Concerning the Persian Version of Lilavati (1587)

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(Communicated by Dr. N. R. Ray)

I

At once one of the most delightful and significant treatises in the whole history of mathematics, the Lilavati of Bhaskari-carya is a subtle exposition in which the analytical and poetical elements of thought are attractively combined. No one interested in Oriental literature can fail to fall in love with it. It forms the first part (Patigundadhyaya) of the Siddhanta-siromani, completed by Bhaskara, of Bedar, in the Decan around 1150 A.D. and contains, in addition to the fundamentals of arithmetic, certain problems involving simple and quadratic equations, a treatment of permutations and combinations, arithmetical and geometrical progressions, various problems in plane and solid geometry, and an exposition on the indeterminate equation of the first degree—a field in which both the Chinese and Hindus had excelled for at least half a millennium.

Since its composition in the middle of the twelfth century, Lilavati has inspired a number of commentaries, translations, and editions, of which the following may be noted:

1. Ganitatakamud, which has been dated 1387 A.D.
2. Ganitamritasaraga, by Gangadhaara, c. 1420 A.D.
3. The commentary of Moshadeva (prior to 1473 A.D.).
4. Lakshmi-sisla, c. 1501 A.D.
5. Ganitamritapada, c. 1541 A.D.
6. Dvivedanta, by Ganea Daivajna, 1549 A.D.
7. Pujyavakhyaya, by Viresvara-patni.
8. Lilavati-bhasana, son of Viresvara. (A later commentary than that of Suryadesa.)
9. The Persian translation made by the poet Faiz, 1587 A.D.
10. Mitabhisheka, by Rangabhatta, probably c. 1630 A.D.

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1 The Siddhanta-siromani has four parts, entitled Patigundadhyaya (Arithmetic), Gohitadhyaya (Geometry and Trigonometry), Ganitadhyaya (Astronomy) and Bhagvanadhyaya (Algebra), though the division of subject-matter is not fixed.
2 This list is incomplete; see also e.g. "Descriptive Catalogue of MSS in Mathla" (Benares-Sheet), III, 374-396, Poona, 1937.
II

There is little doubt from the foregoing that Lilâvati, and indeed the whole Šiddhânta-siromani, was widely used and appraised in India. It was held in esteem in the time of Akbar, who ordered his poet laureate (Malik uš-Shu’arat), the Shâhid Abûl-Fazl (Faizī or Fauzyâ) to prepare a translation in the Court language, Persian. This translation was completed A.H. 986 (1587 A.D.), and is mentioned in the A’m-i-Abkar, which is the third volume of the Akbar Nâmah compiled by Abul-Fazl. Faizī, who was the elder brother of Abul-Fazl by some four years, was instrumental in introducing him to Akbar, whose trusted advisor he later became.

The Persian commentary based on Lilâvati and entitled Badâ‘î-funun was written A.D. 1663-64 by Dharma Narayana. It was published in the Kâtyâ and dedicated to Abul-Fazl. It would seem, therefore, that during the reign of the Mughal emperors the mathematical work of Bhâskara received general recognition in learned circles.

We have noted 38 copies of Faizī’s version in the British Museum (MS. Add. 5419, dated A.D. 1777), the Library of the India Office (MSS. 1909 dated A.D. 1666 and Shahjâhânâbad, 1809 dated A.D. 1777, and 1800

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1 Cat. of the Persian MSS. in the Library of the India Office (H. Eltû, I. pp. 14-15 [1903].
3 W. Perach, Berlin Cat. p. 1041.
4 T. J. W. Wister and Ahshad Mirza. Vol. XVII.

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Furthermore, we assumed a number, viz. ninety-six, and divided it by each one of the remainders. The quotient obtained with four (as divisor) is twenty-four, with six is sixteen, with ninety-six is one, and with one is ninety-six. Hence the price of a ruby is twenty-four and the price of an emerald is sixteen, the price of a pearl is one, and the price of a diamond is ninety-six. Hence each man possessed articles of price 233 nishkas.¹

Another solution is this:—we multiplied all the remainders together, so that when 4 was multiplied by 6 it gave 24, and when 24 was multiplied by 96 it gave 2304, and when this was multiplied by unity it remained the same. Then we divided the whole product by each remainder. The quotient in the first case was 570, and in the second 354, in the third 24, and similarly in the fourth 2304. According to the calculation each of them possessed articles worth 5592 dramma² and when we convert these dramma into nishkas there are 233 nishkas, as has been mentioned in the first method.

The second discussion concerns the Rule of Three, and is taken from Lilavati, Chap. III, Sec. VI.³ The given thing of which the quantity is to be found is named pahal,⁴ secondly the price is named parmaṇa,⁵ and thirdly the value of the thing by which we estimate the amount of it is called achiha⁶.

Example. If two and a half palas⁷ of saffron is bought for three-ninths of a nishka, what can be bought for nine nishkas? I write parmaṇa ⁹/⁷, pahal ⁵/⁴, and achiha ⁹/₁, in this way:

<table>
<thead>
<tr>
<th>3</th>
<th>5</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

We multiplied pahal, which is ⁵/⁴, by achiha, which is ⁹/₁, and obtained ⁵³/₁₂ as product. We then divided it by ³⁷/₁₂ which is parmaṇa, and the quotient is ⁵² palas and ² karshas⁸ of saffron for ⁹ nishkas.

The text then proceeds to the Rule of Three, inverse proportion. As an instance, if a girl of sixteen years can be bought for ³² akrājas⁹ (gold coins), how many akrājas will be required to buy a girl of twenty years? Also an ox which has been worked for two years can be bought for ⁴ nishkas. What will be the cost of an ox which has been worked for six years?

1 If we put k = 96 (the L.m., of 1, 4, 8, 96), then a = 24, b = 16, c = 1, d = 96; whence the total

⁹a + ⁸b + ⁷c + ⁶d (say) = 233.

MS. ١١٠٤. I has nishkas, C, has nishkas.

2 If we make k = ⁴/₂₈, ⁹/₁, then a = 570, b = 384, c = 24, d = 2304. The total stock of each man = ⁴₄₈ + ⁴₄₈ + ⁴₄₈ + ⁴₄₈ (say) = 5592.

MS. ١١٠٤. ١١٠٤. ١١٠٤. ١١٠٤.

3 C, p. 33. ١١٠٤. ١١٠٤. ١١٠٤.

4 MS. ١١٠٤. ١١٠٤. ١١٠٤. ١١٠٤.

5 MS. ١١٠٤. ١١٠٤. ١١٠٤.

6 MS. ١١٠٤. ١١٠٤. ١١٠٤.

7 MS. ١١٠٤. ١١٠٤. ١١٠٤.

8 MS. ١١٠٤. ١١٠٤. ١١٠٤.

9 MS. ١١٠٤. ١١٠٤. ١١٠٤.

The terms parmaṇa, pahal, and achiha now conform to inverse proportion.

² MS. ١١٠٤.

³ This problem is followed by a similar one relating to the 'touch and weight of gold'. See C, p. 34. B, p. 41. A further example is also given which relates to the vessel used in the measurement of stones.

⁴ MS. ١١٠٤. In the MS., a dot is used to represent the unknown quantity which it is required to evaluate.
We brought the 5 under the 16, and took the zero under the 100, in this way:

<table>
<thead>
<tr>
<th>1</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

'After that we multiplied 12 by 16, which became 192. Then we multiplied 192 by 5, and obtained 960. This is the capital. Then we multiplied 1 by 100, giving 100. This became the interest. As the capital was greater than the interest, I divided it into 100. The quotient was 9$. So the interest on 16 rupees for 12 months, according to the above conditions, is equal to that sum.

The other example is like this: the time is not known but the capital and interest are given. First of all we wrote down the capital, which is 100 rupees (capital) and 5 rupees as interest for one month, then we added the capital and interest as interest, thus:

<table>
<thead>
<tr>
<th>1</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>48/5</td>
</tr>
</tbody>
</table>

'After that we put the sum of 48/5 below the 100; and the 5 which is now below the 100, we brought below the 16, like this:

<table>
<thead>
<tr>
<th>1</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>48/5</td>
<td>5</td>
</tr>
</tbody>
</table>

Next we multiplied 100 by 48, which became 4800, below which is 5 (i.e., $\frac{4800}{5}$). Then we divided the new fraction by 50. Then quotient was 12. Thus we found that the number of months is 12.'

The example used to illustrate the Rule of Seven

'There is a sheet of high-quality silk which is of length 8 cubits and width 3 cubits; and eight such sheets can be bought for 100 nishkas. What then will be the price of another sheet of the same quality which is $\frac{3}{2}$ cubits long, $\frac{1}{2}$ cubit wide?

'1 write the achādi like this:

<table>
<thead>
<tr>
<th>3/1</th>
<th>7/2</th>
</tr>
</thead>
<tbody>
<tr>
<td>8/1</td>
<td>1/2</td>
</tr>
<tr>
<td>8/1</td>
<td>1/1</td>
</tr>
<tr>
<td>100/1</td>
<td></td>
</tr>
</tbody>
</table>

The solution is then obtained, as in the Rule of Five, by the method of proportion, and is of nishka 0, dramma 14, panas 9, kākini 1, cowry shells 68. It is not proposed to indicate here the development of this method in the Rule of Nine and the Rule of Eleven; in the case of nine factors the text follows the Sanskrit original in giving the example involving the cost and dimensions of wooden planks, and in the case of eleven factors the same example is repeated but with two groups of cooles, differing in number, to transport them.

The next discussion deals with barter, and the following example is given:

'If 300 mangoes can be bought for 16 panas, and 30 pomegranates for 1 panas, find how many pomegranates of this kind can be obtained for 10 of these mangoes.'

1 C, p. 27. B, p. 44.
2 MS. دستم
3 Let $x$ denote the required price. Then, using the areas of silk, we have $8 \times 3 \times 8 \times 1 \times 1 \times \frac{1}{3} = 100 : x$; whence $x = \frac{1 \times 7 \times 1 \times 100}{8 \times 3 \times 8 \times 2 \times 3}$. MS.
4 Answer obtained using 4 kākini (MS. kākini = 1 panas)
16 pana
16 dramma
1 nishka
(MS. has $\frac{1}{2}$ nishka in place of cowry shells.)

MS. Folio 118 gives tables of quantities which correspond with those of Lilavati, Chap. I.

1 C, p. 57. H, p. 45. The 'fingers' measurement, $\frac{1}{4}$ finger occurs.
2 MS. In Sanskrit the distance in leagues over which the planks have to be carried is different in this case.
I write the *parmañ* and *achha* thus:

\[
\begin{array}{c|c}
16 & 1 \\
300 & 30 \\
10 & 1 \\
\end{array}
\]

Then, the amount which is in the middle of *parmañ*, i.e. 300, and the amount which is in the middle of *achha*, i.e. 30, we interchanged in position, thus:

\[
\begin{array}{c|c}
16 & 1 \\
300 & 30 \\
10 & 1 \\
\end{array}
\]

Then, in accordance with the foregoing method, I find that 16 pomegranates are equivalent to 10 mangoes. 1

The next illustration is taken from *Lilavati*, Chap. IV, 2 and deals with Investigation of Mixtures, being a problem on capital and interest in which both these values have to be found. 3

For example, on 100 rupees it has been decided to pay 5 rupees per month (as interest). After one year 1000 rupees were paid (altogether). 4 If I wish to know the capital and the accrued interest, I write in this way:

\[
\begin{array}{c|c}
1 & 12 \\
100 & 1 \\
5 & \\
\end{array}
\]

In this way, as with the Rule of Five, we brought 5, which is under *parmañ*, below 1, which is *achha*; and brought 0, which is below 1, underneath 100. All the numbers of *achha* and *parmañ* should then be multiplied among themselves separately. The product of *parmañ*, which is 100 × 1, is 100. The product of *achha*, which is 12 × 5, is 60. We then divided 60 by 100; the quotient is 3. Next we added the supposed number, which is 1, to it. The total came to 3 1/3 = 3.5. Next we multiplied 1000, which is given, with unity, which is the supposed number. It became 1000. Then we divided 1000 by 3 1/4, which became 250, which is the interest.

Two further examples follow which involve (i) a sum of money lent in three portions at different rates of interest for different periods, and (ii) the sharing of an aggregate sum of money earned jointly by three partners who each supplied a different initial capital.

The next two problems concern purchase and sale in the bazaar: it will suffice to quote one of them, as they follow closely *Lilavati*, Chap. IV, Section III:

As another instance, one can obtain one *pala* of camphor for two *nichka*, and one *pala* of sandal for 1/24 *dramma* 5 and 1/2 *pala* of aloeswood for 1/4 *dramma*. A man gave a *nichka*, and asked for one part of camphor, 16 parts of sandal, and 8 parts of aloes. The answers in price and

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1 There are two instances of proportion, as noted by Śāryādīva, in this problem—

(i) To find the number of pomegranates *z* obtainable for 16 pana.

\[
30 : z :: 1 : 16
\]

Thus 30 × 16 pomegranates are equivalent to 300 mangoes.

(ii) To find the number of pomegranates *y* required.

\[
300 : 10 :: 30 : y
\]

Thus 30 × 16 pomegranates are equivalent to 300 mangoes.

2 By the modern method, if the principal = P, interest = I, total amount = A, rate per cent per month = r for a time in years = T, then,

\[
I = \frac{P \times r \times T}{100}
\]

\[
A = P + I = P \left(1 + \frac{r \times T}{100}\right)
\]

3 C, p. 35. B, p. 41.

4 Meaning capital plus accrued interest.

5 MS. jāyā.
quantity for camphor, sandal and aloe respectively are 14 2/9 (dranna),
4/9 (pala); 8/9, 64/9; 8/9, 32/9.

Finally, a question is taken from Lilavati, Chap. IV, Section II, on
fractions, and relates to the familiar exercise about the cistern:—There is
a tank into which water flows from four different directions. One of its
inlets is such that if we leave it to itself, it will fill the tank in one day.
The second one fills it in 1/2 day, the third one in 1/3 day, and the fourth one
in 1/4 day. If we open all four of them together, in what portion of a day
would they fill it?

It is written in this way:—

\[
\begin{array}{cccc}
1 & 1 & 1 & 1 \\
1 & 2 & 3 & 6 \\
\end{array}
\]

We divided every number written below by the number written above.
According to the previously-mentioned method of the division of fractions,
we obtained then

\[
\begin{array}{cccc}
1 & 2 & 3 & 6 \\
1 & 1 & 1 & 1 \\
\end{array}
\]

After this we added the numbers written above, which are quotients,
thus obtaining 12. Next we divided unity by 12. Thus it is 1/12th part
of a day, which is a half-nickel, in which it is filled.

We conclude in the name of God, virtue, and victory.

Such are the main features of this Manchester MS.; dealing only
with a part of Lilavati, and that part which concerns primarily business
transactions, it is in the tradition of commercial arithmetic first established
by the medieval Sanskrit treatises.

IV

This paper, in its small way, calls further attention to the Hindu
genius for mathematics, to its unique contribution to mathematical
thought and symbolism which has survived in virtue of its universality.
It became known to mediaeval Islam, and thence to Latin Christendom,
long before Bhaskara, and was studied with a rare sympathy and
enthusiasm by Al-Biruni; whilst we find it again in the Persian
works in the days of the Mughal emperors, when Europeans in India were,
had they appreciated it, indeed at the fountain-head. Not, however, until
the early nineteenth century did the editions of E. Strechey, J. Taylor,
and H. T. Colebrooke see the light of day.

We conclude by expressing our appreciation of the generous assistance
afforded by the staff of the John Rylands Library, particularly the kindly
co-operation of the late Dr. Henry Guppy. Also we are indebted to Roshan
Zamir of the Education Dept., Tabriz, Azerbaijan, who kindly helped in
checking the translation.

* The process amounts to this:—If the inlets separately can fill the tank in 1,
2, 3, and 4 day, then they also separately fill 1/2, 1/3, and 1/4 parts of the tank in
one day, and thus collectively fill 12 parts of the tank in one day, i.e. collectively they
fill the tank in \( \frac{1}{2} \) day.

\* MS. نجکری

CONSTRUCTION OF FERTILITY TABLES FOR INDIA AND
STUDIES THEREFROM

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The provisional figure of total population in India has been given as
356-89 millions* on 1st March, 1951. This records an increase of 13.4% over
1941 figures. The rate of increase in this intercensal period also is
rather high as in previous one. Fear of over-population is therefore
haunting the mind of every intelligent citizen. Many social and economic
difficulties now being experienced have been attributed to this rapid
growth of population.

Growth of population depends mainly on the age and sex composition
of the population and the fertility rate prevailing in the country. In
India among 356-89 millions of population there are now 173-51 millions
of females. Sex-ratio thus works out at 946 females for every 1,000 males
in 1951 as against 940 in 1941. Proportion of females thus shows an
increase on the whole but its influence on growth of population cannot be
ascertained unless the age-distribution of this increased female population
is known because such increase must be in child-bearing ages in order to
be effective. This age-distribution will become known when the census
report will be published. Even then it would not be possible to measure
accurately the future contribution of this increase to population unless
the rate of fertility among women is known for all ages of child-bearing
period. But the census authorities, for reasons best known to them, have
so far refrained from collecting information in a form suitable for fertility
studies. It is therefore apparent that diagnosis of the real cause of rapid
growth of population in India may not be feasible from census figures.

In the absence of such complete enumerations, we must have recourse
to sample surveys. But sample studies also have been very rare in this
direction in India. Only one fertility table has been published so far
but that also from a few hundreds of families in Cochin, a very small State
in India. As the conditions in Cochin differ greatly from those obtaining
in other parts in India and as the size of the sample was very small, these
rates could not be used to represent the fertility of this vast subcontinent.

For fertility studies in India, we have always been using the fertility
tables of foreign countries like Ukraine or Japan. But the use of such
tables in conjunction with our female population is not likely to yield
satisfactory results because the fertility curve of India has a different
shape from that of the above foreign lands. The population studies in
India have always faced this difficulty and the necessity of a fertility table
derived from our own population has long been recognized but no all-out
attempt has as yet been made.

The main object of this paper has therefore been to derive specific
fertility rates for different provinces of India, to study the influence of
age-difference and financial position of the couple on these rates, to
construct a combined fertility table and to study population trends.

* Exclusive of Jammu and Kashmir State and certain tribal areas.