Did you know Bhaskaracharya? What was he famous for and when did he live?

Bhaskaracarya was a mathematician-astronomer of exceptional abilities. He was born in 1114 AD.

Mathematics became the hand-maiden of astronomy and, from the time of Aryabhata I, it began to be incorporated in astronomical treatises. Thus all components of mathematics came to be developed: geometry, trigonometry, arithmetic and algebra. The great astronomers had to be great mathematicians too.

The great astronomer-mathematicians of the Siddhanta period, in a chronological order were: Aryabhata I, Varahamihira, Brahmagupta, Aryabhata II, Sripati, Bhaskara II (known popularly as Bhaskaracarya), Madhava, Parameśvara and Nilakantha. These great scientists, except the last three, grew in different parts of this vast sub-continent. Perhaps such isolated growth may explain the apparent abruptness in astronomical and mathematical development in India. Even before Bhaskara made his mark on Indian Jyotisa, there were three distinct schools, the Saura, the Arya and Brahma. Bhaskara was respected and studied even in distant corners of India. Bhaskara was perhaps the last and the greatest astronomer that India ever produced.

Brahmagupta was Bhaskara's role model and inspirer. To Brahmagupta he pays homage at the beginning of his *Siddhanta-siromani* and most of his astronomical elements are taken from the *Brahmasphuta siddhanta* or the *Rajamrganka* belonging to the same school. Bhaskara improved upon him not through any great original contribution but by the thoroughness with which he could and did analyse the rationale of the calculations. Bhaskara's exhaustiveness was so profound that his works have not only eclipsed lesser works but even the works of his great master Brahmagupta himself.

Bhaskara was born in Saka 1036 (1114 A.D.) and composed his masterpiece *Siddhanta-siromani* at the age of thirty-six. His second work, the *Karanakutuhala* was composed at the age of sixty-nine. The sharpness of his keen mind seems to have remained undiminished over a long period. Some details of his family too are known through the Goladhaya of the *Siddhanta-siromani*. His father, Mahesvara, was an astronomer well-versed in all the branches of learning and was Bhaskara's guru. He belonged to the *Sandilya gotra*, and lived in Vijjadavida near the Sahya Mountain. This information is corroborated and supplemented by two stone inscriptions in temples, one at Patan and the other at Behal. The first of these mentions five ancestors and the son and grandson of Bhaskara. Amongst the ancestors is Trivikrama, who is the poet Trivikrama, author of the *Damayantikatha* and the son Laksmidhara is said to have been called away from Patan to be the court pandit of king Jaitrapala. The grandson Cangadeva was astrologer royal to the Yadava king Singhana of Devagiri and got the incription engraven, established an institute for the study and propagation of Bhaskara's works. Bhaskara's whole family was one of scholars and most of the members were astrologers attached to royal houses. Curiously enough, there is no mention of Bhaskara himself being the astrologer royal to any king. Perhaps he disdained the work of a professional astrologer as is hinted at in the story of Bhaskara writing his work *Lilavati* for the sake of his daughter, elaborated with delicate fancy by Edna E Cramer in her *Mainstream of Mathematics*. The inscription at Behal gives the lineage of another son Sripat of Mahesvara. In the Persian translation of Bhaskara's *Lilavati*, prepared at the behest of the Mughal emperor

Akbar, Bedar is mentioned as the native place of Bhaskara. But this place is not near the Sahya Mountain. Vijjadavida, the now unknown native place of Bhaskara, must have been somewhere near Patan.

Bhaskara is known for his two main works: a 'Siddhanta' text, the '*Siddhanta-siromani*' and a 'Karana' text, the '*Karanakutuhala*'. The former is in four parts, viz. (i) *Patiganita or Lilavati*, (ii) *Bijaganita*, (iii) *Grahaganita*, (iv) *Goladhyaya*. Of these, the first two are usually treated as separate treatises. The *Lilavati* deals with arithmetic and geometry; it is said that the name is after his daughter Lilavati, who was according to her horoscope to remain unmarried. There is a story which says that Bhaskara put to use all his astrological knowledge to find out an auspicious moment for her marriage, and on the marriage day had a water-clock fixed up as to hit the exact time favourable for her happy marriage, but his efforts were foiled by the child-bride herself. Impelled by girlish curiosity she kept on running to the water-clock and bending to peer at it. In one of these visits to the water-clock, a pearl loosened from her neck and got stuck to the hole of the water-clock. The auspicious moment passed unnoticed and the girl had to remain unmarried. To console her and perpetuate her name Bhaskara called his treatise on arithmetic and geometry by her name. According to others, *Lilavati* was the name of Bhaskara's wife. More probably Bhaskara was attracted by this fanciful name. Bhaskara was known not only for his mathematical scholarship, but also for his poetic inclinations.

In the *Lilavati* the eight mathematical operations (*parikarmastaka*), addition, subtraction, multiplication, division, squaring, cubing, extraction of square and cube-roots are dealt with first. The operations with zero (*sunyaparikarma*) follow. Then come *vyastavidhi* (method of inversion), *istakarma* (unitary method), *sankramana* (finding a & b when a+b and a-b are known i.e., the method of elimination), *vargasankramana* (finding a & b from a-b and a2 -b 2), *vargakarma* (finding a & b so that a2+b2-1 and a2-b2-1 may be perfect squares), *mulagunaka* (problems involving square roots i.e. those which lead to quadratic equations); *trairasika* (rule of three); *bhandapratibhandaka* (barter), *misravyavalara* (mixtures), *srenivyavahara* (series); *ankapasa* (permutations and combinations) and *kuttaka* (indeterminate analysis). In fact, some of these topics like series, permutations and combinations and indeterminate analysis more properly come under algebra.

The section on geometry (*ksetraganita*) opens with the enunciation of the theorem of the square of the hypotenuse. But the enunciation is algebraic rather than geometrical and leads on to the solution of rational right triangles and height and distance problems. The condition for given lengths of sides forming the sides of a closed rectilinear figure is then given. The rules for calculating the attitude, area, etc., of triangles and different types of quadrilaterals come next. After criticising Brahmagupta's rule for finding the diagonals of quadrilaterals, he gives his method of getting a rational quadrilateral by the juxtaposition of rational right triangles and shows how the diagonals are then easily found. Circles are dealt with next, a very satisfactory approximate formula for calculating the arc in terms of the chord and vice versa were given, so also are given the correct expressions for the volume and surface of a sphere. Though Sridhara before Bhaskara gave the correct expression for the volume, Bhaskara's is in more general terms. The sides of rectilinear figures with 3,4….9 sides are calculated next. *Khatavyavahara* (section on excavations) and *krakacavyavahara* (shadow problems) cover some interesting problems. Except for the section on permutations and combinations, the scheme is the same as that in the

mathematical chapters of the *Brahmasphutasiddhanta*. But Bhaskara's treatment is always richer and more comprehensive.

The topics in the *Bijganita* are fundamental operations with positive and negative quantities (*ghanarnasadvidham*) with zero (*sunyasadvidham*), with symbols (*varnasadvidham*) and with surds (*karanisadvidham*); indeterminate simple equations (*kuttaka*); indeterminate equations of second degree (*vargaprakrti*) equations in one unknown (*elavarna-samikaranam*); solving quadratic equations by completing the square (*madhyamaharanam*); equations in more than one unknown (*anekavarnasamikaranam*), equations involving products of the unknowns (*bhavita*).

In the *Grahaganita* and *Gola* chapters after the preliminary disquisition on the importance of the astronomy of the heavenly bodies, the *Prasnadhyaya* (chapter of questions) asks a series of questions which are discussed in the succeeding chapters. The third chapter, *Bhuvanakosa* (the universe), asserts the unsupported situation of the earth in space and explains how beings exist on the surface of the round earth. The measurement of the circumference, surface area and volume of the earth is next dealt with. A good approximation for p (namely p = 3.1416) is used for these calculations. The fifth chapter titled *Madhyamagativasana* treats of the mean motions of the sun, moon and the planets. Like the other Indian astronomers, Bhaskara too ignores the thesis put forward by Aryabhata that the stationary stars appear to move to the observer on the revolving earth. The next chapter, *chedyakadhikara*, describes the true motions of the heavenly bodies. The true motions, says Bhaskara, can either be represented as motion along a small circle or as motion centre moves along the circumference of a circle whose centre does not lie at the centre of the earth. The diagrammatic representation of the true motions is termed 'chedyaka'. In the *Golabandhadhikara*, Bhaskara explains how models of the celestial sphere with the orbits of the sun, moon and the planets are constructed by the teacher of astronomy. The eighth chapter, *Triprasnavasana*, tells us how to know the time of sunrise, the relative lengths of the day and night in different seasons and at different latitudes, and the latitude of a place. The next three chapters are devoted to eclipses. The *Yantradhyaya* describes the astronomical instruments used for observing the heavenly bodies but ends it with the remark that intelligence is a better tool than all these, which is significant from an astronomer who seems to have neglected practical observation. Some machines with no astronomical application are also included in the list. One chapter is devoted to a poetic description of the seasons, another to astronomical questions and their answers.

The last chapter is again devoted to *Jyotpati* (calculation of sine), which gives different methods of calculating the sine. The whole text of the *Grahaganita* and the *Goladhyaya* has a commentary (*vasana*) attached to it, written by Bhaskara himself.

Amongst Bhaskara's noteworthy contributions to mathematics is his finding of the correct volume of a sphere by dividing it into pyramids with their apexes at the centre and bases on the surface of the sphere and of the correct surface area of a sphere by cutting up the surface into concentric rings and into lenticular strips and adding up their areas. How exactly this is done is not clear, but some scholars think it is by a sort of differentiation-integration. Bhaskara's treatment of the mathematics of the zero is remarkable for the undoubted realisation that any quantity divided by zero is infinity (termed *Khahara* or *Khahara* having zero as divisor) and for

the implied concept of the infinitestimal. In the calculation of the instantaneous motions of planets he seems to anticipate differentiation.

Bhaskara took all the astronomical elements from the older works, chiefly the *Brahmasphutasiddhanta* and the *Rajamrganka*. The high fame the *Siddhantasiromani* has achieved is due to the fact that every thing of Indian astronomy is to be found in it. The period between Aryabhata I and Bhaskara II was the golden age of Indian Jyotisa. It saw the production of many astronomical works, but they were all pushed to the background, by the brilliant exposition and the analytical power displayed by Bhaskara. So thorough and so sure was his treatment that no need for further improvement was felt. The later works were almost all commentaries on Bhaskara's works.

Unlike most of the scientific works originating in the south, Bhaskara's works were not unknown in the north. They were studied with great assiduity. Many of the developments in mathematics are embedded in the commentaries on the *Lilavati*. The *Siddhantasiromani* too enjoyed great popularity. Bhaskara calculated the equinoctial shadow at any place and the new corrections to be applied to the calculation of the time of sunrise. The precession of the equinoxes too was accepted by Bhaskara, though later astronomers allowed Bhaskara's correct theory to be perverted. All this shows beyond doubt that Bhaskara was blessed with a remarkably active brain. Bhaskara's works have served as reference books in every nook and corner of India.

The *Karankutuhala,* like other Karana works, is a manual for easy astronomical calculations. Even now it is used for making calendars in many parts of India.

The number of commentaries testifies to the popularity of Bhaskara's works. The *Lilavati* has the largest number. Some amongst these like the *Gantikaumudi* of Narayana Pandita, son of Nrsimha, are almost independent works bringing in much new matter. Others like *Kriyakramakri* while faithfully commenting on the *Lilavati* verses one by one, supplement the information contained in the *Lilavati* with many new discoveries. More than twenty commentaries on the *Lilavati* have already been brought to light. There may be more. On the other sections of the *Siddhantasiromani* and on the *Karanakutuhala* too there are a number of commentaries. In 1577 A.D. the *Lilavati* was translated into Persian and the *Bijaganita* in 1665. In modern times Colebrooke translated the *Lilavati* and the *Bijaganita*. The *Lilavati* is rated so high in popular opinion that there is a saying that a person well up in the *Lilavati* will be able even to find out the exact number of the leaves on a tree.

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