics and astronomy. A few scholars like Keshav Daivadnya, Ganesh Daivadnya Madhav, Sawai Jai Singh and others tried to keep the flame of knowledge burning in that dark period.

Birth and Education of Bhaskaracharya :

Ganesh Daivadnya has bestowed a very apt title on Bhaskaracharya. He has called him 'Ganakchakrachudamani', which means, 'a gem among all the calculators of astronomical phenomena.' Bhaskaracharya himself has written about his birth, his place of residence, his teacher and his education, in Siddhantashiromani as follows, ' A place called 'Vijjadveed', which is surrounded by Sahyadri ranges, where there are scholars of three Vedas, where all branches of knowledge are studied, and where all kinds of noble people reside, a brahmin called Maheshwar was staying, who was born in Shandilya Gotra (in Hindu religion, Gotra is similar to lineage from a particular person, in this case sage Shandilya), well versed in Shroud (originated from 'Shut' or 'Vedas') and 'Smart' (originated from 'Smut') Dharma, respected by all and who was authority in all the branches of knowledge. I acquired knowledge at his feet'. From this verse it is clear that Bhaskaracharya was a resident of Vijjadveed and his father Maheshwar taught him mathematics and astronomy. Unfortunately today we have no idea where Vijjadveed was located. It is necessary to ardently search this place which was surrounded by the hills of Sahyadri and which was the center of learning at the time of Bhaskaracharya. He writes about his year of birth as follows, 'I was born in Shake 1036 (1114 AD) and I wrote Siddhanta Shiromani when I was 36 years old.' Bhaskaracharya has also written about his education. Looking at the knowledge, which he acquired in a span of 36 years, it seems impossible for any modern student to achieve that feat in his entire life. See what Bhaskaracharya writes about his education,

'I have studied eight books of grammar, six texts of medicine, six books on logic, five books of mathematics, four Vedas, five books on Bharat Shastras, and two Mimansas'. Bhaskaracharya calls himself a poet and most probably he was Vedanti, since he has mentioned 'Parambrahman' in that verse.

SIDDHANTASHIROMANI

Bhaskaracharya wrote Siddhanta Shiromani in 1150 AD when he was 36 years old. This is a mammoth work containing about 1450 verses. It is divided into four parts, Lilawati, Beejaganit, Ganitadhyaya and Goladhyaya. In fact each part can be considered as separate book. The numbers of verses in each part are as follows, Lilawati has 278, Beejaganit has 213, Ganitadhyaya has 451 and Goladhyaya has 501 verses. One of the most important characteristic of Siddhanta Shiromani is, it consists of simple methods of calculations from Arithmetic to Astronomy. Essential knowledge of ancient Indian Astronomy can be acquired by reading only this book. Siddhanta Shiromani has surpassed all the ancient books on astronomy in India. After Bhaskaracharya nobody could write excellent books on mathematics and astronomy in lucid language in India. In India, Siddhanta works used to give no proofs of any theorem. Bhaskaracharya has also followed the same tradition. Lilawati is an excellent example of how a difficult subject like mathematics can be written in poetic language. Lilawati has been translated in many languages throughout the world. When British Empire became paramount in India, they established three universities in 1857, at Bombay, Calcutta and Madras. Till then, for about 700 years, mathematics was taught in India from Bhaskaracharya's Lilawati and Beejaganit. No other textbook has enjoyed such long lifespan.

BHASKAR'S MATHEMATICS

Lilawati and Beejaganit together consist of about 500 verses. A few important highlights of Bhaskar's mathematics are as follows,Terms for numbers In English, cardinal numbers are only in multiples of 1000. They have terms such as thousand, million, billion, trillion, quadrillion etc. Most of these have been named recently. However, Bhaskaracharya has given the terms for numbers in multiples of ten and he says that these terms were coined by ancients for the sake of positional values. Bhaskar's terms for numbers are as follows:
eka(1), dasha(10), shata(100), sahastra(1000), ayuta(10,000), laksha(100,000), prayuta (1,000,000=million), koti(107), arbuda(108), abja(109=billion), kharva (1010), nikharva (1011), mahapadma (1012=trillion), shanku(1013), jaladhi(1014), antya(1015=quadrillion), Madhya (1016) and parardha(1017).

Kuttak

Kuttak is nothing but the modern indeterminate equation of first order. The method of solution of such equations was called as 'pulverizer' in the western world. Kuttak means to crush to fine particles or to pulverize. There are many kinds of Kuttaks. Let us consider one example. In the equation, $ax + b = cy$, a and b are known positive integers. We want to also find out the values of x and y in integers. A particular example is, $100x + 90 = 63y$ Bhaskaracharya gives the solution of this example as, x = 18, 81, 144, 207… And y=30, 130, 230,330… Indian Astronomers used such kinds of equations to solve astronomical problems. It is not easy to find solutions of these equations but Bhaskara has given a generalized solution to get multiple answers.

Chakrawaal

Chakrawaal is the "indeterminate equation of second order" in western mathematics. This type of equation is also called Pell's equation. Though the equation is recognized by his name Pell had never solved the equation. Much before Pell, the equation was solved by an ancient and eminent Indian mathematician, Brahmagupta (628 AD). The solution is given in his Brahmasphutasiddhanta. Bhaskara modified the method and gave a general solution of this equation. For example, consider the equation $61x2 + 1 = y2$. Bhaskara gives the values of x = 22615398 and y = 1766319049
There is an interesting history behind this very equation. The Famous French mathematician Pierre de Fermat (1601-1664) asked his friend Bessy to solve this very equation. Bessy used to solve the problems in his head like present day Shakuntaladevi. Bessy failed to solve the problem. After about 100 years another famous French mathematician solved this problem. But his method is lengthy and could find a particular solution only, while Bhaskara gave the solution for five cases. In his book 'History of mathematics', see what Carl Boyer says about this equation,
'In connection with the Pell's equation $ax2 + 1 = y2$, Bhaskara gave particular solutions for five cases, a = 8, 11, 32, 61, and 67, for $61x2 + 1 = y2$, for example he gave the solutions, x = 226153980 and y = 1766319049, this is an impressive feat in calculations and its verifications alone will tax the efforts of the reader' Henceforth the so-called Pell's equation should be recognized as 'Brahmagupta-Bhaskaracharya equation'.

Simple mathematical methods

Bhaskara has given simple methods to find the squares, square roots, cube, and cube roots of big numbers. He has proved the Pythagoras theorem in only two lines. The famous Pascal Triangle was Bhaskara's 'Khandameru'. Bhaskara has given problems on that number triangle. Pascal was born 500 years after Bhaskara. Several problems on permutations and combinations are given in Lilawati. Bhaskar. He has called the method 'ankapaash'. Bhaskara has given an approximate value of PI as 22/7 and more accurate value as 3.1416. He knew the concept of infinity and called it as 'khahar rashi', which means 'anant'. It seems that Bhaskara had not notions about calculus, One of his equations in modern notation can be written as, d(sin (w)) = cos (w) dw.

BHASKAR'S ASTRONOMY

Ganitadhyaya and Goladhyaya of Siddhanta Shiromani are devoted to astronomy. All put together there are about 1000 verses. Almost all aspects of astronomy are considered in these two books. Some of the highlights are worth mentioning. Earth's circumference and diameter Bhaskara has given a very simple method to determine the circumference of the Earth. According to this method, first find out the distance between two places, which are on the same longitude. Then find the correct latitudes of those two places and difference between the latitudes. Knowing the distance between two latitudes, the distance that corresponds to 360 degrees can be easily found, which the circumference of is the Earth. For example, Satara and Kolhapur are two cities on almost the same longitude. The difference between their latitudes is one degree and the distance between them is 110 kilometers. Then the circumference of the Earth

is 110 X 360 = 39600 kilometers. Once the circumference is fixed it is easy to calculate the diameter. Bhaskara gave the value of the Earth's circumference as 4967 'yojane' (1 yojan = 8 km), which means 39736 kilometers. His value of the diameter of the Earth is 1581 yojane i.e. 12648 km. The modern values of the circumference and the diameter of the Earth are 40212 and 12800 kilometers respectively. The values given by Bhaskara are astonishingly close.

Aksha kshetre

For astronomical calculations, Bhaskara selected a set of eight right angle triangles, similar to each other. The triangles are called 'aksha kshetre'. One of the angles of all the triangles is the local latitude. If the complete information of one triangle is known, then the information of all the triangles is automatically known. Out of these eight triangles, complete information of one triangle can be obtained by an actual experiment. Then using all eight triangles virtually hundreds of ratios can be obtained. This method can be used to solve many problems in astronomy.

Geocentric parallax

Ancient Indian Astronomers knew that there was a difference between the actual observed timing of a solar eclipse and timing of the eclipse calculated from mathematical formulae. This is because calculation of an eclipse is done with reference to the center of the Earth, while the eclipse is observed from the surface of the Earth. The angle made by the Sun or the Moon with respect to the Earth's radius is known as parallax. Bhaskara knew the concept of parallax, which he has termed as 'lamban'. He realized that parallax was maximum when the Sun or the Moon was on the horizon, while it was zero when they were at zenith. The maximum parallax is now called Geocentric Horizontal Parallax. By applying the correction for parallax exact timing of a solar eclipse from the surface of the Earth can be determined.

Yantradhyay

In this chapter of Goladhyay, Bhaskar has discussed eight instruments, which were useful for observations. The names of these instruments are, Gol yantra (armillary sphere), Nadi valay (equatorial sun dial), Ghatika yantra, Shanku (gnomon), Yashti yantra, Chakra, Chaap, Turiya, and Phalak yantra. Out of these eight instruments Bhaskara was fond of Phalak yantra, which he made with skill and efforts. He argued that ' this yantra will be extremely useful to astronomers to calculate accurate time and understand many astronomical phenomena'. Bhaskara's Phalak yantra was probably a precursor of the 'astrolabe' used during medieval times.

Dhee yantra

This instrument deserves to be mentioned specially. The word 'dhee' means ' Buddhi' i.e. intelligence. The idea was that the intelligence of human being itself was an instrument. If an intelligent person gets a fine, straight and slender stick at his/her disposal he/she can find out many things just by using that stick. Here Bhaskara was talking about extracting astronomical information by using an ordinary stick. One can use the stick and its shadow to find the time, to fix geographical north, south, east, and west. One can find the latitude of a place by measuring

the minimum length of the shadow on the equinoctial days or pointing the stick towards the North Pole. One can also use the stick to find the height and distance of a tree even if the tree is beyond a lake.

A GLANCE AT THE ASTRONOMICAL ACHIEVEMENTS OF BHASKARACHARYA

- The Earth is not flat, has no support and has a power of attraction.
- The north and south poles of the Earth experience six months of day and six months of night.
- One day of Moon is equivalent to 15 earth-days and one night is also equivalent to 15 earth-days.
- Earth's atmosphere extends to 96 kilometers and has seven parts.
- There is a vacuum beyond the Earth's atmosphere.
- He had knowledge of precession of equinoxes. He took the value of its shift from the first point of Aries as 11 degrees. However, at that time it was about 12 degrees.
- Ancient Indian Astronomers used to define a reference point called
- 'Lanka'. It was defined as the point of intersection of the longitude passing through Ujjaini and the equator of the Earth. Bhaskara has considered three cardinal places with reference to Lanka, the Yavakoti at 90 degrees east of Lanka, the Romak at 90 degrees west of Lanka and Siddhapoor at 180 degrees from Lanka. He then accurately suggested that, when there is a noon at Lanka, there should be sunset at Yavkoti and sunrise at Romak and midnight at Siddhapoor.
- Bhaskaracharya had accurately calculated apparent orbital periods of the Sun and orbital periods of Mercury, Venus, and Mars. There is slight difference between the orbital periods he calculated for Jupiter and Saturn and the corresponding modern values. About 800 years back an intelligent mathematician and astronomer was born in Maharashtra. Unfortunately Maharashtrians have hardly taken cognizance of such a great man. It is good to see that new amateur astronomy clubs and centers are being named after him and there are awards given in his name. The concepts and methods developed by Bhaskaracharya are relevant even today.