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ASTRONOMICAL INSTRUMENTS IN CLASSICAL SIDDHĀNTAS

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This paper is a comprehensive study of the development of astronomical instruments during Classical Siddhānta period (c.5-12th Century AD). Several astronomical instruments were described in the chapter called *Yantra adhyāya* and some other sections of astronomical Siddhāntas. From these sources, historical development of several types of instruments, which were devised by Indian astronomers, can be traced. And also the methods of astronomical observation using instruments were explained by Bhāskra II, etc. These facts show the importance to investigate the observational astronomy in India.

This paper consists of the following sections.

1. Introduction,
2. Primary Sources,
3. The Gnomon,
4. The Graduated Level Circle and Orthographic Projection,
5. The Staff,
6. The Circle instrument and its Variants,
7. The Celestial Globe and the Armillary Sphere,
8. The Clepsydra and Water Instruments,
9. The Phalaka yantra,
10. Methods of Observation,
11. Conclusion.

1. INTRODUCTION

In this paper, I propose to present a comprehensive description of the development of astronomical instruments described in Classical Siddhāntas. What I call "Classical Siddhāntas" are the astronomical Siddhāntas (fundamental texts) from Āryabhaṭa's to Bhāskara II's, i.e. those which were composed between the 5th century and the 12th century AD. In my previous paper,[1)] I described two earliest Indian astronomical instruments, gnomon and clepsydra, used in Vedāṅga period. In Classical Siddhānta period, several astronomical instruments were used besides the gnomon and clepsydra.

Prior to this Classical Siddhānta period, Greek astrology and astronomy were introduced into India. According to David Pingree, a Greek astrological text was translated into prose Sanskrit by Yavaneśvara in AD 149/150, and it was versified as the *Yavana-jātaka*[2)] by Sphujidhvaja in AD 269/270. Only the last chapter (chap.79) of the *Yavana-jātaka* is devoted to mathematical astronomy, and the rest is devoted to astrology. In the last chapter, astronomical theory of the Greeks (*Yavanas)* is explained, but the name of the sage Vasiṣṭha is also mentioned, and it seems that some tradition of Indian old astronomy ascribed to Vasiṣṭha is also partly mentioned there. Another Sanskrit work which suggests Greek influence in the field of mathematical astronomy is the *Pañca siddhāntikā* of Varāhamihira (6th century AD). It is a compilation based on five earlier astronomical works, viz. the *Paitāmaha–*, *Vasiṣṭha–*, *Pauliśa–*, *Romaka–*, and *Sūrya-siddhānta.* Among them, the *Paitāmaha-siddhānta* is purely based on Vedāṅga astronomy, and the *Vasiṣṭha-siddhānta* seems to be a mixture of Indian traditional astronomy developed from Vedāṅga astronomy and newly introduced foreign elements. The *Pauliśa-siddhānta* uses the eccentric theory, which may have been introduced from Greece, and also the longitudinal difference in time between "Yavana" and Avanti(=Ujjain) is given as 7 1/3 *nāḍīs*, which corresponds to 44^0, as mentioned in the *Pañca-siddhāntikā* (III.13). This "Yavana" must be Alexandria. The *Romaka-siddhānta* also uses the eccentric theory, and also the Metonic cycle of intercalation and the length of a tropical year which is the same as that of Hipparkhos. In the *Pañca-siddhāntikā* (XV.23),"Romaka-viṣaya" (country of Romaka) is mentioned to be the place 90° west of Laṅkā (the place on the equator whose longitude is the same as Ujjain). "Romaka-viṣaya" must be connected with Roman Empire. The *Sūrya-siddhānta* is the most developed Siddhānta among the five Siddhāntas quoted in the *Pañca-siddhāntikā,* and the eccentric and epicyclic theory, characteristic theory of Greek astronomy, is used there. The process of Greek influence of astronomy into India has been discussed by several people, notably by David Pingree.[3)] The extent of Greek influence is, however, still controversial.

The possibility of astronomical observations in this period is also controversial. Roger Billard[4)] analyzed Sanskrit astronomical works by the methods of mathematical statistics, particularly the method of least squares, and showed that these works are based on actual astronomical observations in India. That the statistical methods of R. Billard is mathematically sound was maintained by B.L. van der Waerden[5)] and also by Raymond Mercier.[6)] David Pingree, however, attacked Billard's conclusion, and argued that astronomical observation had not been carried out in ancient India, and that astronomical constants in Classical Siddhāntas had been derived from Greek sources.[7)] The analysis of astronomical constants in Classical Siddhāntas is beyond the scope of the present paper, and I shall not go into detail of the controversy of Billard and Pingee. However, I would like to point out one thing. One of the reasons why D. Pingree does not believe in actual astronomical observation in ancient India is as follows.

> "...They (statistical figures in Billard's book – quoter) show that Āryabhaṭa's mean longitudes were most nearly correct at precisely the time when the tropical and the Indian sidereal zodiacs coincided – in the early 6th century A.D. He (Billard – quoter) would attribute this circumstance to Āryabhaṭa's precise observations, even though there was no tradition of observational astronomy in India prior to Āryabhaṭa; but Āryabhaṭa could only observe true , not mean longitudes. It is not an easy matter to deduce the mean longitudes

from these true longitudes; and for Āryabhaṭa, with his rather clumsy and innacurate planetary models, it would have been impossible to arrive at results as good as Billard shows his to be... " (David Pingree)[8]

I do not think that this Pingre's argument is justified. We shall see in the last section of this paper that the methods to derive astronomical constants from actual astronomical observations were explained by Bhāskara II in detail, and that the methods are theoretically correct. There is no reason to think that some of these methods were not known to earlier Indian astronomers. So, it will be necessary to investigate the possibility of actual astronomical observations in ancient India more seriously.

There is one thing which we should keep in mind regarding this Classical Siddhānta period. Whatever the extent of early Greek influence may be, the transmission was limited for a short period (from ca. 2nd century AD to ca. 4th century AD), which is prior to Classical Siddhānta period, and the Classical Siddhānta period itself was rather free from foreign influence. Astronomy was developed in India in its own way in this period, and established itself as an independent discipline.

Sanskrit texts on the theory of mathematical astronomy are classified into three classes according to their epoch as follows.

(1) Siddhānta — (Its epoch is the beginning of the *kalpa),*

(2) Tantra — (Its epoch is the beginning of the *kali-yuga)*), and

(3) Karaṇa — (Its epoch is any convenient year selected by the author).

A *mahā-yuga* is a period of 4320000 years. According to traditional theory, a *kalpa* is 1000 *mahā-yugas,* and a *mahā-yuga* is divided into four period, viz. *kṛta-yuga* (1728000 years), *tretā-yuga* (1296000 years), *dvāpara-yuga* (864000 years), and *kali-yuga* (432000 years). (Āryabhaṭa's theory is different from the traditional theory, and a *kalpa* is 1008 *mahā-yugas,* and a *mahā-yuga* is divided into four equal periods.) The beginning of the current *kali-yu*ga is Friday, February 18, 3102 BC of Julian calender.

The Siddhāntas may be considered as the most fundamental texts of Hindu astronomy. Usually, a Siddhānta consists of two parts, i.e. the *Grahagaṇita-adhyāya* (Chapter on the calculation of the position of planets) and the *Gola-adhyāya* (Chapter on spherics). The *Graha-gaṇita-adhyāya* further consists of several chapters, viz. the *Madhyama-adhyāya* (Chapter on mean motion), the *Spaṣṭa-adhyāya* (Chapter on true motion), the *Tri-praśna-adhyāya* (Chapter on three problems, i.e. direction, place, and time.), and also chapter on lunar and solar eclipses, lunar phases, helical rising and setting, conjunction of planets and stars, and planetary nodes etc. The *Gola-adhyāya* also consists of several chapters on spherics.

Most of Sidhāntas have a chapter on astronomical instruments. i.e. the *Yantra-*

adhyāya as a part of the *Gola-adhyāya*. This is our chief source material. The word "*yantra*" means instrument. Besides the *Yantra-adhyāya,* a section on armillary spheres (*Gola-bandha*) is included in the *Gola-adhyāya*. Besides, some informations of astronomical instruments are found in the *Tri-praśna-adhyāya* etc.

For general information about astronomical instruments in this period, the works of S.B. Dikshit,[9] S.R. Das,[10] L.V. Gurjar,[11] and R.N. Ray[12] may be consulted.

As we shall see in the next section, many Siddhāntas have been translated into English. However, two very important Siddhāntas, the *Brāhma-sphuṭa-siddhānta* of Brahmagupta and the *Siddhānta-śekhara* of Śrīpati, have not been translated into English. So, special attention will be paid to these two works, and all verses in the *Yantra-adhyāya* of these two works will be discussed in this paper.

2. Primary Sources

i) The Āryabhaṭīya

Āryabhaṭa (born AD 476) wrote the *Āryabhaṭīya* (AD 499),[1] which is the earliest Sanskrit work on astronomy whose author and date are definitely known. This is the fundamental text of one of the schools of Hindu astronomy, the *Ārya-pakṣa*. The epoch of this work is the sunrise of the beginning of the current *mahā-yuga*. This is a special feature of this work. This work is well known by Āryabhaṭa's rotating earth theory.

This work was translated into English by P.C. Sengupta (1927), W.E. Clark (1930), and K.S. Shukla (1976).[2]

In our connection, the *Āryabhaṭīya* has a description of a rotating celestial globe (IV.22). The gnomon is also mentioned in the section of mathematics (III.14-16).

Among several commentaries on the *Āryabhaṭīya,* the following have been published: the commentary of *Bhāskra* I (AD 629),[3] of Someśvara (AD 10-12th century)[4], of Sūryadeva Yajvan (b.AD 1191),[5] of Parameśvara (AD 15 th century),[6] and of Nīlakaṇṭha Somayājin (b. AD 1444).[7] Some of them give informations about further detail of instruments, the method of observation etc.

ii) The Āryabhaṭa-siddhānta

Āryabhaṭa wrote another work, the *Āryabhaṭa-siddhānta*, which is a text of another school of Hindu astronomy, the Ārdha-rātrika-pakṣa. This is a school of midnight system, and one civil day begins at midnight.

Unfortunately this work has been lost, but its fragment on astronomical instruments is found in Rāmakṛṣṇa Ārādhya's commentary *Subodhinī* (AD 1472)[1] on the *Sūrya-*

siddhānta. This fragment has been edited and translated into English by K.S. Shukla.[2] The following astronomical instruments are described there.

(1) The *chāyā-yantra* (shadow instrument),
(2) the *dhanur-yantra* (semi-circle instrument),
(3) the *yaṣṭi-yantra* (staff)
(4) the *cakra-yantra* (circle instrument),
(5) the *chatra-yantra* (umbrella instrument),
(6) the *toya-yantrāṇi* (water instruments),
(7) the *ghaṭikā-yantra* (clepsydra),
(8) the *kapāla-yantra* (clepsydra)'
(9) the *śaṅku-yantra* (gnomon).

Similar instruments ascribed to Āryabhaṭa have also been mentioned by Tamma Yajvan (=Tammaya) in his commentary *Kāmadogdhrī* (AD 1599)[3] on the *Sūrya-siddhānta.* Some related informations can also be had from Mallikārjuna Sūri's commentary (AD 1178)[4] on the *Sūrya-siddhānta.*

iii) The Pañca-siddhāntikā

The *Pañca-siddhāntikā* of Varāhamihira is a compilation based on five earlier works, viz. the *Paitāmaha-*, *Vasiṣṭha-*, *Romaka-*, *Pauliśa-*, and *Sūrya-siddhānta.*

This work was edited and translated by Thibaut and Dvivedin, and also by Neugebauer and Pingree.[1]

The *Pañca-siddhāntikā* has an independent chapter on astronomical instruments, i.e. the 14th chapter entitled *Chedyaka-yantrāṇi* (graphic calculations and instruments). The contents of this chapter are as follows.

(1) Calculations and observations on a graduated level circle (vv.1-11)
(2) the V-shaped staffs (vv.12-13),
(3) the locus of the gnomon shadow (vv.14-16),
(4) the definition of terms (vv.17-18),
(5) the *kapāla* (hemispherical sundial) (vv.19-20),
(6) the *cakra* (circle instrument) (vv.21-22),
(7) the *gola* (celestial globe) (vv.23-25),
(8) explanation of *ayana* (the sun's northern and southern courses) (vv.26),
(9) water instruments (vv.27-28),
(10) calculation of the terrestrial longitudinal distance (vv.29-30),
(11) the clepsydra (vv.31-32),
(12) conjunction of the moon with stars (vv.33-38), and
(13) visibility of the star Agastya (Canopus) (vv.39-41).

iv) The Mahā-bhāskarīya

Bhāskara I, a follower of Ārya-pakṣa wrote a commentary (AD 629) on the *Āryabhaṭīya* of Āryabhaṭa, the *Mahā-bhāskarīya*, and the *Laghu-bhāskarīya.*

The *Maha-bhāskarīya* was edited and translated by K.S. Shukla.[1] It has a description of a circular platform with graduated circumference in order to observe the sun's amplitude at the time of sunrise (III.56-60(i)).

The commentaries on the *Mahā-bhāskarīya*, one by Govinda-svāmin with a super-commentary by Parameśvara[2] and the other by Parameśvara[3], have been published.

The *Laghu-bhāskarīya* was also edited and translated by K.S. Shukla.[4]

v) The Brāhma-sphuṭa-siddhānta

Brahmagupta (born AD 598) wrote a Siddhānta work *Brāhma-sphuṭa-siddhānta* and a Karaṇa work *Khaṇḍa-khādyaka.*

The *Brāhma-sphuṭa-siddhānta*[1] (AD 628) is the fundamental text of one of the schools of Hindu astronomy, Brāhma-pakṣa. It has a chapter on astronomical instruments, i.e. the *Yanta-adhyāya* (Chapter XXII).

Brahmagupta explained the purpose of the *Yantra-adhyāya* as follows (XXII. 1-4; I tentatively follow the verse number in Sudhākara Dvivedin's ed.)[2].

मध्याद्यमिह यदुक्तं तत् प्रत्यक्षमिव दर्शयति यस्मात्।
तस्मादाचार्यत्वं गोलविदो भवति नान्यस्य।।1।।

आचार्यैर्न ज्ञातः श्रीसेणार्यभटविष्णुचन्द्राद्यैः।
गोलो यस्मात् तस्मात् ब्राह्मो गोलः कृतः स्पष्टः।।2।।

गणितज्ञो गोलज्ञो गोलज्ञो ग्रहगतिं विजानाति।
यो गणितगोलबाह्यो जानाति ग्रहगतिं स कथम्।।3।।

गोलस्य परिच्छेदः कर्तुं यन्त्रैर्विना यतो ऽशक्यः।
संक्षिप्तं स्पष्टार्थं यन्त्राध्यायं ततो वक्ष्ये।।4।।

"As the mean motion etc., which has been explained here, is shown clearly as if it is presented before the eyes by one who knows spherics, the title of *ācārya* (teacher) is endowed on him and none else.

Ācāryas like Śrīṣena, Āryabhaṭa, Viṣṇucandra etc. did not understand spherics. So, the spherics as taught by Brahmā is made clear (by me).

One who knows mathematics knows spherics, and one who knows spherics understands the motion of planets. If one is ignorant of mathematics and spherics, how can he know the motion of planets?

As it is impossible to discuss spherics without instruments, I shall tell a short chapter on instruments clearly.''

Brahmagupta listed the names of instruments as follows (XXII.5 7).[3]

सप्तदशकालयन्त्राण्यतो धनुस्तुर्यगोलकं चक्रम्।
यष्टिः शङ्कुर्घटिका कपालकं कर्त्तरी पीठम्॥5॥

सलिलं भ्रमो ऽवलम्बः कर्णश्छाया दिनार्धमर्कोऽक्षः।
नतकालज्ञानार्थं तेषां संसाधनान्यष्टौ॥6॥

सलिलेन समं साध्यं भ्रमेण वृत्तमवलम्बकेनोर्ध्वम्।
तिर्यक्कर्णेनान्यैः कथितैश्च नव प्रवक्ष्यामि॥7॥

"There are seventeen instruments in order to determine time, namely:

(1) the *dhanus* (semi-circle instrument),
(2) the *turya-golaka* (quarant),
(3) the *cakra* (circle instrument),
(4) the *yaṣṭi* (staff),
(5) the *śaṅku* (gnomon),
(6) the *ghaṭikā* (clepsydra),
(7) the *kapālaka* ("bowl", the hemispherical sundial),
(8) the *karttarī* ("scissors", a kind of equatorial sundial), and
(9) the *pīṭha* ("seat", a horizontal circle with a vertical gnomon).

And also (10) water, (11) compasses (12) plumb line, (13) set square, (14) shadow, (15) midday, (16) the sun, and (17) the latitude. Among them, these eight instruments are accessories for the purpose of knowing time.

A level ground is obtained by means of water, a circle is obtained by a pair of compasses, a vertical line is obtained by a plumb line, and an oblique line is obtained by a set square and other instruments.

I shall explain the remaining nine instruments."

Besides those nine instruments, Brahmagupta described water instruments and the self rotating instrument also.

The last half verse of this chapter is as follows (XXXII.57(ii).[4]

अध्यायो द्वाविंशो यन्त्रेष्वार्यास्त्रिपञ्चाशत् ।।57।।

"This is the 22nd chapter. There are 53 verses in *ārya* metre on instruments."

It is quite strange that this verse is numbered 57 in Sudhākara Dvivedin's edition, because this verse itself says that there are 53 verses in this chapter. In Ram Swarup Sharma's edition of the text (with various readings),[5)] the first three verses of the *Yantra-adhyāya* in Dvivedin's edition have been transferred into the previous chapter (*Gola-adhyāya*), and the vs.33 in Dvivedin's edition has been excluded. Thus the total number of the verses has become 53 in R.S. Sharma's edition. However, the previous chapter *Gola-adhyāya* already has 70 verses (without the first 3 verses of the *Yantra-adhāya)*, and the 70th verse clearly states that there are 70 verses in the *Gola-adhāya.* So, R.S. Sharma's numbering is also questionable. In this paper, I tentatively follow Dvivedin's numbering, but it does not mean that Dvivedin's numbering is doubtless.

The synopsis of the *Yantra-adhyāya* of the *Brāhma-sphuṭa-siddhānta* is as follows.

(1) Introduction (vv.1-4),
(2) list of instruments (vv.5-7),
(3) the *dhanus* (vv.8-16),
(4) the *turya golaka* (vs.17),
(5) the *cakra* (vs.18),
(6) the *yaṣṭi* (vv.19-38),
    (6-1) the staff in general (vv.19-23),
    (6-2) the V-shaped staffs (vv.24-26),
    (6-3) determination of directions by the staff (vs.27),
    (6-4) spherics and the staff (vv.28-31),
    (6-5) surveying by the staff (vv.32-38),
(7) the *śaṅku* (vv.39-40),
(8) the *ghaṭikā* (vs.41),
(9) the *kapālaka* (vs.42-43 (i)),
(10) the *karttarī* (vv.43(ii)-44),
(11) the *pīṭha* (vs.45),
(12) water instruments (vv.46-52),
(13) the self rotating instrument (vv.53-57(i)), and
(14) conclusion (vs.57(ii)).

vi) the Khaṇḍa-khādyaka

The *Khaṇḍa-khādyaka* (AD 665) is a Karaṇa work of Brahmagupta. Its first part follows Ārdha-rātrika-pakṣa, and the second part gives Brahmagupta's own correction. It was edited with Āmarāja's commentary by Babuā Miśra (1925), and was edited with Pṛthūdaka-svāmin's commentary and translated into English by P.C. Sengupta (1941 and 1934), but they do not give information about astronomical instruments. Full text was edited with Bhaṭṭotpala's commentary (AD 969) and translated into English by Bina Chatterje (1970),[1] and its second part contains a description of the *śaṅku* and the *ghaṭikā* (*Uttara.* III.2-3).

This description is actually the same as that of the *Brāhma-sphuṭa-siddhānta* (XXII. 39 and 41). Bhaṭṭotpala's commentary is helpful to interpret the text.

vii) The Śiṣyadhī-vṛddhida-tantra

Lalla (sometime between the 8th century and the early 11th century) wrote the *Śiṣyadhī-vṛddhida-tantra.*[1] Its commentaries written by Bhāskara II[2] and Mallikārjuna Suri[3] have been published.

The *Śiṣyadhī-vṛddhida-tantra* has a chapter on astronomical instruments, i.e. the *Yantra-adhāya* (Chapter XXI). Unfortunately, neither of the two commentaries is extended to the *Gola-adhyāya* which includes the *Yantra-dhyāya.* The *Śiṣyadhī-vṛdhida-tantra* has ben translated into English by Bina Chatterjee, but the *Yantra-adhyāya* was left untranslated by her. So, the English translation of the *Yantra-adhyāya* was supplied by K.S. Shukla in the published English translation.[4]

Lalla listed the names of instruments as follows (XXI.53).[5]

गोलो भगणश्चक्रं धनुर्घटी शङ्कुशकटकर्तर्यः।
पीठकपालशलाका द्वादशयन्त्राणि सह यष्ट्या।।53।।

"(1) The *gola* (armillary sphere),
(2) the *bhagaṇa* (a ring in the equatorial plane),
(3) the *cakra* (circle instrument),
(4) the *dhanus*(semi circle instrument),
(5) the *ghaṭī* (clepsydra),
(6) the *śaṅku* (gnomom),
(7) the *śakaṭa* ("cart", the V-shaped staffs),
(8) the *kartarī* ("scissors", an equatorial semi circle with a perpendicular gnomon),
(9) the *pīṭha* ("seat", a horizontal circle with a vertical gnomon),
(10) the *kapāla* ("bowl", a horizontal semi circle with a vertical gnomon),
(11) the *śalākā* ("needle", a variation of the staff), and
(12) the *yaṣṭi* (staff) are the twelve instruments."

Lalla also listed some accessories in the next verse. Besides the above twelve instruments, Lalla described the water instruments, the self rotating globe, and the shadow instrument also.

And also, the *Śiṣyadhī-vṛddhida-tantra* has the *Gola-bandha-adhikāra* (Chapter on the construction of the armillary sphere) as an independent chapter (Chapter XV).

viii) The Vaṭeśvara-siddhānta

Vaṭeśvara wrote the *Vaṭeśvara-siddhānta* (AD 904). It consists of the *Gaṇitādhyāya* and the *Golādhyāya,* but the *Golādhyāya* is extant in fragments only. Unfortunately, the *Yantrādhyāya* has not been found in those fragments. This text was edited and translated into English by K.S. Shukla.[1]

Its *Tripraśnādhyāya* (Chapter III), one chapter in the *Gaṇitādhyāya*, mentions some instruments, such as the gnomon, the staff, and the "triangle-instrument" (a variation of the staff).

ix) The modern Sūrya-siddhānta

The modern *Sūrya-siddhānta* is the fundamental text of one of the schools of Hindu astronomy, Saura-pakṣa. This is one of the most popular Sanskrit astronomical texts. This is called "modern" only for convenience' sake in contrast with the *Sūrya-siddhānta* in the *Pañca-siddhāntikā* of Varāhamihira. This text was translated into English by Bapu Deva Sastri, and also by Ebenezer Burgess (and W.D. Whitney).[1]

The modern *Sūrya-siddhānta* has a chapter on astronomical instruments (Chapter XIII) entitled *Jyotiṣa-upaniṣad-adhyāya.* In this chapter, the armillary sphere is explained in detail, but the other instruments are only briefly mentioned. It only lists the names of the *śaṅku*, *yaṣṭi*, *dhanus*, and *cakra* (XIII. 20). The *kapālaka* (clepsydra) is briefly explained (XIII. 23). The *nara-yantra* ("man instrument"), which is actually the same as the *śaṅku* (gnomon), is also mentioned (XIII.24). The self-rotating globe and the water instruments are also mentioned, but the method of their construction is not explained explicitly.

Among several commentaries on the *Sūrya-siddhānta,* those of Parameśvara (AD 1432)[2] and Raṅganātha (AD 1603)[3] have been published. Kamalākara's commentary was also published recently (Varansai, 1991), but it does not contain the chapter of astronomical instruments. There are several other commentaries which are still in the form of manuscripts. Among them, those of Mallikārjuna Sūri (AD 1178),[4] Rāmakṛṣṇa Ārādhya (AD 1472),[5] and Tamma Yajvan (AD 1599)[6] give important informations about the *Āryabhaṭa-siddhānta* of Āryabhaṭa.

x) The Siddhānta-śekhara

Śrīpati wrote the *Sidhāanta-śekhara*[1] (the 11th century). It has a chapter on astronomical instruments, i.e. the *Yantrādhyāya* (Chapter XIX). Although the commentary written by Makkibhaṭṭa (AD 1377) has ben published, this commentary does not extend to the *Yantrādhyāya*, and a commentary written by Babuāji Miśra has been supplied in the printed edition.[2]

The *Yantrādhyāya* of the *Siddhānta-śekhara* begins as follows (XIX.1-2).[3]

शक्यः परिच्छेदविधिर्विधातुं
यन्त्रैर्विना नो समयस्य तज्ज्ञैः।
तेषां स्वयंवाहकपूर्वकाणा–
मतः प्रवक्ष्ये खलु लक्षणानि।।1।।

अद्भिः समा भूर्वलयं भ्रमात्तु
त्र्यस्रं च कर्णाच्चतुरस्रयुक्तम्।
लम्बोऽध ऊर्ध्वार्जवसिद्धये स्यात्।
बीजानि तैलाम्बुरसाः ससूत्राः।।2।।

"As a man who knows it (astronomy) cannot establish the rule of division of time without instruments, I shall tell the description of the instruments such as the self rotating instruments etc.

A level ground is obtained with the help of water (ap), a circle is drawn by a pair of compasses *(bhrama)*, a triangle and quadrangular are drawn by a set square *(karṇa)*, a plumb line *(lamba)* is for settling a straight vertical line, and the seeds *(bīja)* (i.e. the essential requirements for water instruments) are oil *(taila)*, water *(ambu)*, mercury *(rasa)*, and string *(sūtra)*".

At the end of the *Yantrādhyāya*, Śrīpati listed the names of instruments as follows (XIX.27).[4)]

गोलश्चक्रं कार्मुकं कर्त्तरी च
कालज्ञाने यन्त्रमन्यत् कपालम्
पीठं शङ्कुः स्याद् घटी यष्टिसंज्ञं
गन्त्री यन्त्राण्यत्र दिक्सम्मितानि।।27।।

"(1) The *gola* (armillary sphere),
(2) the *cakra* (circle instrument),
(3) the *kārmuka* (= *dhanus* or "bow", semi-circle instrument),
(4) the *karttarī* ("scissors", an equatorial semi-circle with a perpendicular gnomon),
(5) the *kapāla* ("bowl", a horizontal semi-circle with a vertical gnomon),
(6) the *pīṭha* ("seat", horizontal circle with a vertical gnomon),
(7) the *śaṅku* (gnomon),
(8) the *ghaṭī* (clepsydra),
(9) the *yaṣṭi* (staff), and
(10) the *gantrī* (= *śakaṭa* or "cart", V shaped staffs).

These are the instruments to know time, and the instruments to know directions."

Besides these ten instruments, Śrīpati describes the self-rotating globe, the water instruments, and the shadow instrument also in the *Yantrādhyāya.*

The synopsis of the *Yantrādhyāya* of the *Siddhānta-śekhara* is as follows.

(1) Introduction (vs.1),
(2) list of accessories (vs.2),
(3) the armillary sphere (vv.3-6),
(4) the self-rotating globe (vv.7-8),
(5) water instruments (vv.9-11),
(6) the *cakra* (vv.12-13 (i)).
(7) the *cāpa* (= *dhanus*, semi circle) (vs. 13 (ii)),
(8) the *karttarī* (vs.14),
(9) the *kapāla* (vs. 15 (i)),
(10) the *pīṭha* (vv.15 (ii)-17),
(11) the *śaṅku* (vs. 18),
(12) the *ghaṭī* (vv. 19-20),
(13) the *yaṣṭi* (vv.21-23 (i)),
(14) the shadow instrument (vv.23 (ii)-25),
(15) the *śakaṭa* (vs.26), and
(16) list of instruments.

Besides the *Yantrādhyāya,* Śrīpati described the armillary sphere in the section of the *Gola-bandha* (XVI. 29-39) in chapter 16 (*Gola-varṇana-adhyāya).*

xi) The Siddhānta-śiromaṇi

Bhāskra II (born AD 1114) wrote the *Siddhānta-śiromaṇi* (AD 1150). It consists of two parts, viz. the *graha-gaṇita-adhyāya* and the *Gola-adhyāya.* The *Graha-gaṇita-adhyāya* was translated into English by D Arkasomayaji, and the *Gola adhyaya* was translated into English by Bapu Deva Sastri and Lancelot Wilkinson.[1] It has Bhāskara II's own commentary *Vāsanā-bhāṣya.* The commentary *Śiromaṇi-prakāśa* of Ganeśa (on the *Graha-gaṇita*),[2] the commentary *Vārttika* (AA 1621) of Nṛsiṁha (on the both of *Graha-gaṇita* and *Gola),*[3] and the commentary *Marīci* of Munīśvara (on the both)[4] have been published.

The *Golādhyāya* of the *Siddhānta-śiromaṇi* includes a chapter on instruments, i.e.. the *Yantrādhyāya* (Chapter XI). At the beginning of this chapter, Bhāskara II lists the names of instruments as follows (XI.2).[5]

गोलो नाडीवलयं यष्टिः शंकुर्घटी चक्रम्।
चापं तुर्यं फलकं धीरेकं पारमार्थिकं यन्त्रम्।।2।।

"(1) The *gola* (armillary sphere),
(2) the *nāḍī-valaya* (a ring in the equatorial plane),

(3) the *yaṣṭi* (staff),
(4) the *śaṅku* (gnomon),
(5) the *ghaṭī* (clepsydra),
(6) the *cakra* (circle instrument),
(7) the *cāpa* (semi circle instrument),
(8) the *turya* (quadrant),
(9) the *phalaka* ("board", an instrument to calculate time graphically from the sun's altitude), and
(10) the *dhī* ("intelligence", a variation of the staff), which is the unique best instrument".

Besides the above listed instruments, Bhāskara II described the self rotating instrument in the *Yantrādhyāya.*

And also, the *Golādyāya* of the *Siddhānta-śiromaṇi* has an independent chapter on the armillary sphere, i.e. the *Gola-bandha-adhikāra* (Chapter.VI).

xii) Minor anonymous Siddhāntas

Four anonymous Siddhāntas have been collected and published by V.P. Dvivedī under the title of *Jyautiṣa-siddhānta-saṅgraha.*[1] The title of the Siddhāntas are the *Soma-siddhānta,* the *Brahma-siddhānta,* the *Pitāmaha-siddhānta,* and the *Vṛddha-vasiṣṭha-siddhānta.*

There are also other anonymous siddhāntas, such as the *Vasiṣṭha-siddhānta*[2] and the *Romaka siddhānta.*[3]

The *Soma-siddhānta* mentions the armillary sphere. The *Brahma-siddhānta* mentions the water instruments etc. The *Vṛddha-vasiṣṭha-siddhānta* mentions some interesting instruments, such as the semi-circle instrument, the quadrant on which graphical calculation is done, the horizontal (not vertical) gnomon, the clepsydra, and water instruments.

xiii) Additional remarks.

Besides the above mentioned works, the *Mahā-siddhānta* of Āryabhaṭa II [1] is also considered to be a classical Siddhānta. (Its date is controversial.) It does not have chapter on instruments.

The tradition of Classical Siddhānta astronomy was well preserved in South India in later period also, and several interesting works were composed there.[2] Among them, the *Gola-dīpikā* I (AD 1443)[3] of Parameśvara contains a section on the armillary sphere. According to K.V. Sarma,[4] there are some other Kerala works on practical astronomy also.

3. The Gnomon

i) Introduction

The gnomon is an ancient device for determining the east west direction as well for knowing time. Already in Vedāṅga period, the gnomon was used, and mentioned in the *Kātyāyana-śulba-sūtra,* the *Artha-śāstra* etc.[1] It is usually a simple vertical stick. However, the horizontal gnomon is mentioned in the *Vṛddha-vasiṣṭha-siddhānta.* The gnomon is usually called *śaṅku*, and sometimes called *nara-yantra* (man-instrument) in Sanskrit.

ii) The shape and material of the śaṅku-yantra

a) The Āryabhaṭa-siddhānta

The *Āryabhaṭa-siddhānta* of Āryabhaṭa, quoted by Rāmakṛṣṇa Ārādhya in his commentary on the *Sūrya-siddhānta*, describes there types of the *śaṅku-yantra* as follows.[1]

तले द्व्यङ्गुलविस्तारः समवृत्तो द्वादशोच्छ्रयः।
सारदारूमयः शङ्कुर्द्वितीयो द्वादशाङ्गुलः।।32।।

सूच्यग्रस्थूलमूलो ऽन्यस्तदुत्सेधस्तलाग्रयोः।
सतिर्यग्वेधसूच्योयोस्तु लम्बसूची स्फुटो नरः।।33।।

तुल्याग्रस्तलवृत्तो ऽन्यः शङ्कुः स्याद् द्वादशाङ्गुलः।
या व्यक्ता शङ्कुभा यन्त्रात् सा व्यक्तैव नतप्रभा।।34।।

"(The first kind of gnomon is) two *aṅgulas* in diameter at the bottom, uniformly circular (i.e. cylindrical), twelve *aṅgulas* in height, and made of strong timber.

The second kind of gnomon is twelve *aṅgulas* (in height), pointed at the top, and massive at the bottom (i.e. conical in shape). (Associated with it is) another true gnomon of the same height, mounted vertically on two horizontal nails fixed (to the previous gnomon) at the top and bottom thereof.

Another (third) kind of gnomon (which is more handy) is that having equal circles at the top and bottom (i.e. cylindrical) and of twelve *aṅgulas.*

Whatever shadow of the gnomon is seen to be cast by this instrument is indeed the projection of the (Sun's) zenith distance (i.e. the R sine of the Sun's zenith distance)." (Translated by K.S. Shukla.)[2]

In this text, three types of the gnomon have been described by Āryabhaṭa.

Rāmakṛṣṇa Ārādhya continues to explain astronomical instruments after the quotation from the *Āryabhaṭa-siddhānta,* but unfortunately the original manuscript from which our Lucknow-manuscript was copied seems to have been very defective and broken off at the middle of the description of the gnomon. Nevertheless, the following passage may be quoted, because the second type of the gnomon is explained more plainly.[3)]

अन्यस्तृतीय (द्वितीय?) शङ्कुमूले द्व्यङ्गुलविस्तारा द्वादशाङ्गुलोच्छ्र
..................[gap]................ तन्मूलाग्रयोरितर्यक् छिद्रद्वयं कृत्वा तच्छिद्रद्वये
तिर्यक् शकलद्वयं संस्थाप्य तच्छायाग्र ....[gap]................
छिद्रद्वयं कुर्यात् शङ्कुस्तम्भ इत्युच्यते। तच्छलाग्रच्छिद्रयोः
सूक्ष्मशलाकाद्वादशाङ्गुला श्लक्ष्णा सा वृत्त सूची

"Another third (second?) type of the gnomon is two *aṅgulas* in diameter at the bottom, and twelve *aṅgula* in height,.... (gap)... After making two holes horizontally at the base and top, two nails should be horizontally fixed to the previous two holes, and shadow end (?)... (gap)..Two holes should be made. It is called the supporter of the [true] gnomon. To the holes at the tip of the previous two nails, a smooth round and fine twelve *aṅgula* needle (i.e. the true gnomon) [should be fixed] ..."

Tamma Yajvan also described three types of the gnomon, which had originally been mentioned in the *Āryabhaṭa-siddhānta,* in his commentary on the *Sūrya-siddhānta* as follows.[4)]

शङ्कुस्त्रिविधः। एको मूलाग्रत्य (तुल्य?) परिधिद्वादशाङ्गुलोच्छ्रयो
द्व्यङ्गुलविस्तारः। अपरो द्व्यङ्गुलविपुलो द्वादशाङ्गुलसूच्यग्रः
आद्यन्तयोस्तिर्यक्स्थितः सुषिरद्व्ययुक्तः तत्सुषिरयोस्तिर्यक्
शलाकाद्वययुक्तः स्तम्भवत्कार्यः। तच्छलाकाद्वयाग्रलम्बितसूत्रं
नरशङ्कुः। द्वादशाङ्गुलः सूक्ष्ममूलाग्रपरिधिः कल्पितद्वादशाङ्गुलः
सर्वजनव्यवहारयोग्यो ऽपरः तृतीयब[ः]

"The gnomon *(śaṅku)* is of three kinds.

One type is cylindrical in shape, twelve *aṅgulas* in height, and two *aṅgulas* in diameter.

Another type of two *aṅgulas* in diameter at the bottom, twelve *aṅgulas* in height, pointed at the top, and having two horizontal holes at the bottom and top with horizontal needles fixed to the holes, should be got constructed like pillar *(stambha).* A string *(sūtra),* which is suspended vertically between the two nails, is the [true] gnomon *(nara-śaṅku).*

The third type, which is fit for the use of all people, is cylindrical in shape with small diameter, and is twelve *aṅgulas* in height''.[5)]

From the above sources, we can summarize the shape and size of the three types of Āryabhaṭa's gnomon as follows. (See Fig. 1.)

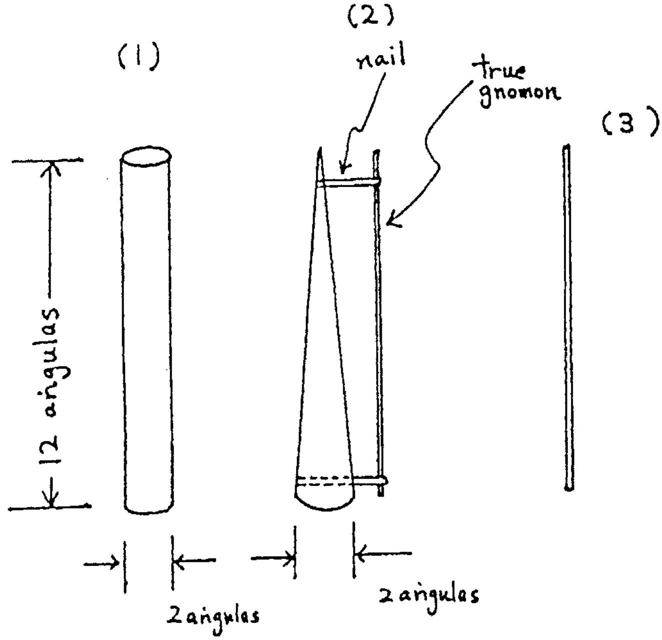


Fig. 1. Āryabhaṭa's gnomon (reconstructed)

The 1st type : A cylinder made of timber, 2 *aṅgulas* in diameter and 12 *aṅgulas* in height.

The 2nd type: A conical gnomon, which is 2 *aṅgulas* in diameter at the base and 12 *aṅgulas* in height, and a needle attached there as the 'true gnomon"(for ascertaining verticality).

The 3rd type: A cylinder which is 12 *aṅgulas* in height, and has small diameter.

b) Bhāskara I's commentary on the Āryabhaṭīya

Bhāskara I described three types of the gnomon in his commentary on the *Āryabhaṭīya* (II.14) as follows.[6)]

(1) The 1st type :

द्वादशाङ्गुलशङ्कुर्मूलत्रिभागे चतुरस्रो, मध्यत्रिभागे त्र्यस्रिः,
उपरित्रिभागे शूलाकार इति।

"Twelve-*aṅgula* gnomon whose one third at the bottom is of the shape of right prism, one third in the middle is of the shape of a traingular prism, and one thrid at the top is a spike".

Bhāskara I criticized this Ist type that it is difficult to ascertain its verticality.

(2) The 2nd type :

चतुरस्रश्चतुर्दिशमवलम्बकसाधनसम्भवात्कोटिद्वयेन
छायाग्रहणादभीष्टकोट्यां दिग्ग्रहणसिद्धिरिति।

"A right prism [whose four sides are] directed towards the four directions. For ascertaining the verticality, the shadow of two uprights are made coincided, and the direction [of the sun] is ascertained to be in direction of this desired upright."

Bhāskara I criticized this 2nd type that it is difficult to make it, and that it should be rotated at every moment because the sun in moving.

(3) The 3rd type (Āryabhaṭa school's gnomon) :

प्रशस्तदारूमयो ह्यसुषिरो राजिग्रन्थिव्रणवर्जितो भ्रमसिद्धो
मूलमध्याग्रान्तरालतुल्यवृत्तो नाल्पव्यासो नाल्पायामश्च प्रशस्तः।

"An excellent cylindrical gnomon, made by a rotating machine, made of excellent timber, free from holes, streaks, knots and wounds, with large diameter and large height."

After instructing to ascertain the verticality of this gnomon, Bhāskara I added as follows.

शङ्कोरुपरि केन्द्रे विष्कम्भार्धाधिकान्या समवृत्ता शलाका
मध्यप्रसाधिनी लोही दार्वी वा क्रियते।

"At the top of the [supporting] gnomon, at its centre, another cylindrical needle made of metal or timber, whose height is greater than the radius of the supporter, is attached centrally".

This is Āryabhaṭa school's gnomon according to Bhāskara I. This type of the gnomon is different from any type of the gnomon described in the *Āryabhaṭa-siddhānta*. (See Fig.2)

As regards the height of the gnomon, Bhaskara I wrote that it could be of any length with any number of divisions, although some people had said that it should be 12 *aṅgulas*.

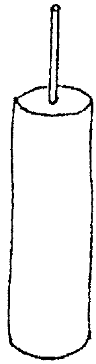


Fig. 2. Āryabhaṭa school's gnomon according to Bhāskara I (reconstructed)

c) Brahmagupta's gnomon

Brahmagupta described a conical gnomon in his *Brāhma-sphuṭa-siddhānta* (XXII. 39) as follows,[7)]

मूले द्व्यङ्गुलविपुलः सूच्यग्रो द्वादशाङ्गुलोच्छ्रायः।
शङ्कुतलाग्रविद्धो ऽग्रवेधलम्बादृजुर्ज्ञेयः।।39।।

"It is two *aṅgulas* in diameter at the bottom, pointed at the top, twelve *aṅgulas* in height, and pierced at the base and top of the gnomon (*śaṅku*). By the plumb line from the hole at the top, verticality should be ascertained".

The same verse appears in Bhaṭṭotpala's version of the *Khaṇḍakhādyaka* of Brahmagupta (Uttara..III.2). The following is a quotation from Bhaṭṭotpala's commentary.[8)]

तलाग्रविद्ध इति। तलादुपरि कियत्यपि दूरे वेधः कार्यः। समस्तिर्यक्। एवमग्रादप्यधः। ततो वेधः। यः शलाकाग्रे प्रवेश्योपरितः शलाकायामुभयतो ऽवलम्बकद्वयं तुल्यप्रमाणमवलम्बयेद्यावदधः शलाका तिर्यगित्यर्थः। एवमृजुः स्पष्टो धार्यः। इति।

"[The text reads :] 'pierced at the base and top'. Above the base, at any distance, a hole should be made, which is smooth and horizontal. Similarly, below the top, [another] hole [should be made]. After inserting the tip of two needles [to the previous holes], two plumb lines, which are of equal length, should be hung down from above on both sides of the needle. This is for [ascertaining] the horizontality of the needle. Thus correct verticality is kept."

According to this commentary, the conical gnomon is the main body of this instrument, and the two needles are used only for ascertaining the verticality of the conical gnomon. So, this is similar to the second type of Āryabhaṭa's gnomon. (see Fig. 3)

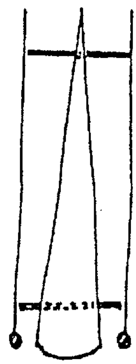


Fig. 3. Brahmagupta's gnomon (reconstructed)

Brahmagupta wrote to determine time by the gnomon as follows (XXII.40).[9)]

छायां दृग्ज्यां दृष्टिं (यष्टिं) छायाकर्णमवलम्बकं शङ्कुम् ।
परिकल्प्य शङ्कुयन्त्रे योज्यं घटिकादि यष्ट्युक्तम् ।।40।।

"Considering the shadow (*chāyā*) as the R. sine of the sun's zenith distance *(dṛg-jyā)*, the hypotenuse of the shadow [and the gnomon] (*chāyā karṇa)* as the Radius (*yaṣṭi),* and the upright *(avalambaka*, i.e. the gnomon) as the R. sine of the sun's altitude (*śaṅku*), the *ghaṭikās* etc. should be obtained from the gnomon (*śaṅku-yantra*) as was told in the section of the *yaṣṭi*".

Here Brahmagupta refers to the verse (XXII.23) of the same work, where he wrote to determine time from the R.sine of the sun's altitude. (See the section of the *yaṣṭi* below). The method of calculation is, however, not given there. The method of calculation to determine time is a topic of the *Tri-praśna-adhyāya* rather than the *Yantra-adhyāya* . (We shall discuss the method of calculation later.)

d) Lalla and Śrīpati

Lalla described the gnomon in his *Śiṣyadhī-vṛddhida-tantra* (XXI. 31 33). The description of its shape and material is as follows (XXI. 31 32).[10)]

भ्रमसिद्धः सममूलाग्रपरिधिरतिसुगुरूसारदारूमयः ।
ऋजुरव्रणराजिलाञ्छनस्तथा च समतलः शङ्कुः ।।31।।

वृत्तः षडङ्गुलानि द्वादशदीर्घश्चतुर्भिरवलम्बैः।
स्याप्यः (स्थाप्यः) सुसमः प्रथमे जलेन च सुसमीकृते फलके।।32।।

"The gnomon, constructed with the help of the revolving machine, having equal periphery at the top and bottom, made of very heavy and strong timber, perfectly straight and free from scars, streaks and spots, uniformly circular, six *aṅgulas* in circumference and twelve *aṅgulas* in height, should be set up on a (horizontal) board, (already) levelled by means of water, in vertical direction with the help of four plumbs". (Translated by K.S. Shukla)[11)]

Śrīpati also described a similar gnomon in his *Siddhānta-śekhara* (XIX. 18) as follows.[12)]

भ्रमविरचितवृत्तस्तुल्यमूलाग्रभागो
द्विरदरदनजन्मा सारदारूद्भवो वा।
गुरु ऋजुरवलम्बादव्रणः षट्कवृत्तः
समतल इह शस्तः शङ्कुरर्काङ्गुलः स्यात्।।18।।

"The approved gnomon (*śaṅku)* is a cylinder made by a rotating machine, having uniform measure at the bottom and top, made of tusk of an elephant or strong timber; and it is heavy, made straight by a plumb line, free from scars, uniformly six *aṅgulas* in circumference, and twelve *aṅgulas* in height."

If π is roughly considered to be three, the gnomon of Lalla and Śrīpati can be said to be the same as the first type of the gnomon of the *Āryabhaṭa-siddhānta* , whose diameter is two *aṅgulas.*

e) Bhāskara II's gnomon

Bhāskara II described a cylindrical gnomon in his *Siddhānta-śiromaṇi* (*Gola*, XI.9) as follows.[13)]

समतलमस्तकपरिधिर्भ्रमसिद्धो दन्तिदन्तजः शङ्कुः।
तच्छायातः प्रोक्तः ज्ञानं दिग्देशकालानाम्।।9।।

"The gnomon has equal periphery at the bottom and top, is constructed by a rotating machine, and is made of ivory.

From its shadow, the knowledge of the direction, the place (i.e. the latitude of the observer), and the time is declared".

Bhāskara II did not mention the size of the gnomon.

f) Conclusion

There were several variations of the gnomon in early age, but the cylindrical 12-*aṅgula* gnomon has become the standard gnomon. The variations of the gnomon described by several authors in Classical Siddhānta period can be tabulated as below.

	shape	diameter (*aṅgulas*)	height (*aṅgulas*)	material
Āryabhaṭa's				
Ist type	cylinder	2	12	timber
2nd type	cone and a "true gnomon"	2 at the base	12	——
3rd type	cylinder	small	12	——
Bhāskara I's				
Ist type	a combination of a right prism, a triangular prims, and a spike			
2nd type	a right prism			
3rd type	cylinder with a needle	large	large	timber
Brahmagupta	cone	2	12	——
Lalla	cylinder	6/π	12	timber
Śrīpati	cylinder	6/π	12	ivory or timber
Bhāskara II	cylinder	——	——	ivory

iii) The theory of the gnomon

The use of the gnomon is explained in detail in the chapter on "three problems" (direction, place, and time), i.e. the *Tri-praśna-adhyāya* of the Siddhāntas. Since it is beyond the scope of the present paper to discuss about the *Tripraśnādhyāya* extensively, I would like to confine myself to the discussion of two practical topics, i.e. the determination of directions, and the determination of time. The determination of directions is explained in architectural literature also, and we shall briefly discuss this topic also.

a) The determination of directions

The method of determination of east west direction by using the gnomon is already mentioned in the *Kātyāyana-śulba-sūtra* (I.2), which is one of the Vedāṅga literature. In Classical Siddhānta period, some astronomers further devised the correction due to the change of the sun's declination within a day.

(1) Indian circle method

Let us first see the method of the determination of directions without the correction due to the change of the sun's declination. The following method is based on the description in the *Sūrya-siddhānta*(III. 1-4)[1] (See Fig. 4)

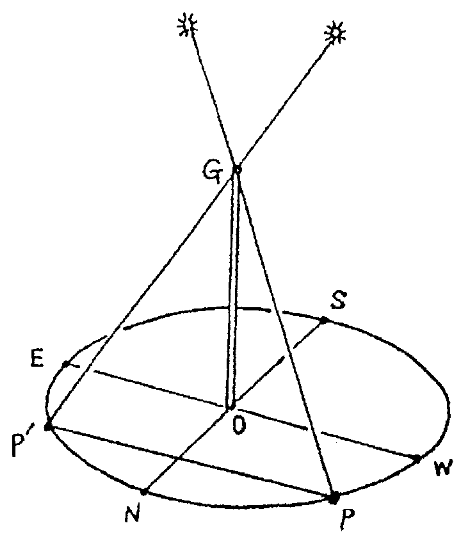


Fig. 4. Indian circle method

Firstly, one should draw a ground level circle (ENWS in the figure), whose radius is equal to the height of the gnomon (i.e. 12 *aṅgulas)*, on a horizontal stony surface or hard plaster. Then, a 12-*aṅgula* gnomon (GO) should be erected vertically at its centre. Then one should mark two points (P and P[1]) on the circle, where the tip of the shadow touches in the forenoon and afternoon respectively. Their perpendicular bisector (NS) is the north-south line. The perpendicular bisector of this north-south line is the east-west line (EW).

This method is the same as that of the *Kātyāyana-śulba-sūtra*, and well known as Indian circle method.

(2) Corrected Indian Circle method.

Before discussing the correction due to the change of the sun's declination, we should see the relationship between the gnomon-shadow, the sun's amplitude, the sun's declination, and the observer's latitude.[2] (See Fig.5, where Fig.5(a) is a diagram of the ground level circle, and Fig.5(b) is an orthographic projection of the celestial sphere onto the plane of the meridian.)

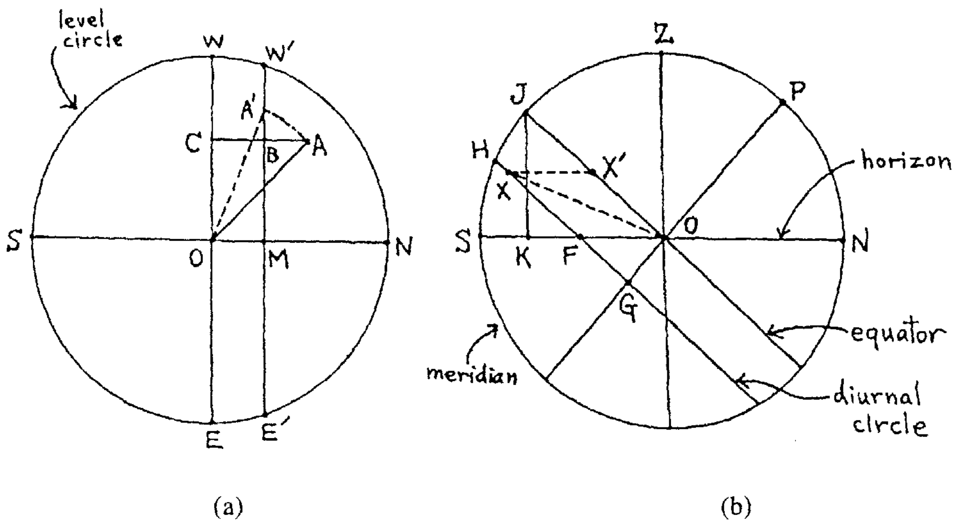


Fig. 5. (a) Diagram on the level circle (b) Orthographic projection of the clectial sphere onto the plane of the meridian

Firstly, let us see the relationship between the shadow and the sun's amplitude (angular distance between the east cardinal point and the sun's rising point). The R.sine of the sun's amplitude (OF in Fig. 5 (b) is called *agrā*. Let OM (in Fig.5 (a)) be the equinoctial midday shadow, when the gnomon is erected vertically at the point O. Then the line E' 'MW' is the locus of the tip of the equinoctial shadow which is a straight line, if the change of the sun's declination is neglected. Let X (in Fig. 5 (b)) be the position of the sun at arbitrary time of arbitrary day, X' the point on the equator whose altitude is the same as X, the point A (in Fig. 5 (a)) the tip of the shadow when *the sun is at* X, *and* A' *the tip* of the shadow when the sun is at X'. Here, OA = OA', because the altitude of X and X' is the same. Now, the orthographic projection of XX' on the plane of the meridian is equal to the *agrā* (OF), as is evident from Fig. 5(b), and the orthographic projection of AA' on the plane of meridian is equal to AB, as is evident from Fig. 5 (a). Two triangles OXX', where O is the centre of the celestial sphere and X and X' are the points on the celestial sphere, and GAA', where G is the top pf the gnomon, are similar. Considering these facts, we have the following proportion:

$agr\bar{a} : \overset{\frown}{AB} = XX' : \overset{\frown}{AA'} = R : h,$

where R is the Radius of the celestial sphere, and h is the hypotenuse of the shadow, that is :

$$h = \sqrt{g^2+OA^2},$$

where g is the height of the gnomon. Accordingly,

$$AB = \frac{h}{R} \times agr\bar{a}. \qquad (1)$$

Sometimes, this amount AB is also called *agrā* (for the circle whose radius is equal to the hypotenuse of the shadow). (See *Sūrya-siddhānta*, III.7.)

Now let us see the relationship between the *agrā,* the sun's declination, and the observer's latitude. In Fig. 5 (b), two triangles OJK and FOG are similar. Since the angle ZOJ is equal to the observer's latitude, the segment JK is equal to the R.cosine of the observer's latitude. It is clear from the figure that the segment OG is equal to the R. sine of the sun's declination. Therefore,

$$JK : OJ = OG : OF,$$

or

$$R.\cos \varphi : R = R.\sin \delta : agr\bar{a},$$

where φ is the observer's latitude, and δ is the sun's declination. Accordingly,

$$agr\bar{a} = \frac{R \times R.\sin \delta}{R \cos \varphi} \qquad (2)$$

Now let us discuss the method of the correction applied to Indian circle method due to the change of the sun's declination.

As far as we can judge from extant sources, the earliest Indian astronomer who explicitly mentioned the correction due to the change of the sun's declination is Caturvedācārya Pṛthūdakasvāmin (fl. AD 864). He rightly pointed out in his commentary on the *Brāhma-sphuṭa-siddhānta* (III.1)[3)] of Brahmagupta that the east-west line determined by the usual method should be corrected according to the difference of the *agrā* at the time of the shadow's entry into and exit from the circle.

Śrīpati also explained the method of the correction. He wrote the process of the calculation in his *Siddhānta-śekhara* as follows (IV.2-3).[4)]

याति भानुरपमण्डलवृत्ताद्दक्षिणोत्तरदिशोरनुवेलम् ।
तेन सा दिगनृजुः प्रतिभाति स्यादृजुः पुनरपक्रममौर्व्या ।।2।।

छायानिर्गमनप्रवेशसमयार्कक्रान्तिजीवान्तरं
क्षुण्णं स्वश्रवणेन लम्बकहृतं स्यादङ्गुलाद्यं फलम् ।
पश्चाद्बिन्दुमनेन रव्ययनतः संचालयेद्व्यत्ययात्
स्पष्टा प्राच्यपरा ऽथवा ऽयनवशात् प्राग्बिन्दुमुत्सारयेत् ।।3।।

"The sun moves towards south or north along the ecliptic every moment. Therefore, the direction [determined by Indian circle method] appears to be incorrect. The corrected direction will be [obtained by applying a correction] further by [using] the R. sine of the declination.

The difference of the R.sine of the sun's declination at the time of the shadow's entry and exit [to and from the level circle] is multiplied by the hypotenuse [of the shadow] and divided by the R. cosine of the terrestrial latitude. The result is [the correction in terms of] *aṅgulas* etc. One should shift the western mark to the opposite direction to the sun's course *(ayana)*. Otherwise, one should shift the eastern mark to the same direction of the sun's course. [Thus] the correct east west line [is obtained]."[5)]

This rule can be expressed as follows:

$$x = \frac{h\ (\text{R.sin}\ \delta_1 \sim \text{R.sin}\ \delta_2)}{\text{R.cos}\ \delta} \qquad (3)$$

where δ_1 and δ_2 are the declination of the sun at the time of the shadow's entry into and exit from the circle respectively, and x is the amount of the correction. (See Fig. 6.) In the figure, the line PP' is the east west direction obtained by the usual method, the segment PQ the amount of the correction, and the line QP' the corrected east-west direction., The amount of the correction x is equal to the difference of the north-south component of the shadow at the two moments, as is evident from the figure. From the equations (1) and (2), we can prove the formula (3) as follows. Let A and A' be the amount of the *agrā* at the two moments. Then we can express the amount x as follows. From the equation (1),

$$x = \frac{h}{R} \times A \sim \frac{h}{R} \times A'.$$

Then from the equation (2),

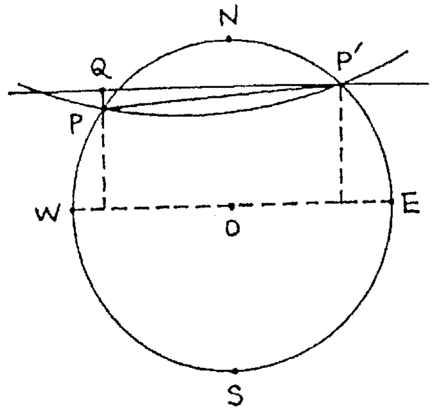


Fig. 6. Fixation of East-West Line

$$x = \frac{h \times R \times R.\sin \delta_1}{R \times R.\cos \varphi} - \frac{h \times R \times R.\sin \delta_2}{R \times R.\cos \varphi}$$

Therefore,

$$x = \frac{h\ (R.\sin \delta_1 - R.\sin \delta_2)}{R \cos \varphi}$$

Hence proved.

After Śrīpati, Bhāskara II[6] etc. also explained the method of the correction due to the change of the sun's declination.

(3) Three-shadow method

There is another method of determination of directions which can be called three-shadow method. Varāhamihira wrote in his *Pañca-siddhāntikā* (XIV.14-16) as follows.[7] (See Fig.7.)

नाभ्यासन्नच्छायाग्रमङ्कयेत् त्रिस्ततो लिखेन्मत्स्यौ।
तन्मत्स्यवदननिःसृतसूत्रद्वयपाततुल्येन।।14।।

सूत्रेण बिन्दुकत्रयसंस्पर्शसमेन मण्डलं यत् स्यात्।
तेन कदाह्नि च्छाया शङ्कोर्गच्छत्यमुंचन्ती।।15।।

तन्मण्डलमध्याद्यच्छङ्कुतश्च दक्षिणोत्तरं भवति।
तच्छङ्कुविवरमुदगास्थितं च माध्यन्दिनी छाया।।16।।

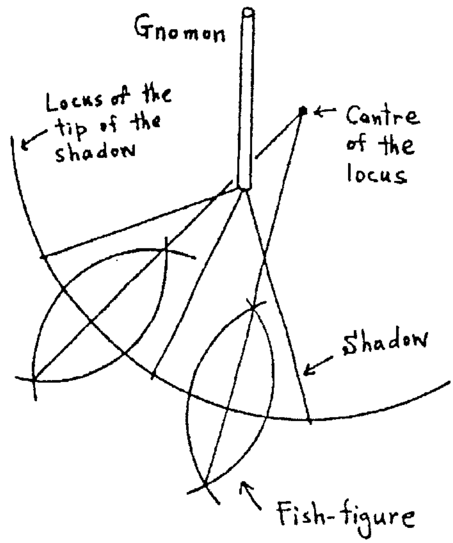


Fig. 7. Three-shadow method

"One should thrice mark the tip of the shadow near the centre (i.e. near the tip of the midday shadow), and then draw two fish figures (in order to draw the perpendicular bisectors of the marks). [Draw] a circle with a string with the centre which is the intersection of the two lines produced from the mouths of these fish figures, in such a way that the circle touches the three marks. On that day, the shadow of the gnomon moves on that [circle] without leaving it.

The line joining the centre and [the base of] the gnomon is the north south line. Its (the circle's) distance from the gnomon in the north direction is the midday shadow.''[8)]

This method is evidently inexact, because the locus of the tip of the shadow is hyperbola and not circle, but this approximation can be used without much error near the midday shadow. Bhāskara II, in his *Siddhānta-śiromaṇi* (*Gola*,XI.38(ii)),[9)] rightly criticized the theory that regards the locus of the shadow as a circle.

b) The determination of directions in architecture

The Indian circle method was also used in *śilpa-śāstra* (architecture) for the determination of directions. P.K. Acharya[10)] made well-known the method written in Chapter IV of the *Mānasāra* (ca. 500-700 AD?). According to the *Mānasāra,* the gnomon is made of wood, and its length may be 24, 18, or 12 *aṅgulas*, and its width at the base should be 6, 5, and 4 *aṅgulas* respectively. It tapers from the bottom towards the top, and its width at the top is 2, 1, and 1/3 *aṅgula* respectively. Alternatively, 9-*aṅgula* gnomon could be used. As regards the method of the observation, the *Mānasāra* (VI.11(d)-15) reads as follows.[11)]

स्वीकरीकृतभूतले । ।11 । ।

तन्मध्ये बिन्दुतत्त्वज्ञो शङ्कुयामद्वयेन च ।
भ्रामयेन्मण्डलं कुर्यात्तन्मध्ये शङ्कुमर्पयेत् । ।12 । ।

पूर्वाह्णे शङ्कुतश्छायां पश्चिमे मण्डलान्तकम् ।
तत्रैव बिन्दुसंज्ञाश्च कुर्यात्तु शिल्पि (ल्प) वित्तमः । ।13 । ।

[अ] पराह्णे शङ्कुतश्छायां पूर्वदिङ्मण्डलान्तके ।
पूर्ववद्बिन्दु (न्दुं) संस्थाप्य पश्चाच्छङ्कुं त्यजेत्ततः । ।14 । ।

शङ्क्वायामषडाधिक्यनवत्यंशविभाजिते ।
तस्यांशेन अपच्छायां त्यक्त्वा प्राचीं नयेत्ततः । ।15 । ।

"In the centre of the selected site the expert geometrician should describe a circle by moving around (a cord of) twice the length of the gnomon (as the radius); and on the centre (of the circle) a gnomon should be fixed.

In the forenoon (at certain time) the chief architect should mark a point (where) the shadow from the gnomon (meets) the circumference in the west. In the afternoon (also) a point should be marked as before (i.e. as in the morning) where the shadow from the gnomon (meets) the circumference in the east. Thereafter the gnomon should be left (to remain) therein.

The length of the gnomon being divided into ninety six parts, (and) the *apacchāyā* being left out of these parts, the (due) east should then be determined.'' (Translated by P.K. Acharya)[12)]

After this, the *Mānasāra* (VI.16-18(i), and 25(ii)-38) gives the amount of the *apacchāyā*, which can be tabulated as follows.

Months	First 10 days	Middle 10 days	Last 10 days	
Meṣa	2	1	0	(aṅgulas)
Vṛṣa	0	1	2	
Mithuna	2	3	4	
Kulīra	4	3	2	
Siṃha	2	1	0	
Yuvatī	0	1	2	
Tulā	2	3	4	
Vṛścika	4	5	6	
Dhanus	6	7	8	
Makara	8	7	6	
Kumbha	6	5	4	
Mīna	4	3	2	

Here, the name of months is given according to the zodiacal sign where the sun stays beginning with Meṣa (Aries)

The method to use this *apacchāyā* is explained in the *Mānasāra* (VI.18 (ii)-21). P.K. Acharya translated this passage as follows.

"The aforesaid *aṅgulas* should be marked in the shadow to the left and right of the centre; (with) that is left after the deduction of these *aṅgulas* the due east line should be drawn.

During the six months (i.e. the northern solstice) beginning with *Makara* (December 21-22) the shadow declines towards the south and during the six months (i.e. southern solstice) beginning with *Kulīra* (June 21-22) the shadow declines towards the north.

In the shadow facing the east-left the left (point) should be marked; thereafter moving towards the east and right the west-left points should be marked. The architect should leave out the *apacchāyā* and draw the east west line.'' (Translated by P.K. Acharya)[13)]

We shall re-examine this text later with the quotation of the original text.

The same table of the *apacchāyā* is also found in another architectural work *Mayamata*[14)] and also in an encyclopedic manual *Īśāna-śiva-gurudevapaddhati* (the late 11th or early 12th century)[15)] of Gurudeva.

The meaning of the *apacchāyā* is controversial. P.K. Acharya thought that the term *apacchāyā* means "the shadow which is displaced or wrongly placed", and supposed that it is for the correction applied to the shadow. He suggested the possibility that it is for the correction due to the change of the sun's declination, but simultaneously pointed out that the values in the given *apacchāyā*-table do not fit for this correction, and he had to conclude that it is difficult to find the correct solution. He pointed out, for example, that the correction should be zero at the solstices if the correction is due to the change of the sun's declination, but the *apacchāyā*-table is not so.[16)]

J.Filliozat thought that the *apacchāyā* means "ombre réduite" (i.e. reduced shadow), that is the midday shadow which has most deduced length. Accordingly, he calculated the latitude of the observer from the length of the *apacchāyā* as the midday shadow of 12-*aṅgula* gnomon, and concluded that the latitude would be 10°, 9° and 5° by the data at the winter solstice, equinox, and summer solstice respectively.[17)]

Bruno Dagens first interpreted in his French translation of the *Mayamata* that the values of the *apacchāyā* are the distance from the tip of the shadow to the east west line.[18)] (What is meant by him is the segment PQ and P'Q' in Fig.8.) However, following J.Fillozat's suggestion, Dagens later abandoned this interpretation.[19)]

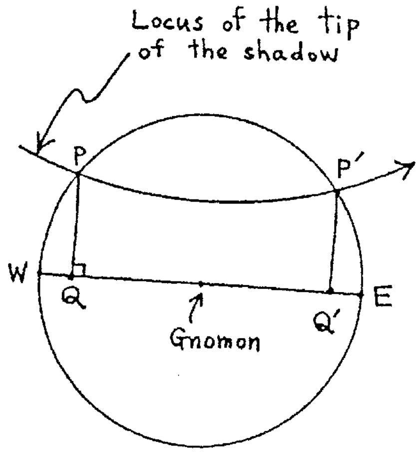


Fig. 8. *Apacchāyā* (Distance from the tip of the shadow to the East-West Line)

Michio Yano interpreted that the *apacchāyā* table represents the "variation of the length of the noon-shadows expressed in a modified linear zig-zag function".[20] This interpretation is practically the same as the interpretation of Filliozat. After pointing out that this linear zigzag function is similar to that of the shadow table in the *Artha-śāstra* (II.20. 39-42), Michio Yano commented as follows.

"It is highly probable that a set of the Babylonian shadow tables was transmitted to India and thereafter handed down to the south undergoing the simple modification (parallel displacement, to use mathematical terms) in order to accommodate itself to the south Indian latitude without changing the fundamental scheme of the linear zigzag function." (Michio Yano)[21]

Bruno Dagens accepted M. Yano's interpretation in his English translation of the *Mayamata,* and commented in its Introduction as follows.

"The orientation of the site is made according to the classic method with a gnomon (*śaṅku,* 6.1-11). On this point the text presents the *apcchyāyā* (6.11b-13 and 27-28) which is said to be a factor that allows for the rectification of the rough indication furnished by the gnomon; in reality, as Michio Yano recently proved, this development on *apacchāyā,* which is found in the same context in several texts, has nothing to do with the orientation but simply gives a method of expressing the variation of the length of the noon shadows in a modified linear function, which method would seem to have its far off origin in Babylonia. The unresolved question remains as to how and why this development has been systematically introduced in the orientation rules." (Bruno Dagens)[22]

As B. Dagens himself is aware, the variation of the midday shadow has nothing to do with the orientation method, and it is quite strange to suppose that it was recorded in the section of the orientating method. Moreover, this interpretation includes too south latitude (10°N or so) of the observation, although it is difficult to suppose that all of these architectural works were used in extreme south only. As regards the hypothesis that the linear zigzag function of the shadow length in the *Artha-śāstra* etc. were transmitted from Babylonia, I have shown in my previous paper that it is groundless.[23)]

I think that B. Dagens's first interpretation in his French translation of the *Mayamaya* was basically correct, and the *apacchāyā*-table was meant to obtain the east-west line which passes through the central gnomon. Practically, the correction must have been considered as PW and P'E in Fig. 8, rather than PQ and P'Q', because the correction of PQ and P'Q' presupposes the knowledge of the line EW which is to be known after the correction.

Let us again read the *Mānasāra* (VI.18(ii)-21) carefully. The original text is as follows.[24)]

छायायां बिन्दुवामे [तु] दक्षिणे चोक्ताङ्गुलं न्यसेत्।।18।।

अङ्गुलान्ते तु यच्छुद्धं प्राचीसूत्रं प्रयोजयेत्।
मकरादि (दौ) च षण्मासे छाया दक्षिणतो भवेत्।।19।।

कुलीरादि (दौ) च षण्मासे छाया चोत्तरतो दिशि।
छायाया अभिमुखे प्रत्यग्वामे वामं (बिन्दुं?) न्यसेद्यते (न्यसेदतः)।।20।।

पूर्वे च दक्षिणे नीत्वा प्रत्यग्वामाङ्गुलान्न्यसेत्।
अपच्छायां त्यजेच्छिल्पी प्राक्प्रत्यक्सूत्र (त्रं) विन्यसेत्।।21।।

I already have quoted P.K. Acharya's translation of the above text, but this text may rather be translated as follows.

"The aforesaid *aṅgulas* should be shifted leftward or rightward from the mark of the shadow [on the circle].

At the end of the [correctional] *aṅgulas*, the correct east [-west] line should be drawn.

During the six months beginning with Makara (Capricorn) (i.e. from the winter solstice) the shadow declines towards the south, and during the six months beginning with Kulīra (Cancer) (i.e. from the summer solstice) the shadow declines towards the north.

Facing the shadow, [if] the [western] mark should be shifted leftward from the western [mark], the eastern [mark] should be shifted rightward taking the *aṅgulas* with which the western [mark was shifted] leftward.

The architect should deduct the *apacchāyā* and draw the east west line."

It appears from this text that the two marks (P and P' in Fig. 8) are shifted towards the same direction (PW and P'E in the figure), and the "correct east-west line" means the east-west line passing though the central gnomon (the line WE in the figure). We can suppose that the seasonal parallel shift of the east-west line from the central gnomon was inconvenient for architects, and this correction was necessary.

Let us estimate the observer's latitude by this interpretation. Firstly, it is necessary to know the radius of the level circle. We have already seen that it is a double of the length of the gnomon in the *Mānasāra* (VI.12). On the contrary, Bruno Dagens thought that the radius of the level circle is the same as the length of the gnomon in the *Mayamata* (VI.7 (ii)-8(i)). The *Mayamata* (VI. 7(ii)-8(i)) reads as follows.[25]

शङ्कुं कृत्वा दिनादौ तु स्थापयेदात्तभूतले ।।7।।
शङ्कुद्विगुणमानेन तन्मध्ये मण्डलं लिखेत् ।

Bruno Dagens translated this text as follows.

"When the gnomon has been made it is set up in the chosen place at sunrise, then a circle is drawn of which the gnomon is the centre and of which the diameter is double the length of the gnomon." (Translated by Bruno Dagens)[26]

Here, it is seen that the phrase *"śaṅku-dviguṇa-mānena"* has been translated as "of which the diameter is double the length of the gnomon". This translation is quite strange, because it is natural to think that the phrase "śaṅku-dviguṇa-mānena" refers to the length of the string by which the circle is drawn, that is the radius of the circle and not diameter. Bruno Dagens explains the reason of this strange interpretation in a footnote of his French translation, which can be rendered into English as follows.

"*Śaṅkudviguṇamānena:* It cannot be related with the radius; because the latter then being two cubits *(aratni),* the circle would tangent at the borders of the levelled square, and there would not remain the space for drawing complementary circles which enable to fix the north south line and the intermediate directions."[27]

Here, Bruno Dagens refers to the largest type of the gnomon, which is indeed one *aratni* (= 24 *aṅgulas*) long (*Mayamata*, VI.4a). The length of a side of the levelled square is indeed one *daṇḍa* (= 4 *aratnis* = 96 *aṅgulas*) (*Mayamata*, VI. 3a). So, B. Dagens is right is saying that the circle of two-*aratni* radius tangents the levelled square. However, this fact does not imply that B. Dagens's interpretation is right. After the east west line is drawn, fish-figures for obtaining the north-south line etc.

can be drawn without much space outside the circle, because there is enough space for the constructions inside the circle. It is rather likely that the dimension of the levelled square was chosen only to contain the circle for the largest gnomon. So, there is no reason to doubt that the phrase *"śaṅku-dviguṇa-mānena"* means "by the *radius* of the double of length of the gnomon", which is the most natural interpretation of Sanskrit.

Now we shall proceed to estimate the observer's latitude. According to *apacchāyā*-table, the *apacchāyā* is zero during the last 10 days of the month Meṣa (Aries) and the first 10 days of the month Vṛṣa (Taurus). It means that the correction is zero when the sun's tropical longitude is 20°-30° and 30°-40°. As the radius of the level circle is a double-length of the gnomon, the sun's altitude is arctan (1/2) when the tip of the shadow touches the circle. From these facts, we can calculate as follows.[28] If the sun's direction is exactly the east or west at the time when its altitude is arctan (1/2) during the days when the sun's longitude is 20°-30°, the most suitable latitude is about 18°-26°. During the days when the sun's longitude is 30°-40°, the most suitable latitude is about 26°-35°.

Now we shall calculate the actual amount of the correction (the chord of PW or P'E in Fig. 8). In order to calculate this amount, we should know the amount of the radius of the level circle, but it is not specified in the text. Since the length of the gnomon may be 24, 18, 12 or even 9 *aṅgulas,* the radius of the circle can be 48, 36, 24 or 18 *aṅgulas.*

Let us see the actual amount of the correction at the winter solstice at the latitude of 18°, 26°, and 35° for various radii.[29]

latitude	r = 48	r = 36	r = 24	r = 18	r = 12
18°	32.2	24.1	16.1	12.1	8.0
26°	38.9	29.2	19.4	14.6	9.7
35°	50.5	37.9	25.3	18.9	12.6

According to this table, only the radius of 12 *aṅgulas* seems to fit the *apacchāyā,* although this radius cannot be derived from the length of the gnomon mentioned in the text. I tentatively assumed that the radius of the level circle is 12 *aṅgulas,* and drew a graph (Fig. 9) of the *apacchyāyā* and the actual amount of the correction at the latitude of 18° and 26°. From this graph, it appears that the *apacchāyā* is very close to the actual correction according to our interpretation, especially at the latitude of 18° or so. It may be that the radius of the circle was regarded to be 12 *aṅgulas* for the calculation of the *apacchāyā* regardless the actual radius of the circle which varies according to the length of the gnomon.

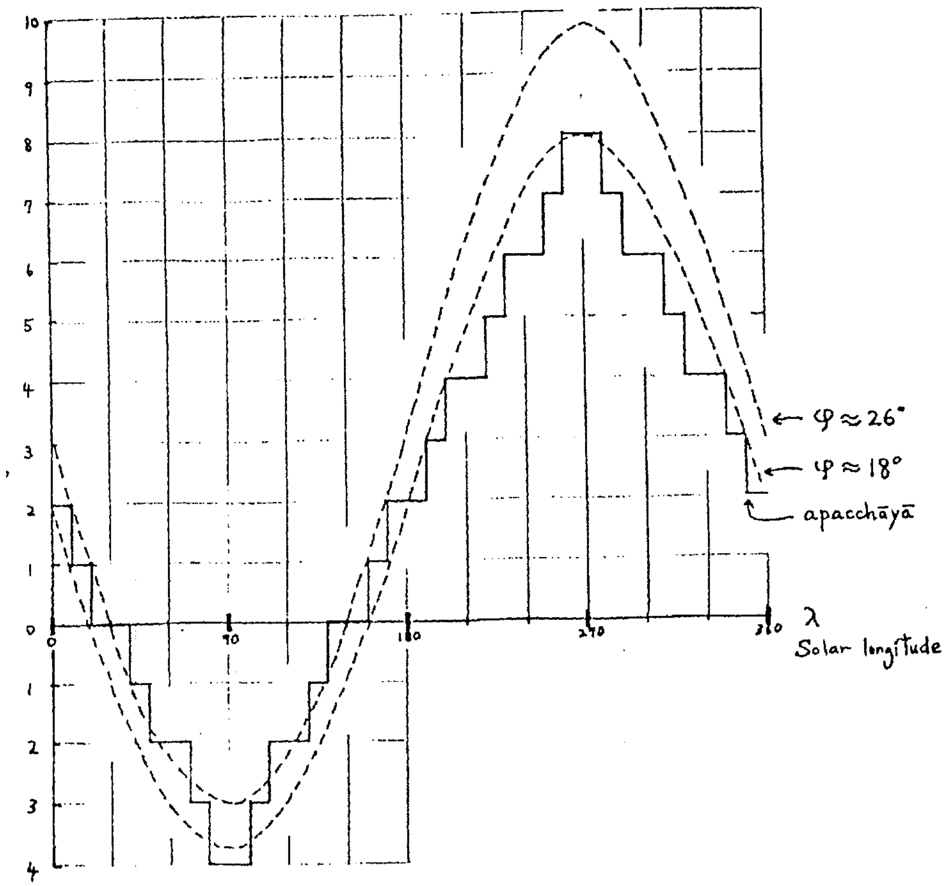


Fig. 9. Amount of the correction *(aṅgulas)* for 12 *aṅgula* radius circle

As far as we interpret that the *apacchāyā* is for the correction when the radius of the level circle is a double the length of the gnomon, the observer's latitude cannot be so much south as was suggested by J. Filliozat etc., because our estimation of the observer's latitude holds whatever the actual amount of *aṅgulas* of the gnomon and circle may be. The fact itself that the radius of the level circle is the double the length of the gnomon also suggests that the latitude of the observer was not so much south. If the radius of the level circle is the same as the length of the gnomon, the tip of the shadow will not cross the circle in winter at higher latitude than 21.5°. Even at slightly lower latitude, the observation will be quite inconvenient in winter, because the tip of the shadow will almost tangent the circle. Thus, larger level circle is more suitable for higher latitude, and the observer at higher latitude will naturally use larger level circle.

c) The determination of time

(1) Rough methods

The theory of the gnomon is explained in the section of mathematics of some Siddhāntas also. The *Gaṇita-adhyāya* (chapter on mathematics) of the *Brāhma-sphuṭa-siddhānta* of Brahmagupta gives a rough method to obtain time as follows (XII.52 (i)).[30)]

छायानरसैकहृतं द्युदलं प्रागपरयोर्द्युगतशेषम् ।

"The half day being divided by the shadow (measured in lengths of the gnomon) added to one, the quotient is the elapsed or the remaining portion of day, morning or evening." (Translated by H.T. Colebrooke)[31)]

This rule can be expressed as follows.

$$\text{time} = \frac{\frac{d}{2}}{1 + \frac{s}{g}}, \qquad \text{——(1)}$$

where *d* is the day-length, *s* the shadow length, and *g* the gnomon length. This rule is based on an old theory of Vedāṅga period mentioned in the *Artha-śāstra* (II.20.39-40) etc., which was originally meant for the summer solstitial day.[32)]

The incorrectness of Brahmagupta's methods was criticized by the commentator Caturvedācārya Pṛthūdakasvāmin (fl. AD 864) that it is incorrect even on the equator.[33)]

Similar rule was also given in the *Gaṇita-sāra-saṅgraha* (IX.8(ii)-9(i))[34)] of Mahāvīra (9th century AD) and also the *Triśatikā* (65)[35)] and the *Gaṇita-pañcaviṃśī* (30)[36)] of Śrīdhara.

Somewhat better rule is found in the *Yavana-jātaka* (79.32)[37] of Sphujidhvaja, which can be expressed as follows.

$$\text{time} = \frac{g \times d/2}{g + s + s'}, \qquad \text{——(2)}$$

where *s'* is the midday-shadow. The *Yavana-jātaka* is based on the doctrine of the Greeks *(Yavanas)* as its title suggests, but it should be noted that the name of Sage

Vasiṣṭha is also mentioned there (97.3), and this fact suggests that some Indian old astronomy may also be recorded there. If we put s' = 0 in the formula (2), it gives the formula (1), and the formula (2) is considered to be the generalized formula for the whole year developed from the formula (1) which is a special formula for the summer solstitial day.[38]

The *Pañca-sidhāntikā* (IV. 48) of Varāhamihira gives the following method which is the same as the formula (2) when the gnomon lenght is 12 *aṅgulas.*[39]

$$nāḍikās = \frac{6 \times d}{12 + s - s'} \qquad \text{------(3)}$$

It is interesting that the *Pañca-siddhāntikā* (II.11) gives the method to obtain *lagna* (rising point of the ecliptic) as follows.[40]

$$lagna = \frac{36}{12 + s - s'} + L\,, \qquad \text{------(4)}$$

where L is the longitude of the sun. As *lagna* minus l roughly corresponds to the time elapsed, and 6 signs to the day-length, this formula is considered to be the same as the formula (3). The formula (4) is attributed to the *Vāsiṣṭha-samāsa-siddhānta,* and this Siddhānta seems to be based on Indian old theory developed from the Vedāṅga astronomy, although new foreign elements like *lagna* and zodiacal signs are mixed there. In this connection, we can also note that the annual variation of the day-length and the annual variation of the length of gnomon-shadow, both of which are similar to those of Vedāṅga astronomy, are also connected with the *Vāsiṣṭha-samāsa-siddhānta* in the *Pañca-siddhāntikā* (chap. II).

The *Gaṇita-sāra-saṅgraha* (IX.15(ii)-16(i)) of Mahāvīra[41] gives the following method to obtain time expressed as portion of a day, which is the same as formulae (2) and (3) in principle.

$$\text{time} = \frac{6}{2 \times (g + s - s')} \qquad \text{------(5)}$$

The above methods are only rough methods, and Varāhamihira and Brahmagupta of course knew the exact method to obtain time as we shall see below.

(2) Exact methods

The exact time can be calculated from the sun's altitude. The principle of the calculation has been explained in the *Tripraśnādhyāya* of Siddhāntas. This calculation can be explained as an application of the orthographic projection of the celestial sphere onto the plane of the meridian and onto the plane of the equator. (See Fig. 10 (a and b)). The process of the calculation is as follows.

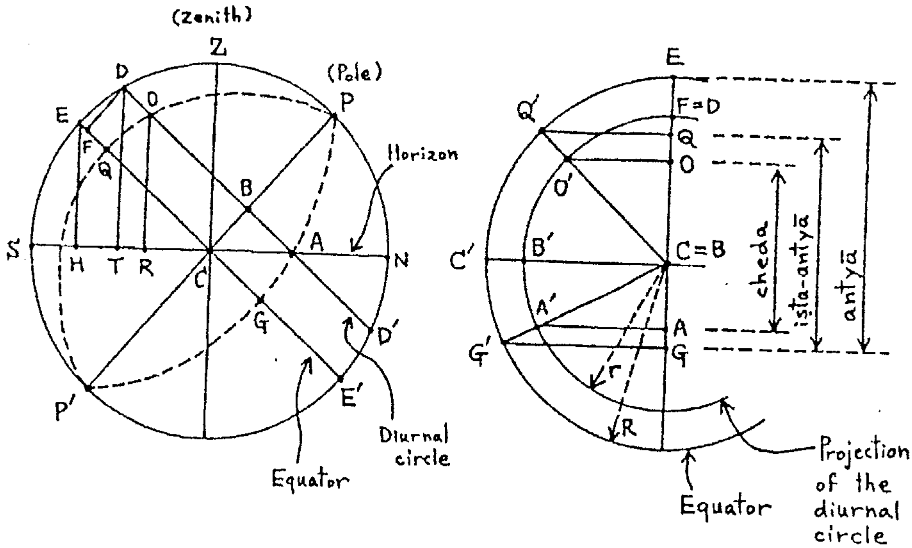


Fig. 10. (a) Orthographic projection onto the plane of the meridian; O: Projection of the sun. (b) Orthographic projection onto the plane of the equator; O′: Projection of the sun.

Firstly, the R sine of the sun's altitude should be calculated from the length of the shadow of the 12-*aṅgula* gnomon. Let *a* be the sun's altitude, and *s* the length of the shadow of the 12-*aṅgula* gnomon. Then,

$$\text{R.sin a} = \text{R} \times \frac{12}{\sqrt{12^2 + s^2}} = \textit{śaṅku} \,. \qquad \text{——(1)}$$

The R.sin a is called *śaṅku* (not to be confused with the gnomon itself which is also called *śaṅku*), and is equal to the segment OR in Fig. 10(a).

From the R.sin a, the quantity called *cheda* ("divisor") is calculated. The *cheda* is equal to the segment AO in Fig. 10. Since the angle AOR in Fig. 10(a) is equal to the observer's latitude, it is clear from the figure that

$$\textit{cheda} = \textit{śaṅku} \times \frac{\text{R}}{\textit{lamba}} \,, \qquad \text{——(2)}$$

*where the lamba is the R.cosine of the obs*erver's latitude. The *lamba* is equal to the segment EH in Fig. 10(a).

In order to make correction due to the ascensional difference, the quantity called *kujyā* (= *kṣitijyā*, "earth sine") is used. The *kujyā* is equal to the segment AB in Fig. 10. In the Fig. 10(a), it is clear that

$12 : S = EH : HC = BC : AB$, and

$BC = R.\sin \delta$,

where δ is the sun's declination, and S the equinoctial midday shadow. So,

$$kujy\bar{a} = \frac{S \times R.\sin \delta}{12} . \qquad \text{———(3)}$$

From the *cheda* and *kujyā* obtained as above, the segment BO in Fig. 10 is obtained as :

$$BO = cheda \pm kujy\bar{a} . \qquad \text{———(4)}$$

(The *kujyā* is added when δ is south, and subtracted when δ is north.)

Now, from this amount of BO, the segment CQ, which is the R.sine of the arc corresponding to the time elapsed from or remaining until the sun's crossing with the 6 o'clock line, is obtained. For this purpose, the *dyu-jyā* (*dina-vyāsadala*, "day-radius") is used. Let *r* be the *dyu-jyā*. It is equal to the segment BD in Fig. 10. So, we have

$$r = R.\cos \delta = R - R.\text{vers } \delta . \qquad \text{———(5)}$$

Using this amount, we can calculate the amount of CQ as :

$$CQ = \frac{R}{r} BO. \qquad \text{———(6)}$$

From the amount CQ, its corresponding arc C'Q' is obtained from sine table. To this amount, the ascensional difference *(cara)* (C'G' in Fig. 10(b)) is corrected. The R.sine of the ascensional difference (CG in Fig. 10) is obtained as follows.

$$CG = kujy\bar{a} \times \frac{R}{r} . \qquad \text{———(7)}$$

From this amount, the *cara* is calculated as its corresponding arc. Now, the arc Q'G' in Fig. 10(b) is obtained as follows.

$$Q'G' = C'Q' \pm cara . \qquad \text{———(8)}$$

(The *cara* is added when δ is north, and subtracted when δ is south.)

The result Q'G' is the arc corresponding to the time since sunrise or until sunset. So, the time since sunrise or until sunset in terms of *nāḍīs* is :

$$\text{time} = \frac{60}{360} \text{Q'G'}. \qquad \text{——(9)}$$

This is the method given in the *Pañca-siddhāntikā* (IV.45 47), *Mahā-bhāskarīya* (III.27-28a), *Laghu-bhāskarīya* (III.12-15), *Brāhma-sphuṭa-siddhānta* (III.38-40), *Vateśvara-siddhānta* (III.x.26(ii)-27, and 33), and *Siddhānta-śekhara* (IV.51 52).

By this method, the time until midday or since midday can also be obtained. Using the amount of CQ obtained by the equation (6), the R.versed sine of the sun's hour angle can be calculated as follows. Let h be the sun's hour angle (the angle ECQ' in Fig. 10(b)). Then, we have

$$\text{R.vers h} = \text{R} - \text{CQ}. \qquad \text{——(10)}$$

From this amount, the sun's hour angle *(nata)* is obtained from sine-table. Its corresponding time is the time until midday or since midday. This method is given in the *Brāhma-sphuṭa-siddhānta* (III-40(ii)) and *Siddhānta-śekhara* (IV.52(ii)).

It may be noted here that the *cheda* can be obtained in some alternative ways besides the equation (2). One method to use the "hypotenuse" (of the shadow and gnomon) at equinoctial midday and at desired time of desired day is as follows.

$$\textit{cheda} = \text{R} \times \frac{\text{equinoctial-midday-hypotenuse}}{\text{desired-hypotenuse}} \qquad \text{——(11)}$$

The rationale of this method is as follows. From the definition of the "hypotenuse". we have

$$\text{equinoctial-midday-hypotenuse} = 12 \times \frac{\text{R}}{\text{lamba}}, \text{ and}$$

$$\text{desired hypotenuse} = 12 \times \frac{\text{R}}{\text{R.sin a}},$$

where a is the sun's altitude at desired time. From these relations, it is clear that the equation (11) is equivalent to the equation (2). The equation (11) is used in the *Brāhma-sphuṭa-siddhānta* (III.38-40) and *Siddhānta-śekhara* (IV.51-52) (see above), and also in the *Brāhma-sphuṭa-siddhānta* (III.41-42) and *Śiṣyadhī-vṛddhida-tantra* (IV.31- 32) (see below).

There is an alternative method to obtain time, where the *iṣṭa-antyā* is used. The *iṣṭa-antyā* (or *sva-antyā*, the *antyā* at desired time) is the segment GQ in Fig. 10(b). The *iṣṭa-antyā* at midday (GE in the figure) is simply called *antyā*. There are two methods to obtain the *iṣṭa-antyā*. One method is as follows.

$$\text{iṣṭa-antyā} = GQ = \text{cheda} \times \frac{R}{r}. \qquad \text{———(12)}$$

This relations is clear from Fig. 10(b), because

$$AO : GQ = BO : CQ = BA : CG = r : R.$$

After obtaining the *iṣṭa-antyā*, it is corrected by the R.sine of the ascensional difference (CG in the figure), and the segment CQ is obtained. From the amount of CQ, the time is calculated as before. This is the method given in the *Mahā-bhāskarīya* (III.28b-29), *Brāhma-sphuṭa-siddhānta* (III. 41-42), *Śiṣyadhī-vṛddhida-tantra* (IV. 31-32), *Vaṭeśvara-siddhānta* (III.x.32 and 34), and *Siddhānta-śekhara* (IV.53-54(i)).

By this method, the R.versed sine of the sun's hour angle is obtained as follows.

$$\text{R.vers } h = \text{antyā} - \text{iṣṭa-antyā}, \qquad \text{-----(13)}$$

where *h* is the sun's hour angle. From this amount, the time until midday or since midday is obtained as before. This is the method given in the *Bhāhma-sphuṭa-siddhānta* (III.43), *Siddhānta-śekhara* (IV.54(ii)), and also in the *Sūrya-siddhānta* (III.37-39).

The other method to obtain the *iṣṭa-antyā* is as follows.

$$\text{iṣṭa antyā} = \frac{\text{midday-hypotenuse}}{\text{desired-hypotenuse}} \times \text{antyā} \qquad \text{-----(14)}$$

The rationale of this method is as follows. From the definition of the "hypotenuse", it is clear from Fig.10. that

$$\text{midday-hypotenuse} = 12 \times \frac{R}{DT}, \text{ and}$$

$$\text{desired-hypotenuse} = 12 \times \frac{R}{OR}.$$

Therefore,

$$\frac{\text{midday-hypotenuse}}{\text{desired-hypotenuse}} = \frac{OR}{DT} = \frac{OA}{DA} = \frac{QG}{EG} = \frac{\text{iṣṭa-antyā}}{\text{antyā}}$$

From this relation, the equation (14) can be derived. From the *iṣṭa-antyā*, the R.versed-sine of the sun's hour angle is obtained as :

$$\text{R.vers } h = \text{antyā} - \text{iṣṭa-antyā},$$

and the time is obtained as before. This is the method in the *Brāhma-sphuṭa-siddhānta* (III.44-45), *Khaṇḍa-khādyaka* (I.iii.15-16), *Śiṣyadhī-vṛddhida-tantra* (IV.33), *Vaṭeśvara-siddhānta* (III.x.29), *Siddhānta-śekhara* (IV.55-56), *Siddhānta-śiromaṇi* (*Grahagaṇita*, III.66-68), and *Mahā-siddhānta* (IV.31).

iv) The horizontal gnomon

The gnomon which we have discussed above is the vertical gnomon, and it is the standard gnomon in most of all classical Siddhāntas. The horizontal gnomon is, however, described in the *Vṛddha-vasiṣṭha-siddhānta*.

The *Vṛddha-vasiṣṭha-siddhānta* (III.61-62) reads as follows.[1)]

अथ दिनदलचिह्नां लम्बितां यष्टिकां वा
तदुपरितनशङ्कुं यष्टिकार्कांशतुल्यम्।
विरचय तदधऽऽधो (–धोऽधो?) ऽङ्कं विलोमप्रभाभि–
र्दिनगतघटिकैष्याः शङ्कुभा यत्र लग्ना।।61।।

अर्काहतोन्नतज्या नतजीवाप्ता ऽङ्गुलादिका वामम्।
छाया त्रिज्यार्कहता नतजीवाप्ता च कर्णः स्यात्।।62।।

"Now, a vertical rod (*yaṣṭi*) which has marks of [*ghaṭikās* of] a half-day is made. At its top is a gnomon (*śaṅku*), whose length is one twelfh of the rod. Its lower part is marked for the reversed shadow. The mark where the gnomon-shadow falls indicates the *ghaṭikās* elapsed or to elapse.

The R.sine of the [sun's] altitude is multiplied by 12, and divided by the R.sine of the zenith distance. The result is [the length of] the shadow in reverse in terms of *aṅgulas*.

The Radius is multiplied by 12, and divided by the R.sine of the zenith distance. The result is the hypotenuse."

The "reverse shadow" means the vertical shadow of the horizontal gnomon, in contrast with the ordinary horizontal shadow of a vertical gnomon.

This text can be explained as follows. (see Fig. 11). Let Z be the R.sine of the zenith distance of the sun, and A the R.sine of the altitude of the sun. Then we have the following proportions.

12 : shadow = Z : A, and

hypotenuse : 12 = R : Z.

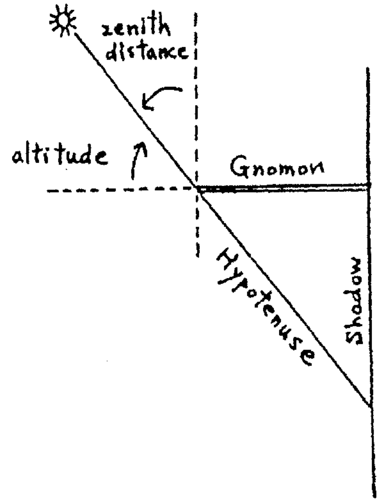


Fig. 11. Sundial

Therefore, we have

shadow = 12 x A/Z, and

hypotenuse = 12 x R/Z.

These are the formulae given in the text. This instrument is a kind of the sundial.

The horizontal gnomon was used in the cylindrical sundial used in Delhi Sultanate and Mughal periods.[2)] The horizontal gnomon of the *Vṛddha-vasiṣṭha-siddhānta* may be a forerunner of that type of later sundial.

4. The Graduated Level Cricle and Orthographic Projection

i) Introduction

We have seen in the previous section that the theory of the gnomon is related with orthographic projection of the celestial sphere. Sometimes orthographic projection of the celestial sphere is represented on the graduated level circle, and astronomical observations and calcuations are carried on with the help of this circle. Let us see some early forms of the orthographic projection of the celestial sphere on the level circle which is related to astronomical observations. In this section, we shall discuss the shadow-instrument of Āryabhaṭa, Lalla, and Śrīpati, and the *chatra-yantra* of Āryabhaṭa, and lastly the observation and graphical calculation on the graduated level circle described by Varāhamihira.

ii) The shadow-instrument

a) The *chāyā-yatra* of Āryabhaṭa

A fragment of the *Āryabhaṭa-siddhānta* of Āryabhaṭa, quoted by Rāmakṛṣṇa Ārādhya in his commentary on the *Sūrya-siddhānta*, has a description of the *chāyā-yantra* (shadow-instrument) as follows.[1]

दिङ्मध्यात्सप्तपञ्चाशदङ्गुलैस्त्रिज्यकांशकैः।
लिखेद्वृत्तं च चक्रांशचिह्नितं सममण्डलम्॥1॥

चराग्रज्याद्युनाडीभिः छायायन्त्राणि साधयेत्।
समवृत्तविदिक्छायाकर्णाभ्यां क्रान्तिदोर्गुणाः॥2॥

समवृत्ते स्वदिश्यग्रां दद्यात् प्राच्यपराशयोः।
चरज्यानामथाग्रांकान् दिङ्मध्यात् स्वदिशि न्यसेत्॥3॥

तदग्रविन्दुतो वृत्तं वृत्तान्ताग्रं लिखेत्तु तत्।
स्वाहोरात्रदलं तत्र स्पष्टा नाड्यः स्वशङ्कुभिः॥4॥

स्वाहोरात्रदलेऽंशाः स्युः षड्गुणा दिननाडिकाः।
अग्रान्ते ऽस्तोदयार्कौ च याम्यार्धे पूर्वपश्चिमे॥5॥

तत्पूर्वापररेखातो दक्षिणार्धं च तत्स्मृतम्।

"1. Construct a perfect circle (*samamaṇḍalaṃ vṛttam*) with radius equal to 57 *aṅgulas*, the number of degrees in a radian, and on (the circumference of) it mark the 360 divisions of degrees.

2. Then construct shadow-instument (for every day of the year) with the help of the Rsine of the Sun's ascensional difference, the Rsine of the Sun's *agrā*, and the *nāḍīs* of the duration of the day (in the following manner) :

   Determine the Rsines of the Sun's declination and of the Sun's longitude from the *samavṛttacchāyākarṇa* (i.e. the hypotenuse of the shadow of the gnomon when the Sun is on the prime vertical) or from the *vidikchāyākarṇa* (i.e. the hypotenuse of the shadow when th Sun is in a mid direction).

3. On the perfect circle (drawn above), lay off the (Sun's) *agrā* in its own direction (north or south) in the east as well as in the west (and at each place put down a point). Again lay off the (Sun's) *agrā* corresponding to the Sun's ascensional difference in its own direction from the centre of the circle.

4. With that end of the (Sun's) *agrā* (as centre) draw a circle passing through the (two) points marked on the circle: this circle denotes the Sun's diurnal circle. On (the southern half of) that circle, put down marks indicating true *ghaṭīs* with the help of the corresponding positions of the gnomon. (In each position the gnomon is to be held in such a way that the end of the shadow may lie at the centre of the circle.)

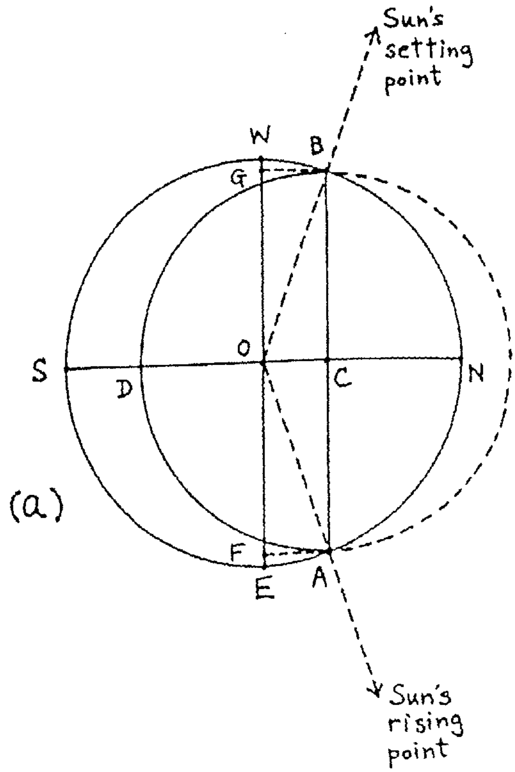
5. These *ghaṭīs* of the day, multiplied by six, are the degrees on the diurnal circle. At the two points marked at the ends of the Sun's *agrā* in the east and west, are the positions of the Sun at rising and setting.

6 (i). Half of the diurnal circle lying towards the south of the rising-setting line (of the sun) is called the southern half of the diurnal circle." (Translated by K.S. Shukla)[2)]

This *chāyā-yantra* can be explained as follows.[3)] (See Fig. 12.) Firstly, one should draw a level circle (SENW) with readius of 57 *aṅgulas*, and graduate its circumference with degrees. The points S, E, N, and W are the south, east, north, and west points respectively. Then he should mark the end of the sun's amplitude (*agrā*) at the points A and B in the east and west in proper direction (north or south). And also, he sould mark the point C on the north-south line at the distance equal to the R.sine of the sun's amplitude (= OC) from the centre O. With the point C as centre, he should draw a circle (ADB) which passes through the points A and B. The semi-circle ADB, which has been drawn towards the south, is called the diurnal circle. The line AB is the sun's rising-setting line. The point A corresponds to the sun's rising point, and the point B the sun's setting point. On the semi circle ADB, the marks of *ghaṭīs* should be made in such a way that at that moment the shadow of the gnomon, which is placed at the mark, lies at the centre of the level circle is O. By these marks the time can be determined, but this figure should be changed every day.

In the above quotation, the construction of the circle ADB, especially the method of fixing the point C, is unique in this *Āryabhaṭa-siddhānta*, and different from the methods of Lalla and Śrīpati which we shall discuss below. So, let us read a part of the comment of Rāmakṛṣṇa Ārādhya, given after the quotation from the *Āryabhaṭa-siddhānta*, in order to confirm the construction of Āryabhaṭa's *chāyā-yantra*. An extract from Rāmakṛṣṇa Ārādhya is as folows.[4)]

अर्काग्राग्रं त्रिज्यावर्गात्त्यक्त्वा मूलं कोटिः स्यात्।
दिङ्मध्यात्कोटिं पूर्वापरयोः प्रसार्य कोट्यग्रादर्काग्रं
स्वदिशि प्रसार्य समवृत्तपरिधौ न्यसेत्।
चरज्यानामर्काग्रं दिङ्मध्यात् स्वदिशि दक्षिणो–
त्तररेखायां प्रसार्य तदग्रे बिन्दुं दद्यात्।
बिन्दुतो वृत्तमर्काग्रबिन्दुद्वयपर्यन्तं दक्षिणभागे
विलिखेत्। तदर्धं वृत्तं स्वाहोरात्रार्धमुच्यते।



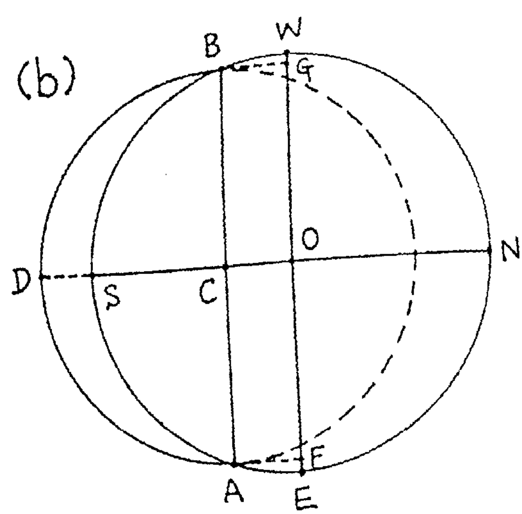


Fig. 12. Āryabhaṭa's Chāyā-yantra

"[The square of] the R.sine of the sun's amplitude (lit.: the end of the sun's *agrā*) is subtracted from the square of the Radius, and the square root [of the result] is the *koṭi*. From the centre of the circle, the *koṭi* is stretched eastwards and westwards; and from the end of the *koṭi*, the end of the sun ['s amplitude] is stretched in its own direction; and one should lay it off at the circumference of the perfect circle. The end of the sun ['s amplitude] corresponding to the sun's ascensional difference is stretched from the centre of the circle in its own direction on the south-north line, one should make a mark at its end. With the end [as centre], one should draw a circle towards the south upto the two marks of the end of the sun ['s amplitude]. This semi-circle is called a half of own diurnal circle."

From the above quotation, the method of construction of the *chāyā-yantra* is clearly understood (See Fig. 12 again). The R.sine of the sun's amplitude is equal to the segments GB and FA, when the Radius is OW, and the *koṭi* is equal to the segments OG and OF.

This *chāyā-yantra* appears to be an attempt to represent the sun's diurnal circle on the level circle, but the represented diurnal circle is not a projection of the actual diurnal circle on the celestial sphere. The represented visible diurnal circle ADB is a semi-circle, although the actual visisble diurnal circle on the celestial sphere is not a semi-circle except in iquinoctial days. And also, the represented diurnal circle is drawn outside the level circle when the sun's declination is south. (See Fig. 12(b).) This also may be considered to be a defect, because it is proper to represent the diurnal circle inside the level circle. These defects are removed in the improved shadow-instrument described by Lalla and Śrīpati as we shall see below.

b) The shadow-instrument of Lalla and Śrīpati

Lalla and Śrīpati described the shadow instrument without naming it. Lalla described the shadow instument in his *Śiṣyadhī-vṛddhida-tantra* (XXI. 44-47) as follows.[5)]

नतजीवाग्राजीवे षष्टिहते त्रिज्यया हृते न्यासे।
षष्ट्यङ्गुलकृतवृत्ते केन्द्रात् प्रागपरतश्चपि (श्चापि)।।44।।

स्वदिशि प्राग्वद् बिन्दुत्रयेण शङ्कुभ्रमं लिखेद् वृत्तम्।
छायावृत्तं दिग्व्यत्ययेन दिक्साधनं कृत्वा।।45।।

अग्राग्राच्छङ्कुभ्रमवृत्ते कालांशकैर्लिखेद् राशिम्।
दिङ्मध्यच्छायाग्रं कृत्वात्र स्थापयेच्छङ्कुम्।।46।।

अग्राग्राच्छङ्कुतलान्तरस्थिता वास्तुमुद्गता भागाः।
कालांशाः षट्कहृता भवन्ति घटिका दिनस्य गताः।।47।।

"44-45. (Severally) multiply the R.sines of the (Sun's meridian) zentith distance and *agrā* by 60 and divide (each product) by the radius. Lay them off, in their own directions, in a circle drawn (on level ground) with 60 *aṅgulas* as radius (and having the directions marked in it), the former from the centre and the latter from the east as well as the west points. And through the three points (thus obtained) draw, as before, the circle denoting the path of the gnomon. By laying off the same in the contrary directions, one may draw the circle (denoting the path of the tip) of the shadow (of the gnomon).

46. Beginning with the end of the *agrā* (laid off in the east), graduate the circle denoting the path of the gnomon with the (appropirate) signs by means of the degrees of time (of their oblique ascension). And then set up a gnomon (on that circle) in such a way that the tip of the shadow may fall at the centre.

47. Then the degrees of time lying from the end of the *agrā* (in the east) up to the foot of the gnomon are the degrees of time of the (Sun's) ascension. These degrees of time as divided by six are the *ghaṭīs* elapsed in the day (since sunrise)."(Translated by K.S. Shukla).[6]

This is a device to represent the sun's circle on the level circle. The same device was described by Śrīpati in his *Siddhānta-śekhara* (XIX.23 (ii)-25) as follows.[7]

अग्राग्रभागान्तभागमौर्वी
कार्येह खल्व (खर्त्व?)ङ्गुलवृत्तजाता।।23।।

न्यस्येदग्रां प्राक्प्रतीच्यग्रतोऽत्र
याम्योदक्स्था मध्यदेशान्तज्या।
साध्यः शङ्कुस्तन्मितिभ्यां भ्रमस्तु
देयस्तस्मिन् स्वोदयात् स्वाग्रकाग्रात्।।24।।

विरचितसमयांशस्तन्मितं शङ्कुमस्मिन्
तदुदरगतभाग्रं स्थापयेदग्रकाग्रात्।
तदवधि विगतास्ते कालभागा भवेयु–
र्दिनगतघटिकाः स्युः कालभागा रसाप्ताः।।25।।

"From the end of the amplitude (*agrā-agra*), the path of the R.sine of the zenith distance [of the sun], which is constructed on the circle of 60-*aṅgula* radius (?), should be drawn.

One should draw it from the east or west end towards the other end. Here, the R.sine of the [midday] zenith distance is measured on the north-south line from the centre.

The gnomon should be made according to the two measures (the amplitude and the R.sine of the zenith distance.). A circle with the graduation of degrees corresponding to the time should be drawn there from the end of the *agrā* that is tyhe rising point. The gnomon thus measured (gnomon whose length corresponds to the R.sine of the sun's altitude) should be erected there in such a way that the tip of the shadow falls at the centre (of the level circle). The distance from the end of the *agrā* up to it (foot of the gnomon) is the degrees corresponding to the time. The degrees divided by 6 is the *nāḍīs* elapsed since sunrise.''

From the above quotations, it is seen that the device described by Lalla and Sripati is basically the same, and is device to represent orthographic projection of the sun's diurnal circle on the level circle.(See Fig.13.)

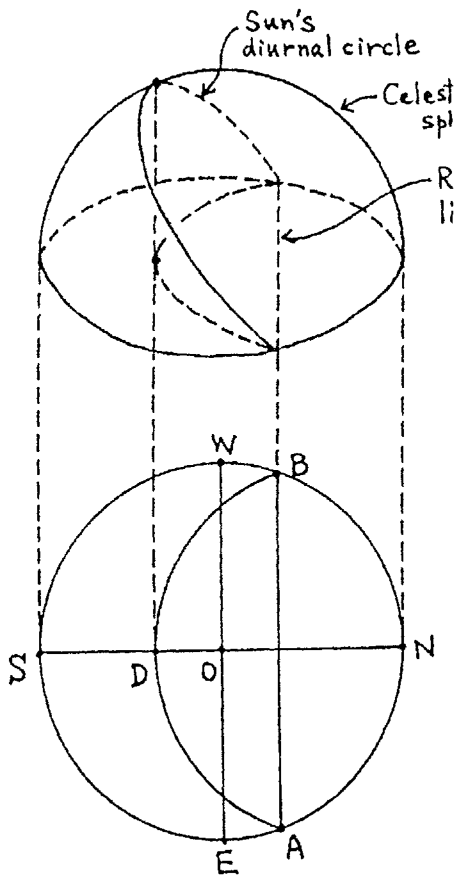


Fig. 13. Lalla and Śrīpati's shadow instrument

In Fig. 13, the circle ESWN is the level circle with the centre O. The circle ADB is the represented diurnal circle, and the points A and B are the sun's risng and setting points respectively. The segment OD is equal to the R.sine of the sun's midday zenith distance. So, it is clear from the figure that three points A, D, and B are exactly orthographic projection of three points of the actual diurnal circle on the celestial sphere at sunrise, midday, and sunset respectively. In reality, orthographic projection of the diurnal circle is ellipse and not circle, but the represented diurnal circle ADB has roughly been considered to be a circle by Lalla and Śrīpati. Practically, this will be a reasonable approximation. On the circle ADB, a gnomon is placed in such a way that its shadow falls on the centre O. If the end of the shadow should be cast on the centre O, as directed by Lalla and Sripati, the height of the gnomon hould be equal to the R.sine of the sun's altitude, and should be changed at every moment. This is probably a device to show sphereics rather than a practical instrument for observation.

iii) Āryabhaṭa's chatra-yantra

In the fragment of the *Āryabhaṭa-siddhānta,* quoted by Rāmakṛṣṇa Ārādhya in his commentary on the *Sūrya-siddhānta,* Āryabhaṭa comments as follows[1], after describing the *chāyā-yantra* and also the *dhanur-,yaṣṭi-*, and *cakra-yantra* which we shall discuss later.

शङ्कुभ्रमप्रकारेण प्रोक्ता नाड्यश्च तत्प्रभा।
अधुना भाभ्रमान्नाड्यः तच्छाया च कथ्यते।।12।।

"Above have been stated the methods of obtaining the shadow of the gnomon and the *nāḍīs* (elapsed in the day) by the movement of the gnomon. Now will be stated the method of finding the *nāḍīs* (elapsed) and also the shadow (of the gnomon) by the movement of the shadow of the gnomon (set up at the centre of the circle)." (Translated by K.S. Shukla).[2]

It appears that the *chatra-yantra* is closely related to the *chāyā-yantra*, and it is also kind of representation of the celestial sphere. The *chatra-yantra* is also constructed on the graduated level circle, but the gnomon is fixed at the centre of the circel.

The description of the *chatra-yantra* ("umbrella instrument") in the *Āryabhaṭa-siddhānta* of Āryabhaṭa, quoted in Rāmākṛṣṇa Ārādhya's commentary on the *Sūrya-siddhānta,* is as follows.[3]

छत्रं वेणुशलाकाभिः कृत्वा चक्रांशसंख्यया।
दिङ्मध्ये समवृत्तञ्च कल्पयेच्छत्रयन्त्रकम्।।13।।

छत्रदण्डञ्च तन्मध्ये व्यासार्धं शङ्कुरेव सः।
स्वाहोरात्रदलं सौम्यं व्यस्ताग्रं भाभ्रमाह्वयम्।।14।।

षड्गुणा दिननाड्यो ऽंशाः सौम्यार्द्धे छत्रयन्त्रतः।
अग्रान्ते ऽर्कोदयास्ते च प्रत्यक्प्राक् प्रभा स्थिता।।15।।

तत्प्रत्यगन्तमस्ताख्यं प्रागन्तमुदयाह्वयम् ।
अस्ताख्याददुयस्यान्तं छायाकालांशकाः स्थिताः ।।16।।

छत्रमध्यस्थशङ्कोस्तु छायैवेष्टप्रभा सदा ।
छायाग्रास्ताख्यमध्यांशा षड्भिर्नाड्यो दिवा गताः ।।17।।

"13. Construct a *chatra-yantra* ("an instrument resembling an umbrella") by bamboo-needles, mark (the circumference of) it with 360 divisions of degrees, and set it at the centre of the (perfect) circle. Or, treat the perfect circle itself as a *chatra-yantra.*

14. The rod of the *chatra-yantra,* in the middle of it, equal to the radius, is the gnomon; the northern half of the diurnal circle drawn through the end-points of the (Sun's) *agrā,* laid off in the contrary direction (in the west and the east), is the so called "path of shadow".

15. The *nāḍīs* of the day, multiplied by six, are the degrees in (the diurnal circle lying in) the north half of the *chatra-yantra.* Towards the end-points of the (Sun's) *agrā*, in the west and the east, falls the shadow at sunrise and sunest respectively.

16. The end (of the Sun's *agrā)* in the west is (therefore) called the "setting point (*asta*)"; and the end (of the Sun's *agrā*) in the east the "rising point (*udaya)*". From the "setting point" to the "rising point" (on the northern half of the diurnal circle) lie (the graduations of) the degrees of time in a *chatra-yantra.*

17. The shadow cast by the gnomon, situated in the middle of the *chatra,* is always the shadow for the desired time. The degrees (on the diurnal circle) intervening between the end of the shadow and the "setting point", divided by six, give the *nāḍīs* elapsed in the day." (Translated by K.S.Shukla)[4)]

Tamma Yajvan also briefly mentioned the *chatra-yantra* of Āryabhaṭa in his commentary on the *Sūrya-siddhānta* as follows.[5)]

छत्रयन्त्रमपि चक्रवदेव कृत्वा मध्ये दण्डयुक्तं कल्पयेत् ।
तद्दण्डं त्रिज्याङ्गुलप्रमाणं कल्पयेत् ।

"The *Chatra-yantra* (i.e. the Umbrella) is constructed like the *Cakra-yantra* with a vertical rod at its centre. The rod should be made as big as there are digits in a radian". (Translated by K.S. Shukla)[6)].

From the above quotations, it is seen that the *chatra-yantra* of Āryabhaṭa consists of a rod and a circle whose circumference is graduated with degrees. The height of the rod is equal to the radius of the circle, and the rod is fixed at the centre of the circle which is horizontal. Āryabhaṭa says to draw so called "path of shadow" on the ground through the end-points of the sun's *agrā,* but evidently this is not the actual path of the shadow, because the actual path of the shadow is hyperbola. It may be that the "path of shadow" is a circle only to indicate the direction of the shadow regardless the length of the shadow. If so, it can be drawn through the end-points of the sun's *agrā*, just like the *chāyā-yantra* of Āryabhaṭa which we have seen before.

The *chatra-yantra* of Āryabhaṭa appears to be similar to the *pīṭha-yantra* of later authors, which we shall see in the section of the circle-instruments and its variants later, at first sight, but there is an important difference between them. The most important feature of the *chatra-yantra* is that the height of the central rod (or gnomon) should be equal to the radius of the level circle. On the contrary, the most important component of the *pīṭha-yantra* is the graduated circle for the observation of azimuth, and the height of the central rod may be arbitrary. So, the *chatra-yantra* of Āryabhaṭa may be considered to be an attempt to represent the celestial sphere in visible form, rather than an instrument for actual observation.

iv) The graduated level circle of Varāhamihira

Varāhamihira described the graduated level circle, where graphical calculations and observations are carried on, in this *Pañca-siddhāntikā* (XIV.1-11). Thibaut and Dvivedin,[1] and Neugebauer and Pingree[2] translated this text, but both interpretations seem to be unsatisfactory, and, even after emending the text, some of the rules have remained obscure. Naugebauer and Pingree commented that some of these procedures are "incorrect". In 1987, I presented a paper in a seminar held at Calcutta,[3] and showed that all the rules of Varāhamihira give either correct values or, at least, reasonable aproximations. In fact, all verses (XIV.1-11) refer to graphical calculations carried on the graduated level circle, chiefly based on the theory of orthographic projection. So, I would like to present my interpretation here, which I first presented at the seminar, in revised form.

In the following quotations of the text, the original reading of the manuscript given in Thibaut and Dvivedin's edition[4] is shown firtly, and my emendation is shown within brackets. The usual *sandhi*-rule is applied, and the *avagraha* is silently added when necessary.

The first four verses (XIV.1-4) have been correctly translated by Thibaut and Dvivedin, and Neugebauer and Pingre. So, it will suffice to quote Thibaut and Dvivedin's translation here. The text (XIV.1-4), as emended by Thibaut and Dvivedin, and their English translation is as follows.

साशीतिकाङ्गुलशतं
विस्तीर्णवृत्तमविषमं धरित्र्याम् ।
समराश्यङ्क चिह्नं
परिधौ सापक्रमं कुर्यात् ।।1।

याम्योदक्समसूत्रा–
दपक्रमांशावगाहिभिः सूत्रैः ।
प्रथमवदंशक्षिप्तं
वृत्तत्रयमालिखेन्मध्यात् ।।2।।

अक्षे क्षिप्तां लेखां
कुर्याच्च भगणचिह्नपर्यन्ताम् ।
अक्षोत्तरलेखान्तर–
मपक्रमांशोत्थमादाय ।।3।।

द्विगुणं प्रसार्य वृत्ते
स्वे दिक् तच्चापांशदलाभ्यस्ताः ।
प्रथमर्क्षचरविनाड्यो
ज्ञेयाः परिशेषयोर्मिश्राः ।।4।।

"1. Draw upon the ground a level circle with a diameter one hundred and eighty aṅgulis long, and mark upon its circumference the signs (degrees, etc.) at equal distances, and also the degrees of declination (of the signs).

2. Further describe from the centre (of the first circle) three other circles, taking for their Radii the string running (at right angles) from the string, which marks the north-south line, to those points on the circumference of the first circle where the degrees of declination (of the signs of the ecliptic) are marked; and mark those circles with the degrees, as you did with the first one.

3. Thereupon draw a line from the centre towards the latitude (i.e. that point of the first circle which marks the latitude of the given place), and lengthen it up to the sphere. Take that piece which is due to the declination (of a given sign of the ecliptic) and is intercepted by the line of latitude and the north-south line;

4. Double it and mark it off on the circle belonging to that sign; multiply half the degrees of the corresponding arc by ten. The result represents the vināḍikās of ascensional differences in the case of the first sign; in the case of the two other signs the vināḍikās come out mixed.'' (Translated by Thibaut and Dvivedin)[5)]

The above text can be explained as follows. (See Fig.14) Firstly, one should draw a level circle (ENWS in Fig 14) with the radius of 90 *aṅgulas,* and graduate its circumference. Then, he should mark three declination-points (one of which is B in Fig.14) corresponding to the declinations for the end-points of the first three zodiacal signs, and measured form the east or west cardinal point. ($E\hat{O}B=\delta$, where δ is the declination.)

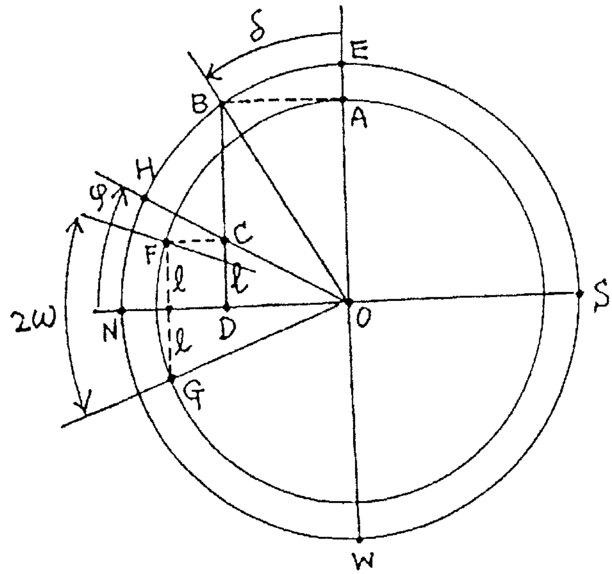


Fig. 14.

Then, he should draw a vertical line (BD) to the north-south line (NS) from the declination-point (B). So, the segment BD is equal to R.cos δ, that is the radius of the diurnal circle on the celestial sphere. The length of the segment BD is taken as the radious (OA) of a circle (AFG) which represents the declination, and such three concentric circles, correponding to three declinations, are drawn with the centre O. These three circles are also graduated with degrees.

Then, he should draw a line (OH), which is inclined by the degrees of the observer's latitude towards the side where declination-points are marked. (NÔH=φ, where φ is the observer's latitude.) Then he should take a segment (DC=l) of the line BD, which is intercepted by the lines ON and OH. Now, the amount of the segment l is expressed as:

l = R.sin δ. tan φ.

Therefore, the amount of l is equal to the *kujyā* ("earth-sine"). (This relation can be understood from Fig.10 (a). There,the segment BC is R.sin δ, the angle ACB is φ, and the segment AB is the *kujyā*. Therefore, the *kujyā* is equal to R.sin δ. tan φ.)

(See Fig.14 again). Now, he should double the segment l, and stretch it on the correspnding declinaton-circle as FG. A half of its corresponding arc is the ascensional difference (ω) of the first sign. Indeed, the amount (*kujyā* x $\frac{R}{r}$) is equal to the R.

sine of the ascensional difference, as we have seen in the previous section of the present paper, and its corresponding arc is the ascensional difference. This calculation has been done graphically. In the case of the 2nd and 3rd signs, the result is the sum of the ascensional differences of the 1st, 2nd, and 3rd signs respectively. This fact is expressed as "mixed" in the text.

Now, let us proceed to the verses (XIV.5-11). As regards these seven verses, I shall present my own translation. The verse (XIV.5) reads:

नाड्यः षड्या (षड्घ्न्यो)* भागा–
स्तज्या (स्तज्ज्या)* व्यासार्धशोधिता छाया।
साध्यंदिनी–(माध्यन्दिनी–)* समेता
नाड्यर्थे सा तया हीना।।5।।

(Note: * – emendations suggested by Thibaut and Dvivedin)

"The *nāḍīs* [since sunrise] multiplied by six are degrees. The R.versed-sine of them (*taj-jyā*) subtracted from the radius and joined to the zenith distance of the sun at midday (*mādhyandinī*) is the R.sine of the sun's zenith distance (*chāyā*). In order to find *nāḍīs*, it (R. sine of the sun's zenith distance at desired time) is to be diminished by that (R.sine of the sun's midday zenith distance)."

In this verse, the meanings of two words, *jyā* and *chāyā*, should be properly understood in this particular context.

In this verse, the word "*chāyā*" does not mean the shadow of a gnomon, but the R.sine of the sun's zenith distance, just like the word *śaṅku* (gnomon) sometimes means the R.sine of the sun's altitude. The absence of the length of the gnomon in this formula is explained only by this interpretation. The word *chāyā* is sometimes used in the same meaning in other works also. For example, the *Śiṣyadhī-vṛddhida-tantra* (XXI.49) reads as follows.[6]

"yaṣṭis trijyā karṇo lambo nā kṛtiviśeṣapadam anayoḥ /
dṛgjyā chāyā prākparalambanipātāntaram bāhuḥ // 49 //"

"The *yaṣṭi* equal to the radius, is the hyotenus; the perpendicular (dropped on the ground form the upper end of the *Yaṣṭi)* is the (great) gnomon (i.e. the Rsine of the Sun's altitude); the square root of the difference of their squares is the (great) shadow, i.e. the Rsine of the (Sun's) zenith distance; and the distance between the east-west line and the foot of the perpendicular is the *bāh*u."(Translated by K.S.Shukla)[7]

From this quotation, it is clear that the word "*chāyā*" sometimes means the R.sine of the sun's zenith distance.

The word *"jyā"*is usually used for the R.sine, and the word *"utkramajyā"* is used for the R.versed-sine. However, it seems that the word *jyā* means R.versed-sine in this verse. Thibaut and Dvivedin suggested that *jyā* means R.versed-sine in this verse, although they took *chāyā* as actual gnomon-shadow. In the next verse (XIV.6), the *jīvā* (=*jyā*) is defined as the distance between the tip of a certain *"chāyā"* and the "horizon", and this fact also obliges us to take *jīvā* or *jyā* in the sense of R.versed-sine. The word *jyā* is sometimes used for the R.versed-sine in other text also. For example, the *Mahā-siddhānta* (IV.31) of Āryabhaṭa II reads as follows.[8]

"carajīvodvṛttaśrutighātaś cāntyāhṛto dyudalakarṇaḥ /
bhakto 'bhīṣṭaśravasā phalonitāntyā natajyā syāt // 31 //"

"Divide the product of the sine of ascensional difference and the hypotenuse [of the shadow when the sun crosses] the east and west hour circle by the day-measure; [the result is] the hypotenuse [of the shadow] at noon. Divide [the same product] by the hypotenuse [of the shadow] at the given time, and reduce the day-measure by the quotient; [the result] is the [versed] sine of the hour-angle".(Translated by Sreeramula Rajeswara Sarma)[9]

This is the method to obtain the R. versed-sine of the sun's hour angle, which we have discussed in the previous section of the present paper. In the above translation of S.R.Sarma, the word "day-measure" is the translation of *antyā,* and the "[versed] sine of the hour-angle" is the translation of *nata-jyā.* So, the above rule can be expresed as follows.

$$\text{"}nata\text{-}jy\bar{a}\text{"} =anty\bar{a} - \frac{\text{midday-hypotenus x antyā}}{\text{desired-hypotenuse}}$$

In this context, it is clear that the word *"nata-jyā"* means the R. versed sine of the sun's hour angle, as was translated by S.R.Sarma. From this quotation, it is clear that the word *"jyā"* sometimes means the R. versed sine.

Assuming the above interpretation, the phrase *"tajjyā vyāsārdha-śodhitā"* can be expressed as folows.

$$R - R.\text{vers}\ (6t) = R.\cos\ (6t), \qquad \text{-----(1)}$$

where *t* is time elapsed since sunrise in terms of *nāḍīs.*

Expression (1) gives the east-west component of the R.sine of the sun's zenith distance, exactly in the equinoctial days, and roughly in other days. (In an arbitrary day, 6t should actually be substituted by the angular distance between the east cardinal point of the horizon and sun.)

Then, Varāhamihira says to "join" *(sametā)* it to the midday-*chāyā*. Presumably he meant that it is to be joined graphically to the midday-*chāyā* perpendicularly, and not arithmetically. Since the variation of the north-south component of the *chāyā* is small around midday, this method gives more or less approximate value of the *chāyā* around midday.

In mathematical expression, Varāhamihira's method gives the following formula.

$$\textit{chāyā} = \text{R.sin}\ \zeta = \sqrt{\{\text{R} - \text{R.vers}\ (6\text{t})\}^2 + \text{M}^2}$$

$$= \sqrt{\{\text{R.cos}\ (6\text{t})\}^2 + \text{M}^2}, \quad \text{-----(2)}$$

where ζ is the sun's zenith distance, and M the R.sine of the sun's midday zenith distance (*chāyā* at midday)

The correct formula for equinoctial days is as follows.

$$\text{R.sin}\ \zeta = \sqrt{\{\text{R.cos}\ (6\text{t})\}^2 + \left(\frac{\text{R.sin}\ (6\text{t})}{\text{R}} \times \text{M}\right)^2} \quad \text{-----(3)}$$

The variation of the north-south component (R.sin (6t)/R) in this formula has been omitted in Varāhamihira's formula (2).

Now, the *Pañca-siddhāntikā* (XIV.6) reads :

छायाहरिज्या–(°हरिजा)*भ्यन्तर–
जीवाचापांशषष्ठभागो यः ।
ता नाड्यः प्राग्यतः (प्राग्याताः)*
पश्चाच्छेषास् नथा (तथा)** प्राप्नो (प्राप्ताः)** ।।6।।

(Note: * – emendations suggested by Thibaut and Dvivedin. ** – emendations suggested by Neugebauer and Pingree.)

"The sixth part of the degrees of the arc corresponding to the R. versed-sine *(jīvā)*, which is the distance between the tip of the *chāyā* (so diminished) and the horizon, is *nāḍīs*. Towards the east, elapsed *nāḍīs* are obtained, and towards the west, remaining *nāḍīs* are obtained."

This is just the reverse of the previous verse (XIV.5). The "*chāyā* (so diminished)" means the east-west component of the R.sine of the zenith distance of the sun. (From the R.sin of the sun's zenith distance at desired time, the midday-*chāyā*, which is considered to be the north-south component of the *chāyā*, is graphically diminished in order to get east-west component of the *chāyā*.) The "distance between the tip of the *chāyā* and the horizon" means the expression (R–*chāyā*). Here, the expression of the "*chāyā* (so diminished)" is (R–R.vers (6t)), as is evident from our discussion on the previous verse (XIV.5). So, the value of the expression (R–*chāyā* (so diminished)) is exactly the R.versed-sine of the arc corresponding to the time elapsed since sunrise or remaining until sunset.

Now, the *Pañca-siddhāntikā* (XIV.7) reads:

तिर्यग्रेखा समद–
क्षिणोत्तरावक्रमांश– (ऽपक्रमांश–)* रेखायाः (°याम्)** ।
तच्चापांशाद्दिघ्नाः (°दिग्घ्ना)*
राश्युदयविनाडिका (°काः)* क्रमंशः (क्रमशः)* ।।7।।

(Note: * – emendations suggested by Thibaut and Dvivedin. ** – emendations suggested by Neugebauer and Pingree.)

"The horizontal line *(tiryag-rekhā)* which is parallel to the north-south line is to be drawn up to the circle which represents the declination (lit.: declination-line, *apakramāṁśa-rekhā*). The degrees of the arc corresponding to it are to be multiplied by ten. [The result is] the *vināḍikā* of the [right-] ascension of the signs in order."

This verse can be explained as follows. (See Fig.15) The word *apakramāṁśa-rekhā* literally means a "declination line". We already have threefold concentric circles which have been drawn under the instruction of vs.2, and can rightly be called "declination-lines". In fact, they are the orthographic projection of the diurnal circles of the end points of the signs onto the plane of the equator. (We should recall that the radii of those circles are R.cos δ, where δ is the declination of the end points of the signs.)

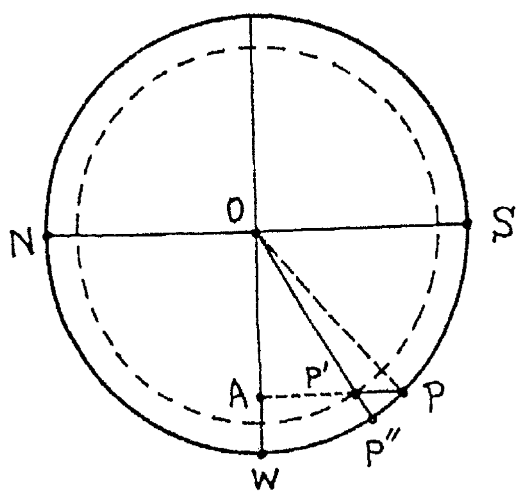


Fig. 15.

Now, let us mark a point P on the circumference of the level circle in such a way that it has 30-degree distance from the west (or east) cardinal point. (WÔP=30° in Fig. 15) Then draw a horizontal line *(tiryag-rekhā)* PP', which is parallel to the north-south line (NS), up to the "declination-line" (dotted circle in Fig.15) which corresponds to the declination of the end point of Aries. Thus, we obtain a point P', which is the orthographic projection of the end point of Aries onto the plane of equator, when the west cardinal point W is considered to be the first point of Aries.

This fact is easily understood as follows. (See Fig. 16.) If P' is the orthographic projection of the end point of Aries, OA=R.cos 30° and OP'= R.cos δ, where δ is the declination of the end point of Aries, as is evident from Fig.16. The segments OA and OP' in Fig.15 are exactly as such. Since this diagrm (Fig.15) is the orthographic projection onto the plane of equator, any point which is projected on the same radial line has the same right ascension. Therefore, the right ascension of the end point of Aries is WÔP' or WÔP''. The arc WP'' is the *"tac-cāpa"* in the text. The right ascension of the other signs are also obtained by the same method. Of course, this method gives the exact value.

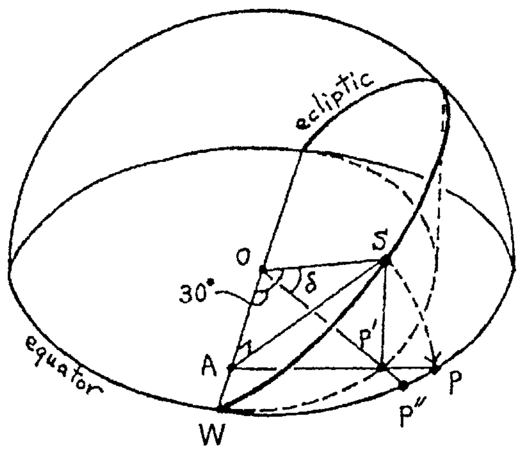


Fig. 16.

Now, the *Pañca-siddhāntikā* (XIV.8) reads:

मध्यानांप्रां (मध्यान्यायां) तथा
छायायामस्वतो (°न्यतो)* गते ततः शंको (शङ्कौ)**।
शंक्वग्रयातंत्सूत्रा (°यातसूत्राद्)**
विषुवान्तर (रं)** याश्चकांद्गदिताः (चक्रादुदिताः)।।8।।

(Note: * – this reading appears in a manuscript, and has been mentioned in the footnote in Thibaut and Dvivedin's edition. ** – emendations suggested by Neugebauer and Pingree)

"A gnomon is to be moved elsewhere in such a way that the tip of its shadow coincides with the centre (of the graduated level circle). From the string which passes through [the centre and] the tip of the gnomon, the equinoctial [midday angular] distance is to be represented on the circle."

This verse is metrically defective. In order to correct it, we should add a particle, say *hi* or *tu*, before *tathā*, and should omit *tataḥ* and *yāś*. Anyway, meaning of the text does not change.

In the word "*viṣuva-antara*", the meaning of "*viṣuva*"can be interpreted in two ways. One is "the equator", and in this case the word "*viṣuvāntara*" means "the distance from the equator". The other is "the equinox", and in this case the word "*viṣuvāntara*" means "the equinoctial distance" or a certain distance which is observed on equinoctial days. When we compare it with the expression *viṣuvacchāyā* which means the equinoctical midday shadow, it seems, most probably, that the "*viṣuvāntara*" is "the equinoctial midday angualr distance which corresponds to the *viṣuvacchāyā*', that is the zenith distance of the midday sun on the equinoctial day. In vs. 10, Varāhamihira says that the *viṣuvāntara* is the *akṣa* (the observer's latitude), and this fact supports our interpretation.

If the above interpretation is correct, this verse tells a method to obtain the sun's midday zenith distance on equinoctial days. At the equinoctial midday, the gnomon is placed in such a way that the end of its shadow coincides with the centre of the level circle. Then the centre and the top of the gnomon is joined by a string. Then the angle between the string and the vertical gnomon is equal to the equinoctical midday zenith distance of the sun. Probably, the string was fallen down on the graduated level circle, and the angle was read from the graduation of the level circle.

A more practical method for the same purpose is explained in the next verse (XIV.9) as follows.

विन्यस्योदक् छायां
छायाग्राच्छङ्कुरपरतः पात्यः (पात्यः)* ।
तत्कर्णसमं मध्यात्
प्रसारयेत्सूत्रमापरिधेः ।।9।।

(Note: * – emendation suggested by Thibaut and Dvivedin.)

"Lay off the [equinoctial midday] shadow [of a gnomon, which is erected at the centre of the level circle,] towards the north, and another gnomon (of the same height) is to be fallen down [perpendicularly to the shadow] from the tip of the shadow. Its hypotenuse should be straightly extended with a string from the centre up to the circumference [of the level circle]".

The meaning of this verse is clear. If the second gnomon is fallen down towards the east, the arc between the east cardinal point of the level circle and the intersection of the string and the level circle indicates the zenith distance of the midday sun.

The next verse is the continuation of this verse. The verse (XIV.10) reads as follows.

तद्विष्वंतर– (°षुवान्तर–)*मक्षो
ऽतो ऽक्षाच्चैवं प्रकल्पयेच्छायाम् ।
इष्टे ऽहनि बुध्या (बुद्ध्वा)* ऽयन–
मक्षादधिकं यदूनं वा ।।10।।

(Note: * – emendations suggested by Thibaut and Dvivedin.)

"Its equinoctial [midday zenith] distance (*viṣuvāntara)* is the terrestrial latitude. Similarly, the [midday] shadow should also be determined from the terrestrial latitude. Knowing the declination [of the sun] on a desired day, which may be greater or smaller than the latitude....(to be continued to the next verse.)"

This verse seems to be a request to obtain the declination of the sun from observation. Knowing midday zenith distance of the sun on equinoctial days, as was instructed in the previous vevse, the difference between the zeniht distance of the midday sun on the desired day and that of the equinoctial day should be obtained. This difference is the sun's declination. Varāhamihira says that it "may be greater or smaller than the latitude", and probably he refers to the direction of the declination rather than its absolute value. When the direction of the declination is south, it is in the same direction as the sun's equinoctial midday zenith distance (i.e. the observer's latitude) on the celestial sphere, and is considered to be "greater" than the latitude. When the declination is north, it is in the opposite direction, and is considered to be "smaller".

The next verse (XIV.11) is a continuation of this verse as follows.

तज्या (तज्ज्या)* तिर्यग्रेखा)
विषुवद्रेखास्थिता स्पृशति यस्मिन् ।
तच्चापांशसमान (°नो)*
ज्ञेयो ऽर्को गोलभागेन ।।11।।

(Note: * – emendation suggested by Thibaut and Dvivedin)

".... place its R.sine (R.sine of the sun's declination) as a horizontal line [perpendicularly] to the line of the equator, [and find out] a point where it touches [the line of ecliptic]. Its corresponding arc is known as [the longitude] of the sun, according to the division of the sphere (according to the quadrant of the ecliptic where the sun is located)'

The most probable interpretation of this verse is that the calculation is carried on a diagram which is the orthographic projection of the ecliptic and the equator onto the plane of solstitial colure. (See Fig. 17).

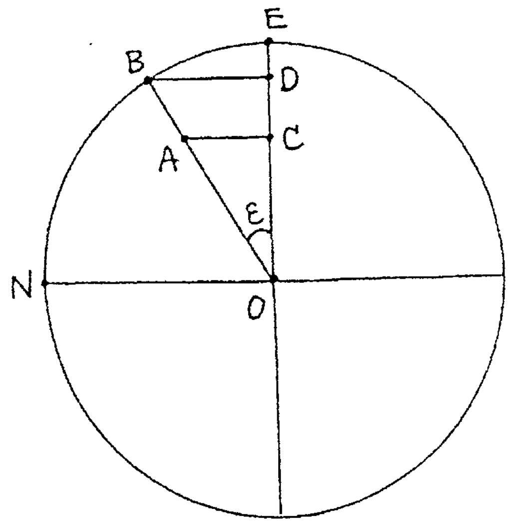


Fig. 17.

In Fig. 17, the line OE is the projection of the equator, and OB the projection of the ecliptic. The first point of Aries is projected on the centre O, and the first point of Cancer is projected on the point B. The angle BOE is the obliquity of the ecliptic (ε), and the segment BD is R.sin ε, when R is the radius of the circle. Let the segment AC be R.sin δ (*taj-jyā* in the text), where δ is the declination of the sun. Then the segment OA is R.sin λ, where λ is the longitude of the sun. This fact is easily understood, because we know the following equation by spherical astronomy (or by Hindu spherics using orthographic projection).

$$\sin \lambda = \frac{\sin \delta}{\sin \varepsilon}$$

Therefore, if R.sin δ is known, R.sin λ can be obtained graphically. Then, the sun's longitude is obtained as:

$$\lambda = \text{arc} \sin \frac{OA}{R}$$

This calculation is also done graphically. Firstly, one should double the length of the segment OA, and place it to the graduated level circle as a chord. Then, he should read its correspondintg arc from the graduation, and halve it. The result is the desired arc sine (*tac-cāpa* in the text).

From the above discussions, it now appears that all the verses (*Pañca-siddhāntikā*, XIV.1-11) refer to graphic calculation on the graduated level circle, and given either an exact value or a reasonable approximation. The rationale of these methods can be understood as an application of the theory of orthographic projection.

v) Conculsion

From the above discussions, it is seen that the celestial sphere was represented on the graduated level circle already at the time of Āryabhaṭa and Varāhamihira. This idea to represent the celestial sphere has an important meaning for the development of the *yaṣṭi* and circular and spherical instruments. The *yaṣṭi*, which we shall discuss below, is a representation of the Radius of the celestial sphere, and it presupposes certain coordinates such as the graduated level circle. The circular instruments, such as the *cakra* described by Āryabhaṭa and Varāhamihira etc., are also another device to represent the celestial sphere in visible form. Āryabhaṭa and Varāhamihira did not describe the armillary sphere, but they described three celestial globe which directly represents the celestial sphere. These instruments are closely related to each other. In this connection, the shadow-instrument of Āryabhaṭa etc., the *chatra-yantra* of Āryabhaṭa, and the graduated level circle of Varāhmihira, which were little noticed by historians of astronomy, should be paid much more attention.

5. The Staff

i) Introduction

The staff (*yaṣṭi-yantra*) is a representation of the Radius of the celestial sphere, and is used for determination of the position of heavenly bodies, and also for terrestrial surveying.

A similar instrument called *nalaka-yantra* (tube instrument) has been described in the *Tripraśnādhyāya* of some Siddhāntas.

A variation of the staff is the *śalākā-yantra* which is a combination of a horizontal staff and another stick which is perpendicular to the former. Another variation is the *śakaṭa-yantra* which is the V-shaped staffs.

Bhāskara II made a staff which is supposed to be in the rectangular coordinates, and called it *dhī-yantra*.

ii) The yaṣṭi-yantra

a) The Āryabhaṭa-siddhānta

Āryabhaṭa described the *yaṣṭi-yantra* in his *Āryabhaṭa-siddhānta*, quoted by

Rāmakṛṣṇa Ārādhya in his commentary on the *Sūrya-siddhānta,* as follows.[1]

वृत्तव्यासदलं यष्टिस्त्रिज्यांशाङ्गुलसम्मिता।।8।।

दिङ्मध्ये ऽर्कोन्मुखी धार्या यष्टिः कर्णस्तदुन्नतिः।
शङ्कुस्तस्यैव मूलात्तु छाया दिङ्मध्यगा सदा।।9।।

यष्ट्यग्रोदयमध्यांशाः षड्भिर्भाज्या दिने गताः।

"The *yaṣṭi-yantra* which is equal in length to the semi-diameter of the (perfect) circle with as many graduations of *aṅgulas* as there are degrees in a radian (i.e.57) should be held at the centre of the circle towards the Sun. The *yaṣṭi* then denotes the hypotenuse, its elevation denotes the gnomon, and the distance from the foot of the gnomon up to the centre of the circle always denotes the shadow (of that gnomon). The degrees intervening between the end of the *yaṣṭi* and the rising point of Sun, divided by six, give the *ghaṭīs* elapsed in the day." (Translated by K.S. Shukla)[2]

It is clear from Fig. 18 that the R.sine of the sun's altitude ("gnomon") and R.sine of the sun's zenith distance ("shadow") can be determind by the staff which represents the Radius.

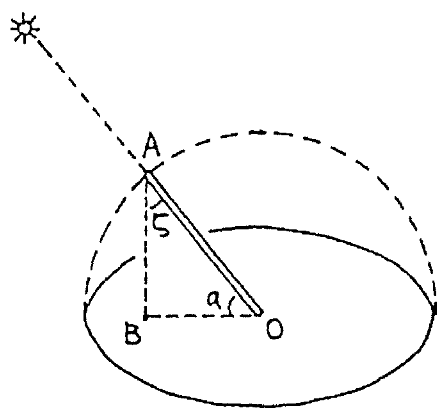


Fig. 18. OA: "Hypotenuse" = Yaṣṭi, AB: "Gnomon" = R.sin a, OB: "Shadow" = R.sin ζ

b) Brahmagupta

Brahmagupta explained the *yaṣṭi-yantra* in detail in his *Brāhma-sphuṭa-siddhānta* (XXII.19-38).[3] Firstly, he explained the method of the observation of the sun (XXII.19-23). The *yaṣṭi* is introduced in vs.19 as follows.

यष्टिस्तिर्यग्धार्या नष्टच्छायावलम्बकः शङ्कुः।
दृग्ज्यान्तरमनुपातात् स्वाहोरात्रार्धमग्रा च।।19।।

"The *yaṣṭi* is to be kept obliquely in such a way that it may not cast shadow. The great gnomon (*śaṅku,* R.sine of the sun's altitude) is the vertical line [dropped from the tip of the *yaṣṭi*]. The distance of it [from the foot of the *yaṣṭi*] is the *dṛg-jyā* (R.sine of the sun's zenith distance). The radius of the diurnal circle and the *agrā* (R.sine of the sun's amplitude) should be calculated by proportion."

The relation between the *yaṣṭi*, *śaṅku* and *dṛg-jyā* is easily understood from Fig.18. The method to calculate the radius of the diurnal circle (= R. cosine of the sun's declination) and the *agrā* is not given here, but given in the *Tri-praśna-adhyāya* as we have discussed under the section of the gnomon.

The graduated level circle is introduced in vs. 20 as follows.

परिलिख्य वृत्तमवनौ यष्टिव्यासार्धमन्यदस्यान्तः ।
स्वाहोरात्रार्धार्धं घटिकाषष्ट्यङ्कितं परिधौ ।।20।।

"Draw a circle, whose radius is equal to the length of the *yaṣṭi*, on the ground. From its centre, draw another circle, whose radius is equal to the radius of the dirunal circle, and mark 60 *ghaṭikās* on its circumference."

Here, the radius of the diurnal circle is of course the radius of the diurnal circle on the celestial sphere. With the help of this circle drawn inside of level circle, an arc on the diurnal circle can be converted into an arc on the great circle which is represented by the level circle. In other words, it is converted into angular distance.

Now, the method to obtain time is given in vs. 21 a follows.

यष्टिव्यासार्धे ऽग्रायष्ट्यग्रान्तरसमज्यया धनुषि ।
घटिका द्वितीयवृत्ते याताः प्रागपरतः शेषाः ।।21।।

"On the sphere whose Radius is equal to the length of the *yaṣṭi,* the distance between the rising (or setting) point and the tip of the *yaṣṭi* is represented by an arc *(dhanus).* On the second circle (i.e. the diurnal circle drawn inside the level circle), the *ghaṭikās* elapsed or remaining, in the forenoon or afternoon respectively, are measured."

Here, the celestial sphere is supposed to be above the graduated level circle.(See Fig.19) On the level circle the sun's rising point B (or setting point B') should be marked. (The angle EOB is equal to the sun's amplitude which has been requested to calculate in vs.19.) When the *yaṣṭi* is directed towards the sun, the arc BA can be measured. This is an arc of the diurnal circle, and it should be converted into the arc of the equator which is a great circle. This conversion is done graphically. The arc BA is placed on the "second circle" drawn inside the level circle (dotted circle in Fig. 19), and its corresponding angle is read from the graduation. The result is equal to the arc corresponding to the time elapsed since sunrise or remaining until sunset.

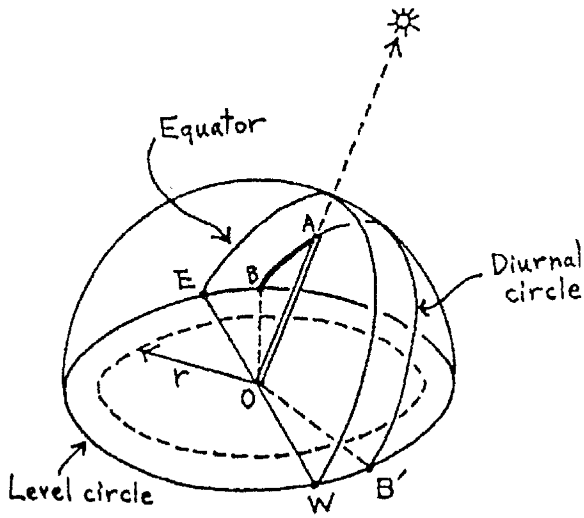


Fig. 19. ∠EOB = ∠WOB' = amplitude, OA: Yaṣṭi, AB: Dhanus, r = radius of the diurnal circle

The same conversion can be done arithmetically also as follows.(XXII.22)

यष्टेः स्वाहोरात्रार्धभाजिता ऽन्तरदलाहता त्रिज्या।
फलचापांशा द्विगुणाः षड्भिर्वा भाजिता घटिकाः।।22।।

"Alternatively, a half of the *yaṣṭi*'s distance (i.e. a half of the chord of the *"dhanus"* obtained above) is multiplied by the Radius, and divided by the radius of the diurnal circle. Then the degrees of its corresponding arc are multiplied by two, and divided by six. The result is *ghaṭikās.*"

Here, a half-chord (i.e. sine) of the great circle is first calculated, and it is converted into arc by a sine-table. The obtained degrees correspond to a half of the time elapsed or remaining. Therefore, it is multiplied by 2. Then the degrees are converted into *ghaṭikās* by multiplying (60/360) = (1/6).

Another method to obtain time is given in vs. 23 as follows.

यष्टिव्यासार्धे वा घटिका शङ्क्वङ्गुलादितो मूलात्।
अवलम्बसूत्रयुक्त्या घटिका दिवसस्य गतशेषाः।।23।।

"Alternatively, on the level circle whose radius is equal to the length of the *yaṣṭi*, the *ghaṭikā* is obtained from the *aṅgulas* of the *śaṅku* (R.sine of the sun's altitude) etc. Placing a string vertically from the foot [of the *śaṅku* up to the tip of the *yaṣṭi* as the representation of the R.sine of the sun's altitude], the *ghaṭikās* elapsed or remaining in a day [are obtained]."

Here, Brahmagupta says to obtain time from the R.sine of the sun's altitude which is determined by the *yaṣṭi-yantra,* but the method of the calculation is not explained here. The method of the calculation is explained in the *Tripraśnādhyāya* as we have discussed under the section of the gnomon.

After the above text, Brahmagupta desrcibed the V-shaped staffs in vss. 24-26. We shall discuss the V-shaped staffs under the division of the *śakaṭa-yantra* later.

In vs. 27, Brahmagupta explains the method to determine the directions by the *yaṣṭi-yantra* as follows.

मध्यधृताया यष्टेर्लम्बकशङ्कू प्रवेशनिर्गमने
क्रान्तिवशात् प्राच्यपरे मत्स्याद्याम्योत्तरे साध्ये ।।27।।

"There are two vertical *śaṅkus* (i.e. strings representing the R.sine of the sun's altitude) dropped from [the upper end of] the *yaṣṭi,* [whose lower end is] fixed at the centre [of the level circle], at the points [where the foot of the *śaṅku* is] entering into and going out from [the level circle]. From the effect of the declination *(krānti-vaśāt),* the east and west should be determined, and from the fish-figure (i.e. perpendicular bisector), the south and north should be determined."

This method is the same as the usual Indian circle method. The meaning of the phrase "from the effect of the declnation" *(krānti-vaśāt)* is not quite clear. The same phrase appears in the *Brāhma-sphuṭa-siddhānta* (III.1) and (XXI.60) also. It might be supposed at first sight that it refers to the correction due to the change of the sun's declination within a day,[4] but it is unlikely. It is more likely that it refers to the parallel shift of the east-west line according to the sun's declination.

Brahmagupta continues to explain spherics in connection with the *yaṣṭi* in vss. 28-31. The verse (XXII. 28) reads as follows.

शङ्कुतलाग्रान्तरयुतिरन्यैकदिशोर्भुजो भुजस्य कृतिम् ।
दृग्ज्याकर्णकृतेः प्रोह्य पदं पूर्वापरा कोटिः ।।28।।

"The difference or sum of the *śaṅkutala* and *agrā,* when their directions are opposite or same respectively, is the *bhuja.* The square of the *bhuja* is substracted from the square of the *dṛg-jyā* which is the hypotenuse, and its square root is the *koṭi* on the east-west line."

This text can be explained as follows. (See Fig.20) In Fig.20, the point Z is the zenith,O the observer, and N,E,S, and W the north, east, south, and west cardianl points on the horizon respectively. Let X be the position of the sun on the celestial sphere, and E'W' the sun's rising-setting line.So, the segment OX may be considered to be the *yaṣṭi.* Draw a vertical line XA from the point X up to its intersection with the ground A, and call the segment XA the *śaṅku* (great gnomon, or the R. sine of the sun's altitude). The distance between the lines EW and E'W' is the *agrā* (R.sine of the sun's amplitude), and is equal to the segment BC in the figure. The distance of A from the rising-setting line E'W' is called *śaṅkutala* (AC) , and the R.sine of the sun's zenith distance is called *dṛg-jyā* (OA).

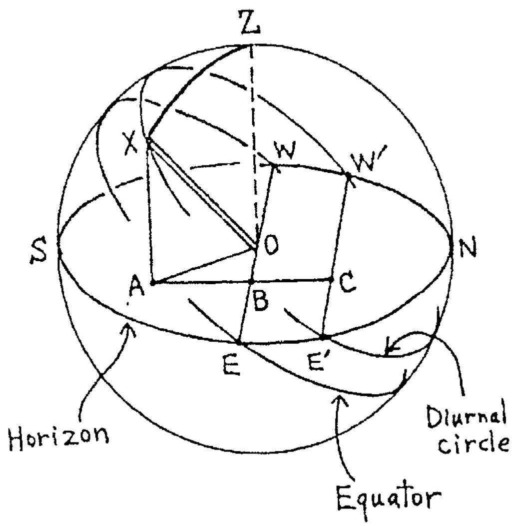


Fig. 20.

In this verse, the triangle AOB is considered, where AB is the *bhuja* (base), AO is the *karṇa* (hypotenuse), and OB is the *koṭi* (upright).

Two rules stated in the text can be expressed as follows.

bhuja = *śaṅkutala* ± *agrā*,

where the *agrā* is added when the sun's declination is south, and substracted when the sun's declination is north.

koṭi = $\sqrt{(dṛg\text{-}jyā)^2 - (bhuja)^2}$.

These two rules are easily understood from Fig. 20.

Brahmagupta continues to explain spherics. The next verse (XXII.29) reads as follows.

उदयास्तसूत्रशङ्क्वन्तरं हृतं शङ्कुना ऽर्कसङ्गुणितम् ।
विषुवच्छायैवं वा विनोदयास्तमयसूत्रेण ।।29।।

"The distance between the rising-setting line and the *śaṅku* (great gnomon) is divided by the length of the *śaṅku* and multiplied by 12. This is the equinoctial midday shadow. It is also knowable without rising-setting line [as explained below]."

As the inclination of the diurnal circle to the ground is the same as the inclination of the equator to the ground, the proportion XA:AC in Fig. 20 is equal to the proportion 12: equinoctial-midday-shadow. Hence the equinoctial midday shadow of 12-*aṅgula* gnomon is obtained as above.

The alternative rule to obtain equinoctial midday shadow is explained as follows (vs.30).

प्राच्यपराशङ्कुतलान्तरद्वयान्तरयुतिः समान्यदिशोः।
द्वादशगुणिता विषुवच्छाया शङ्क्वन्तरविभक्ता।।30।।

"The difference or the sum of the two differences between the east-west line and the foot of the *śaṅku* (great gnomon), when their directions are the same or opposite respectively, is multiplied by 12 and divided by the difference of the two *śaṅkus*. This is the equinoctial midday shadow."

This text can be explained as follows. (See Fig.21) One should determine the *śaṅku* (R. sine of the sun's altitude) with the help of the *yaṣṭi-yantra* twice a day. Then the distance of the foots of the two *śaṅkus* (AA' in Fig. 21) is obtained, as AO+OA' or AO-A'O in Fig.21 according to the directions of A and A'. And also, the difference of the length of the two *śaṅkus* (XD in Fig. 21) is obtained. As the proportion XD:DX' is the same as the proportion 12: equinoctial-midday-shadow, the equinoctial midday shadow of 12-*aṅgula* gnomon is obtained as follows.

$$\text{equinoctial-midday-shadow} = \frac{12 \times AA'}{XD}$$

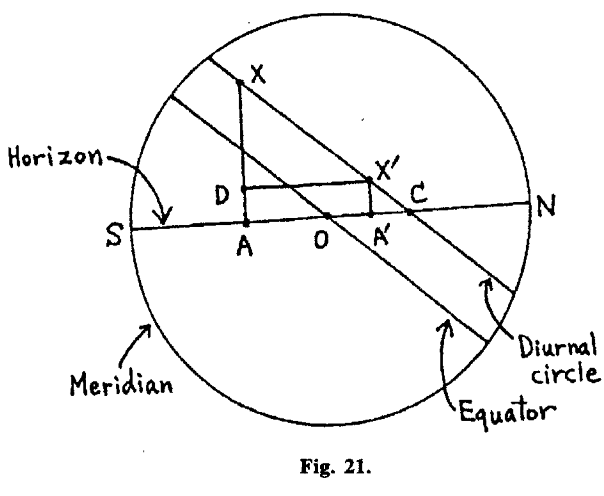


Fig. 21.

Now, Brahmagupta explains the method to determine the sun's declination as follows (XXII. 31).

शङ्कुप्राच्यपरान्तरशङ्क्वग्रैक्यमुदगन्तरं याम्ये।
लम्बगुणं यष्टिहृतं क्रान्तिज्यातो रविः प्राग्वत्।।31।।

"The distance between the *śaṅku* (great gnomon) and the east-west line is added to the *śaṅkvagra* (= *śaṅkutala)* when the sun is in the northern hemisphere, and subtracted when in the southern hemisphere. It is multiplied by the *lamba* (R.sine of the observer's colatitude) and divided by the *yaṣṭi* (Radius). This is the R.sine of the declination (*krānti-jyā).* From this, [the position of] the sun may be known."

This text can be a explained as follows. (See Fig.22.) In Fig.22, the segment OE is equal to the R.sine of the sun's declination *(krānti-jyā)*, and the segment ZD is equal to the R.sine of the observer's colatitude *(lamba)*, because the angle ZOD is the observer's colatitude. As the triangle ODZ is similar to the triangle CEO, we have the following proportion.

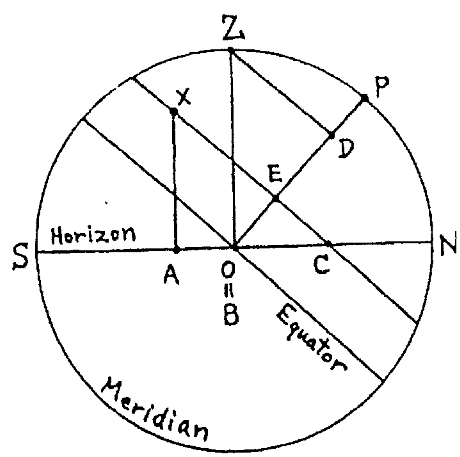


Fig. 22.

OE: OC =ZD:ZO=ZD:R.

Therefore, the *krānti-jyā* OE is obtained as follows.

$$OE = \frac{OC \times ZD}{R}$$

Brahmagupta further explains surveying by the staff as we shall see later.

c) Lalla and Śrīpati

Lalla described the *yaṣṭi-yantra* in his *Śiṣyadhī-vṛddhida-tantra* (XXI. 480-50).[5] He first explains to obtain the R.sine of the sun's altitude (*nṛ*, "man"), and the R.sine of the sun's zenith distance *(chāyā* or *dṛg-jyā).* Then, he says to multiply the distance between the rising-setting line and the foot of the *śaṅku* by 12 and divide by the length of the *śaṅku* (= *nṛ).* This is the equinoctial midday shadow of 12-*aṅgula* gnomon.

Śrīpati also described the *yaṣṭi-yantra* in his *Siddhānta-śekhara* (XIX. 21-23 (i)) as follows.[6]

संसाधिताशं कृतचक्रभागं
विधाय वृत्तं समभूप्रदेशे।
त्रिज्याङ्गुलाङ्कां सुसमां च यष्टिं
नष्टद्युतिं तज्जठरे निदध्यात्॥21॥

तदग्रलम्बः खलु शङ्कुरुक्त–
स्तन्मूलकेन्द्रान्तरमत्र दृग्ज्या।
पूर्वापरात्तद्विवरं भुजः स्या–
च्छङ्क्वग्रमस्तोदयसूत्रमध्यात्॥22॥

शङ्क्वग्रमर्कै 12 र्गुणितं विभक्तं
तल्लम्बकेन स्फुटमक्षभा स्यात्।

"After drawing a circle with marks of cardinal points and graduation of degrees on the levelled ground, the *yaṣṭi*, which is very smooth, [equal to the radius of circle,] and having graduation of *aṅgulas* in a radian, should be placed at its centre in such a way that it does not cast shadow.

The vertical line from its *(yaṣṭi's)* top is called *śaṅku* (great gnomon, R.sine of the sun's altitude).The distance between its *(śaṅku's)* root and the centre [of the level circle] is called *dṛg-jyā* (R.sine of the sun's zenith distance). Its (*śaṅku's* root's) distance from the east-west line is called *bhuja*. Its (*śaṅku*'s root's) distance from the rising-setting line is called *śaṅkvagra.*

The *śaṅkvagra* is multiplied by 12 and divided by the length of the great gnomon. This is exactly the equinoctial midday shadow *(akṣabhā)*".

This description of the *yaṣṭi-yantra* is similar to those of Brahmagupta and Lalla, and the meaning is clear.

The modern *Sūrya-siddhānta* (XII.20)[7] also mentions the *yaṣṭi*, but it mentions its name only without description of the instrument.

d) Bhāskara II

Bhāskara II described the *yaṣṭi-yantra* in his *Siddhānta-śiromaṇi* (*Gola*, XI.28-38(i)).[8] Bhāskara II says to draw a level circle with the marks of four cardinal points and the sun's rising and setting points, and also a circle with the radius of the sun's diurnal circle which is graduated with *ghaṭīs*. Then the arc between the rising (or setting) point of the sun and the tip of the *yaṣṭi* directed towards the sun is converted to *ghaṭīs* with the help of the diurnal circle drawn inside the level circle. Then he says to obtain the *śaṅku* (R.sin of the sun's altitude) and the *dṛg-jyā* (R.sine of the sun's zenith distance) by the *yaṣṭi*. Then he says to calculate the equinoctial midday shadow. One method is to multiply the distance between the *śaṅku* and the rising-setting line by 12, and divide by the *śaṅku*. An alternative method is to use two *śaṅkus* determined in a day. The distance between the foots of these *śaṅkus* is multiplied by 12, and divided by the difference of these *śaṅkus*. This is the equinoctial midday shadow (*palabhā*). Thesse methods are similar to the methods of Brahmagupta.

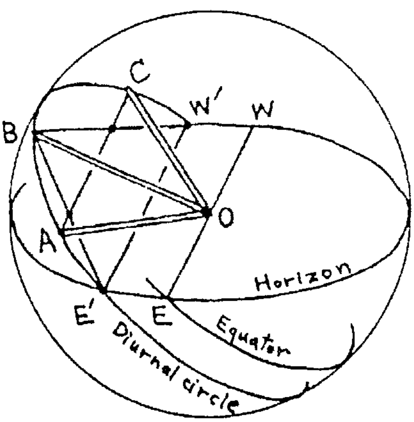


Fig. 23. OA, OB, OC : Yaṣṭis

Additionally, Bhāskara II explained the method to calculate the equinoctial midday shadow, the sun's declination etc. from three observations with the *yaṣṭi*, when the sun's rising-setting line is not known. (XI.33(ii)-38(i)). (See Fig.23). Bhāskara II says to find three *śaṅkus* (i.e. vertical lines dropped from the tip of the *yaṣṭi*) thrice a day, and draw a line (AC in Fig. 23) from the top of the first one (A) to the top of the last one (C).Then he says to draw lines (BE'and BW') from the top of the second one (B) to the eastern and western horizon in such a way that they touch the previous line AC. The two points thus found (E' and W') are joined, and then this line E'W' is the rising-setting line. What he has done is to obtain the cross-cut of the plane of the diurnal circle and the plane of horizon. The distance between the rising-setting line and the centre of the level circle is the R.sine of the sun's amplitude. Bhāskara II then tells to obtain the equinoctial midday shadow as before. It can be calculated because the rising-setting line is already known. Then, Bhāskara II says that the R.sine of the sun's amplitude multiplied by 12 and divided by the *akṣa-karṇa* (hypotenuse of the equinoctial midday shadow of 12-*aṅgula* gnomon) is the R.sine of the sun's declination. And also, he says that the R.sine of the sun's declination multiplied by the Radius and divided by the R.sine of the obliquity of the ecliptic gives the R.sine of the sun's longitude, and it can be converted into the sun's longitude. Evidently, these rules are correct, and can be proved by Hindu spherics.

iii) The nalaka-yantra

The *nalaka-yantra* is practically the same as the *yaṣṭi-yantra*, but the *nalaka-yantra* ("tube instrument") must be a tube, while the *yaṣṭi-yantra* may be a simple stick.

The *nalak-yantra* is mentioned in the *Tripraśnādhāya* of some works, such as the *Śiṣyadhī-vṛddhida-tantra* (IV. 47-48) of Lalla.[1] The *nalaka-yantra* described there is a tube fixed to the direction of a heavenly body so that the observer can see the object through the tube, and the position of the object can be confirmed. The reflection of the heavenly body by the surface of water is also observed by the *nalaka-yantra*. As the expression "*nalakacchidreṇa*" (by the hole of the *nalaka)* occurs in the text, it is clear that it is a hollow tube.

Similar description is found in the *Siddhānta-śekhara* (IV.84-85) of Śrīpati,[2] and the *Siddhānta-śiromaṇi* (*Grahagaṇita*, III.105-108) of Bhāskara II[3] also.

A somewhat similar instrument is the triangle-instrument, which actually is simply called *yantra* or instrument in the text, mentioned in the *Vaṭeśvara-siddhānta* (III.i.26) of Vaṭeśvara.[4] It is an instrument to observe the pole-star. When its hypotenuse is directed towards the pole-star, the base is equal to the length of gnomon, and the upright is equal to equinoctial midday shadow.

iv) The śalākā-yantra

Lalla described the *śalākā-yantra* ("needle instrumnet"),which is used for the determination of the angular distance, in his *Śiṣyadhī-vṛddhida-tantra* (XXI.38-41).[1]

Lalla says to make a needle of as many *aṅgulas* as there are in the radius (i.e. $360/2\pi = 57$), and stretch it from the east to west (BA in Fig.24). Then, at its western end (A), one should lay off another horizontal needle (AC) perpendicularly to the former. Then, observing a rising heavenly body (S) from the tip of the second needle (C) in such a way that the heavenly body clings to the tip of the first needle (B), one should determine the length of the second needle (AC) which indicates the R.sine of the amplitude *(agrā)*. Lalla further says to observe the angular distance between two planets by this instrument. He rightly wrote that the square root of the sum of the squares of the two needles, that is the hypotenuse, is the Radius.

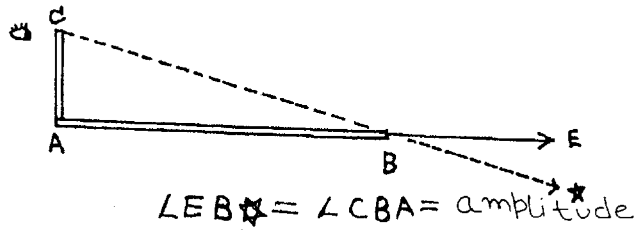


Fig. 24. Śalākā-yantra (reconstructed)

v) The śakaṭa-yantra

The *śakaṭa-yantra* ("cart instrument") is the V-shaped staffs. It was mentioned by Varāhamihira and Brahmagupta as a variation of the *yaṣṭi-yantra* without particular name. The name of the *śakaṭa-yantra* was used by Lalla and Śrīpati.

Varāhamihira described the V-shaped staffs in his *Pañca-siddhāntikā* (XIV.12-13) as follows.[1)]

छेद्यार्धयष्टिवेधादर्केन्द्वोरन्तरांशकार्कांशः ।
स्फुटनष्टतिथिर्ज्ञेया तस्मात् कार्या तथा चान्या ।।12।।

दत्वांशकेषु तेष्वेव भास्करं छेद्यकेन विज्ञातम् ।
स भवति तस्मिन् काले निशाकरश्छेद्यकेनैव ।।13।।

"By the observation with the [V-shaped] staffs whose length is equal to the radius of the level circle (*chedya*), the degrees between the sun and moon [should be obtained, and their] twelfth part [should be taken]. It is known as the true elapsed *tithis*. After this, next *tithi* is also to be counted.

Add [the longitude of] the sun, which is known by [the graphical calculation on] the level circle, to those degrees [of elongation]. The result is [the longitude of] the moon at that time obtained by [the graphical calcultion on] the level circle."

As we have seen in the section of the graduated level circle, the longitude of the sun can graphically be calculated on the graduated level circle as explained in the *Pañca-siddhāntikā* (XIV.10 (ii)-11). And now, the longitude of the moon is obtained by the observation by the V-shaped staffs and the graphical calculation on the graduated level circle.

Brahmagupta also described a similar instrument in his *Brāhma-sphuṭa-siddhānta* (XXII.24-26) as follows.[2)]

यष्टिव्यासार्धाद् भुवि वृत्तं भगणांशकं कृत्वा ।
यष्टी कीलप्रोते मूले प्रथगग्रयोर्बद्धे ।।24।।

ताभ्यां सूर्यशशाङ्कौ वेध्यावग्रस्थितेन सूत्रेण।
सूत्रज्यया ऽन्तरांशा ये ते ऽर्कविभाजितास्तिथयः।।25।।

सूत्रार्धगुणा त्रिज्या यष्टिहृता फलधनुर्द्विगुणितं वा।
रविचन्द्रान्तरमिष्टव्यासार्धोल्लिखितवृत्तस्य।।26।।

"A circle whose radius is equal to the length of the *yaṣṭi* and which has graduation of degrees should be drawn on the ground, and the ends of two *yaṣṭis* are joined to each other at their foot.

The sun and moon should be observed by them with the help of a string which touches the tops [of the two *yaṣṭis*]. The degrees of the angular distance which is known by the chord or string should be divided by 12. This is the *tithis* elapsed.

Alternatively, one may multiply a half of the string by the Radius and divide by the length of the *yaṣṭi*, convert it to degrees (by a sine table), and double it. This is the angular distance between the sun and moon which can also be obtained [graphically] by the graduated level circle of desired radius."

Meaning of this text is clear. (See Fig. 25) The angular distance can directly be measured by this V-shaped staffs and the graduated level circle.

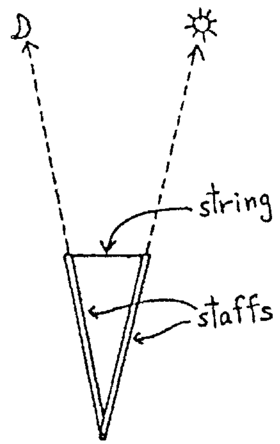


Fig. 25. V-shaped staffs

Lalla described a similar instrument under the name of *śakaṭa-yantra* in his *Śiṣyadhī-vṛddhida-tantra* (XXI.42-43 and 51-52).[3] It is practically the same as the V-shaped staffs of Varāhamihira and Brahmagupta.

Śrīpati also described the *śakaṭa-yantra* in his *Siddhānta-śekhara* (XIX. 26) as follows.[4]

वृत्ते चक्रलवांकिते ऽत्र शकटाकारं शलाकाद्वयं
कृत्वा तेन विवेधयेद्रविविधू लम्बस्य पातस्तयोः।
यावन्तः परिधौ तदन्तरलवाः सूर्यै 12 र्विभक्ता गताः
शुक्ले स्युस्तिथयो भवन्ति बहुले पक्षे च भोग्याः स्फुटम्।।26।।

"After making two sticks whose shape is like a cart (*śakaṭa)* (i.e.V-shaped figure) on a circle graduated with degrees of a circle (360^0), the sun and moon should be observed by them, and their (*śakaṭa's)* fallen figure's (lit: plumb line's) intersection [with the level circle should be found]. The degrees of their distance represented on the circumference [of the level circle] is divided by 12. It is elapsed *tithis* in the case of the white half of a month, and it is remaining *tithis* in the case of the black half of a month exactly".

It is seen that this Śrīpati's description is basicalaly the same as those of Varāhamihira, Brahmagupta, and Lalla.

vi) Terrestrial surveying by the yaṣṭi-yantra

The *yaṣṭi-yantra* can be used for terrestrial surveying also. Its method has been described in the *Brāhma-sphuṭa-siddhānta* (XXII.32-38)[1) of Brahmagupta in detail. Let us see the text one by one. The verse (XXII.32) reads as follows.

अपसृतिरन्यशलाकागुणा शलाकान्तरेण भक्ता भूः।
भूः स्वशलाकागुणिता यष्टिविभक्ता गृहाद्यौच्च्यम्।।32।।

"The distance [between the two needles] is multiplied by the height of the other needle (*anya-śalākā*), and divided by the difference between the heights of two needles. This is the *bhū* ("ground"). The *bhū* is multiplied by the height of the own needle (*sva-śalākā*), and divided by the length of the *yaṣṭi*. This is the height of a house etc."

This is a method to determine the height of a terrestrial object by the *yaṣṭi-yantra* with a vertical needle. (See Fig. 26.)

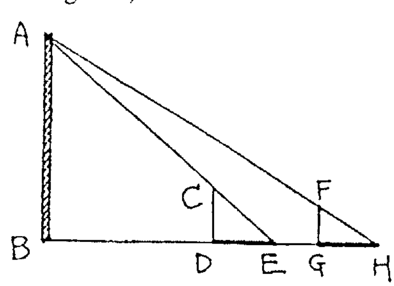


Fig. 26.

In the figure, the segments DE and GH are the *yaṣṭis* of a fixed length. If CD is chosen as the "own needle", then FG is the "other needle". In this case, the segment BE is the *bhū*. The object (house ect.) is AB. The rule stated in the text can be expressed as follows.

$$BE = \frac{DG \times FG}{CD - FG},$$

$$AB = \frac{BE \times CD}{DE}.$$

The second equation can easily be understood from the figure. The first equation can be proved as follows. We have :

$$AB = \frac{BE \times CD}{DE} = \frac{BH \times FG}{GH},$$

$$BH = BE + EH = BE + DG, \text{ and}$$

$$DE = GH.$$

From these relations, we have :

$$BE \times CD = (BE + DG) \times FG.$$

From this equation, the first equation of the text can be derived.

In the above discussion, we selected CD as the "own needle". If FG is chosen as the "own needle", then CD is the "other needle", and the calculation can be done similarly.

Now, the next verse (XXII.33) reads as follows.

दृष्ट्या गुणिता ऽपसृतिर्दृष्टिविशेषेण भाजिता भूमिः।
भूमिः स्वदृष्टिभक्ता शलाकया सङ्गुणोच्छ्रायः।।33।।

"The distance [between the ends of the two *dṛṣṭis*] is multiplied by the [own] *dṛṣṭi* and divided by the difference [between the two *dṛṣṭis*]. This is the *bhūmi*. The *bhūmi* is divided by the own *dṛṣṭi*, and multiplied by the height of the needle *(śalākā)*. This is the height [of the object]."

This is an alternative method to determine the height of a terrestrial object. (See Fig.27.) In this case, the height of the needles CD and FG are the same. The segments DE and GH are called *dṛṣṭis*. If DE is chosen as the "own *dṛṣṭi*", the segment BE is the *bhūmi*. The object is AB. The rule in the text can be expressed as follows.

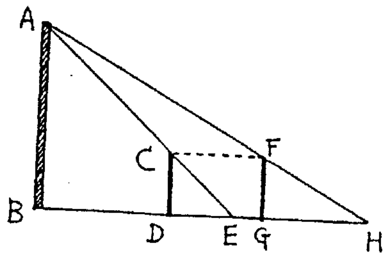


Fig. 27.

$$BE = \frac{DE \times EH}{GH - DE},$$

$$AB = \frac{BE \times CD}{DE}.$$

This method is the same as the calculation of the shadows of two gnomons mentioned in the *Āryabhaṭīya* (II.16), and some other mathematical works, such as the *Līlāvatī* (239). In mathematical works, this type of calculation is explained in the section of *Śaṅkucchāyā* (gnomon-shadow).

Now, the verse (XXII.34(i)) reads as follows.

लम्बनिपातान्तरकं लम्बौच्च्यान्तरविभक्तमधिकगुणम् ।

"The distance between the bottoms of two vertical segments [dropped from the two ends of the *yaṣṭi*] is divided by the difference between the height of two vertical segments, and multiplied by the larger segment (*adhika*). [This is the bhū]."

This method is to determine the distance of the object by observing its root. (See Fig.28.) In the figure, GD is the *yaṣṭi*, where G is the eyepiece. The point C is the root of the object, and the distance CH is the *bhū*. Two vertical segments GH and DF are dropped from the two ends of the *yaṣṭi*. Then the rule stated in the text can be expressed as follows.

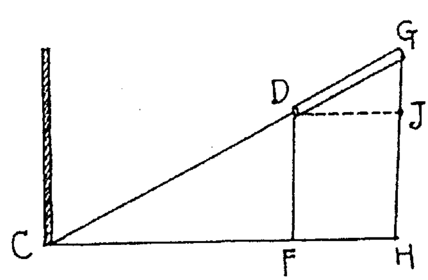


Fig. 28.

$$CH = \frac{FH \times GH}{GJ} .$$

Brahmagupta continues to explain the method to determine the height of the object using the *bhū*, obtained as above, as follows (XXII.34(ii)-35).

भूर्लम्बान्तरगुणिता लम्बनिपातान्तरविभक्ता । ।34 । ।

लब्धोनो दृग्लम्बो दृग्लम्बादग्रलम्बके हीने ।
अधिके ऽधिको गृहोच्च्यं तलाग्रके विद्धया दृष्ट्या । ।35 । ।

"The *bhū* is multiplied by the difference between the height of the two vertical segments, and divided by the distance between the bottoms of two vertical segments. The result is subtracted from the length of the vertical segment from the eye-piece if the vertical segment from the object-piece is shorter than the vertical segment from the eye-piece, and added if longer. This is the height of the house obtained by the observation through the bottom and top [of the *yaṣṭi*]."

This is the method to obtain the height of the object when its distance is known. (See Fig.29) In the figure, GD is the *yaṣṭi* where G is the eye-piece and D is the object-piece. Two vertical segments GH and DF are dropped from these points. The distance CH is the *bhū*, and AC is the object. So, the rule stated in the text can be expresed as follows.

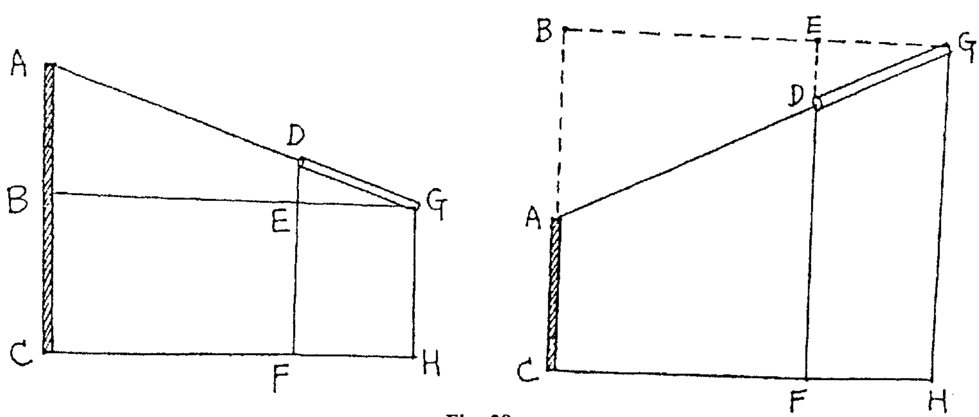


Fig. 29.

$$AB = \frac{CH \times DE}{FH} ,$$

$$AC = GH \pm AB.$$

The rationale of this method is clear.

Brahmagupta explains an alternative method to determine the distance and height of an object as follows (XXII. 36).

दृष्टिर्दृग्लम्बगुणा विभाजिता ऽधःशलाकया भूमिः ।
सकलशलाकागुणिता भूमिर्दृष्ट्या हृतोच्छ्रायः ।।36।।

"The *dṛṣṭi* is multiplied by the vertical segment from the eye-piece, and divided by the lower needle (*adhaḥ-śalākā*). This is the *bhūmi. The bhūmi* is multiplied by the full needle (*sakala-śalākā*), and divided by the *dṛṣṭi*. This is the height [of the object]."

This text can be explained as follows. (See Fig. 30.) In the figure, KL is the *dṛṣṭi*, where L is the eye-piece, KM the "lower needle", and PM the "full needle". The *dṛṣṭi* is kept horizontally, and the needle vertically. The distance CN is the *bhūmi*, and AC is the object. So, the rule in the text can be expressed as follows.

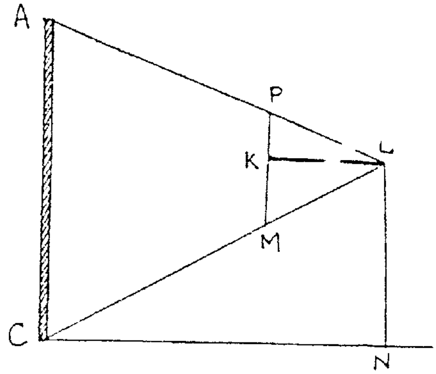


Fig. 30.

$$CN = \frac{KL \times LN}{KM},$$

$$AC = \frac{CN \times PM}{KL}.$$

The rationale of this method is clear.

Now, Brahmagupta explained the method to determine the height of an object when the height of its one part is known as follows (XXII.37).

मित्वा गृहैकदेशं विद्ध्वेष्टशलाकया गृहं सर्वम् ।
प्रथमशलाकाभक्तं मितं द्वितीयागुणितमौच्च्यम् ।।37।।

"After knowing the height of one part of a house and observing by the desired needle, the full height of the house [is determined as follows]. The known part [of the object] is divided by the first needle, and multiplied by the second needle. This is the height [of the object]".

This text can be explained as follows. (See Fig.31) In the figure, AC is the object, where the height of the part BC is known. The segment FG is the *dṛṣṭi*, EF the "first needle", and DF the "second needle". Then the rule stated in the text can be expressed as follows.

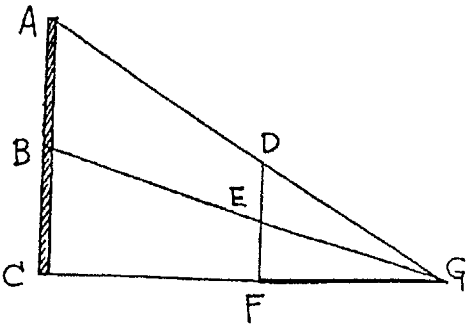


Fig. 31.

$$AC = \frac{BC \times DF}{EF} .$$

The rationale of this method is clear.

Brahmagupta concludes the description of the *yaṣṭi-yantra* as follows (XXII.38).

यष्ट्या हृताच्छलाका त्रिज्याघाताद्धनुर्गृहान्तरकम् ।
यैरुक्तं मूर्खास्ते यतो न दृष्टान्तरं दृग्ज्या ।।38 ।।

"The length of the needle *(śalākā)* is divided by the length of the *yaṣṭi*, and multiplied by the Radius. Thus obtained arc *(dhanus)* is the [angular-] distance of the house. Those who told like this are fools, because the obtained distance is not the *dṛg-jyā* (R.sine corresponding to the object ?)."

The meaning of this verse is not quite clear. (See Fig.32.) In the figure, AB is the object (house etc.) and E the eye. Let FE be the Radius. Probably, the "angular distance of the house" is the angle BEA. If we assume that the *yaṣṭi* is CE, and the needle is CD, the amount (RxCD/CE) becomes the segment FG, which is exactly the R.sine corresponding to the "angular distance of the house". Then, there is no reason to criticize this theory. It may be that the criticized theory is to assume that DE is the *yaṣṭi*, and CD the needle. Then the amount (RxCD/DE) is different from the desired R.sine. Probably, this mistake was criticized by Brahamagupta.

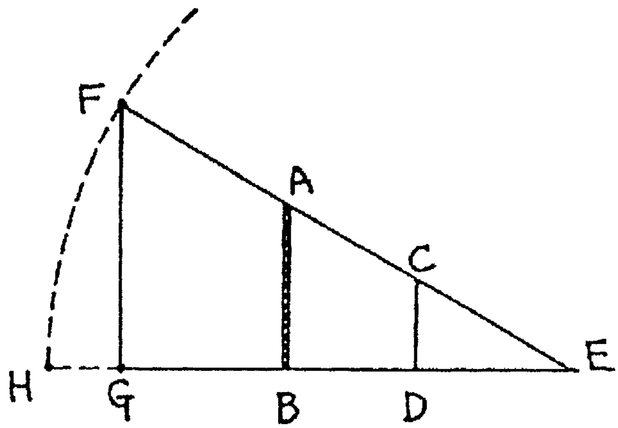


Fig. 32.

vii) The dhī-yantra of Bhāskara II

Bhāskara II described the *dhī-yantra* ("intelligence instrument") in his *Siddhānta-śiromaṇi* (*Gola, XI.40-49*).[1] This is simply a staff which is supposed to be in rectangular coordinates.

Bhāskara II explains to obtain the equinoctial midday shadow by the observation of the pole-star with this instrument, and also to determine the distance and height of terrestrial object by this instrument. The rationale of these calculations is the same as the case of simple staff which we already have discussed.

Bhāskara II praised the *dhī-yantra* in his *Siddhānta-śiromaṇi* (*Gola*, XI. 40-41) as follows.[2]

अथ किमु पृथुतन्त्रैर्धीमतो भूरियन्त्रैः
स्वकरकलितयष्टेर्दत्तमूलाग्रदृष्टेः ।
न तदविदितमानं वस्तु यद्दृश्यमानं
दिवि भुवि च जलस्थं प्रोच्यते ऽथ स्थलस्थम् ।।40।।

वंशस्य मूलं प्रविलोक्य चाग्रं तत्स्वान्तरं तस्य समुच्छ्रयं च।
यो वेत्ति यष्ट्यैव करस्थया ऽसौ धीयन्त्रवेदी वद किं न वेत्ति।।41।।

"But what is the use of numerous instruments described in several works for an intelligent man? By the observation through the bottom and top of the *yaṣṭi* in his hand, there is nothing unknown if it be visible, wherever in the heaven or on the ground. [The object] situated in the water or on the ground is explained [as below].

After observing the root and top of a bamboo, he who finds out its distance and height by means of the *yaṣṭi* in his hand is the knower of the *dhī-yantra.* Tell me what is not found out by him!"

As Bhāskara II says, the staff gives enough informations for Hindu astronomy, if one spares no pains to do complicated calculations. However, it does not mean that other convenient instruments are unnecessary. It seems that Bhāskara II over-praised the *dhī-yantra.*

6. The Circle-instrument and its Variants

i) Introduction

Some instruments can be classified as the variants of the circle-instrument (*cakra-yantra*). Its basic variants are the semi-circle instrument *(dhanur-yantra)* and the quadrant (*turya-golaka-yantra*). A circle kept in the plane of the equator is the *bhagaṇa* of Lalla or the *nāḍī-valaya* of Bhāskara II. The *kartarī* ("scissors") of Brahmagupta is a combination of two semi-circles, one of which is in the plane of the equator, and the other is in the plane of the meridian. Lalla and Śrīpati described a simplified version of the *kartarī*, which consists of a semi-circle in the plane of the equator and a perpendicular gnomon. The *kapāla* ("bowl") described by Varāhamihira and Brahmagupta is the hemispherical sundial. Lalla and Śrīpati described a simplified version of the *kapāla,* which consists of a horizozntal semi-circle and a vertical gnomon. The *pīṭha* ("seat") is a horizontal circle with a vertical gnomon. Similar instrument is the earthern platform desribed in the *Tripraśnādhyāya* of some works.

ii) The cakra-yantra

A fragment of *Āryabhaṭa-siddhānta* of Āryabhaṭa, quoted in Rāmakṛṣṇa Ārādhya's commentary on the *Sūrya-siddhānta,* has a description of the *cakra-yantra* as follows.[1]

भगणांशांकितं चक्रं सरन्ध्रं विषुवत्यथ ।।10।।

धनुः रव्युन्मुखं कृत्वा चापवच्चक्रयन्त्रकम् ।
कल्पयेल्लम्बशङ्कोर्वा छायानाड्यश्च यष्टिवत् ।।11।।

"The *cakra-yantra* ("an instrument resembling a circular hoop") bears (on its circumference) 360 marks of degrees and has (two) holes at the equinoctial points. Pointing the arc of the *cakra-yantra* towards the Sun, like the (arc of the) *dhanuryantra,* the shadow of the gnomon as also the *nāḍīs* elapsed in the day should be ascertained as in the case of the *yaṣṭi-yantra.*" (Translated by K.S. Shukla)[2]

After the quotation of the *Āryabhaṭa-siddhānta,* Rāmakṛṣṇa Ārādhya made a comment on the *cakra-yantra* in his commentary on the *Sūrya-sidhānta.* It may be quoted here, although its manuscript is defective.[3]

चक्रयन्त्रं सुवृत्तं परिधौ भगणांकितै(तं) विषुवत्स्थानादधच्छिद्रं द्वयमुक्तम्। तच्चल-----(gap)----- भिमुखं कृत्वा धनुर्यन्त्रवद्भ्रमयेत्। अथवा यष्टियन्त्रवच्छिद्रालम्बितसूत्रं भूमिलग्नं स्यात्। शङ्कुमू -----(gap)----- मध्यपर्यन्त -----(gap)----- ष्टभा स्यात्। अर्को (अथवा?) ऽप्यर्कोन्मुखछिद्रसूर्याकाग्र– योरन्तरालांशाङ्गुलानि षड्भिभ(र्भ)क्तानि दिनगतघटिकाः स्युः। पूर्वसाधितयाम्यभागस्थित– स्वाहोरात्रार्धवृत्तपर्यन्त एव शेषार्धदिनशेषघटिका ज्ञातव्याः। इति चक्रयन्त्रघटिकाज्ञानम्।

"The *cakra-yantra,* which is well-rounded, marked with degrees on its circumference, and has a couple of holes below the equinoctial points, has been told [by Āryabhaṭa]. Its ...(gap) .. facing towards [the sun], it should be rotated just like the *dhanur-yantra* (it should be rotated around the vertical axis in such a way that it faces towards the sun). Otherwise, just like the *yaṣṭi-yantra,* a string hung from a hole should touch the ground. Gnomon's root ...(gap)... centre and circumference ...(gap)... desired shadow. Otherwise, the *aṅgulals* correponding to the degrees between the hole facing towards the sun and the sun's amplitude (i.e. the sun's rising point) divided by six is *ghaṭikās* elapsed [since susnrise]. On the circumference of the southern half of the diurnal circle which was obtained previously (in connection with the *chāyā-yantra), ghaṭikās* to elapse [until sunset] during the remaining half-day should be known. Thus the knowledge of *ghaṭikās* by the *cakra-yantra."*

Tamma Yajvan also mentioned the *cakra-yantra* of Āryabhaṭa in his commentary on the *Sūrya-siddhānta* as follows.[4)]

चक्रं त्रिज्यांशाङ्गुलवृत्तं परिधौ भगणांशांकितं
विषुवत्स्थानयोः छिद्रद्वययुक्तं स्यात्।

"The *Cakra-yantra* (i.e. the Circle) is a circle (or hoop) whose radius is as many digits in length as there are degrees in a radian, and whose circumference bears 360 marks of degrees as well as two holes one at each equinotical point." (Translated by K.S. Shukla)[5)]

From the above quotations, it is seen that Āryabhaṭa's *cakra-yantra* is a circular hoop which has two holes at diagonal points. (See Fig. 33.) It is held vertically,and rotated around the vertical axis in such a way that the hoop faces towards the sun. After that, the hoop must be rotated around its own axis in such a way that the sun-light passes through the two holes. Then the time can be determined by the angular distance between the sun's rising point and the direction of the hoop (see Fig. 33(a), and the description of Āryabhaṭa's *dhanus* below). Otherwise, strings hung from the two holes to the ground are used to obtain the R.sine of the sun's altitude and of the sun's zenith distance just like *yaṣṭi-yantra* (see Fig. 33(b)). In this case, the diameter between the two holes is considered to be *yaṣṭi.*

Varāhamihira described the *cakra-yantra* in his *Pañca-siddhāntikā* (XIV. 21-22) as follows.[6)]

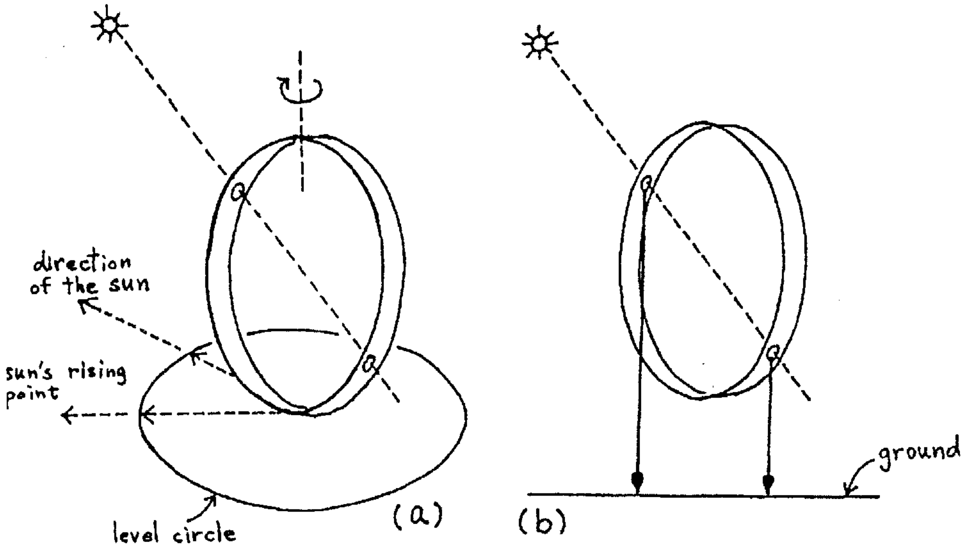


Fig. 33. Āryabhaṭa's cakra (reconstructed)

समभगणाङ्ककचक्रमर्धाङ्गुलवहलमायतं हस्तम् ।
विस्तारमध्यभागे छिद्रं तद्गामि तिर्यक् च ।।21।।

मध्याह्नार्कमयूखं प्रवेश्य सूक्ष्मेण परिधिविवरेण ।
मध्यावलम्बिसूत्रात्तलान्तरांशास्तदन्याक्षः ।।22।।

"The *cakra,* which is half an *aṅgula* wide and a *hasta* in diameter and is marked with degrees evenly, [is made]. It has a hole in the middle of its width.When the ray of the midday sunlight, coming perpendicularly, enter through the fine hole in the circumference, the degrees at the base between [the point illuminated and] the string hung from the centre is the midday zenith distance."[7)]

Varāhamihira's *cakra* is similar to Āryabhaṭa's *cakra,* but it has only one hole, while Āryabhaṭa's *cakra* has two holes. And also, Varāhamihira's *cakra* has a string hung from the centre, while the strings of Āryabhaṭa's *cakra* are hung from the holes on the circumference. It is clear that the sun's zenith distance can easily be determined by Varāhamihira's *cakra.* (See Fig. 34)

Brahmagupta described the *cakra-yantra* briefly in his *Brāhma-sphuṭasiddhānta* (XXII.18) as follows.[8)]

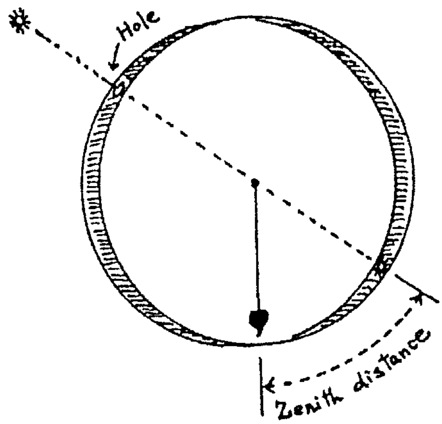


Fig. 34. Varāhamihira's cakra (reconstructed)

परिधौ भगणांशाङ्कं मीनान्तं चक्रतो विद्ध्वा।
चक्रकयन्त्रं मध्याल्लम्बो ऽत्र फलं धनुस्तुल्यम्।।18।।

"The *cakra-yantra* is graduated with degrees and zodiacal signs (lit.: that which ends with pisces) on its circumference. A plumb is hung from its centre. When one observes through the *cakra,* the result is the same as the *dhanus* (semi-circle)".

Here, it is seen that Brahmagupta's *cakra* had a plumb hung from the centre just like the *cakra* of Varāhamihira. Brahmagupta did not mention the hole, and it is not clear whether it had a hole on its circumference or not.

Lalla and Śrīpati described another type of *cakra,* which consists of a circular board and a perpendicular needle at its centre. The *Śiṣyadhī-vṛddhida-tantra* (XXI.20-21) of Lalla reads as follows.[9)]

वृत्तं कृत्वा फलकं षड्वर्गाङ्कं तथा च षष्ट्यङ्कम्।
मध्यस्थितावलम्बं मध्यस्थित्या प्रविष्टोष्णम्।।20।।

तदधोलम्बविमुक्तं गृहादि यत्तदुदितं दिनकरांशात्।
नाड्यः पूर्वकपाले द्युगतास्ताः पश्चिमे द्युदलात्।।21।।

"Construct a circular board, graduate it with the 360 divisions (of degrees) as well as the 60 divisions (of *ghaṭīs),* fix a vertical needle at its centre and set it up in such a way that the Sun may lie centrally in its plane.

Then the signs etc. left behind (since sunrise) by the shadow of the vertical needle denote the signs etc. by which the Sun has ascended (above the horizon). The *nāḍīs* left behind denote the *nāḍīs* elapsed since sunrise, provided the Sun is in the eastern hemisphere. When the Sun is in the western hemisphere, the *nāḍīs* thus obtained should be subtracted from half the duration of the day (to get the *nāḍīs* to elapse before sunset).'' (Translated by K.S. Shukla)[10)]

The original word of the "vertical needle" is *"avalamba"* or *"lamba"*. It may appear at first sight that this *"lamba"* may mean a "plumb", but it is not so. The text requests to determine the sun's altitude, and it must be measured by the shadow of the central 'vertical needle". (See Fig. 35.)

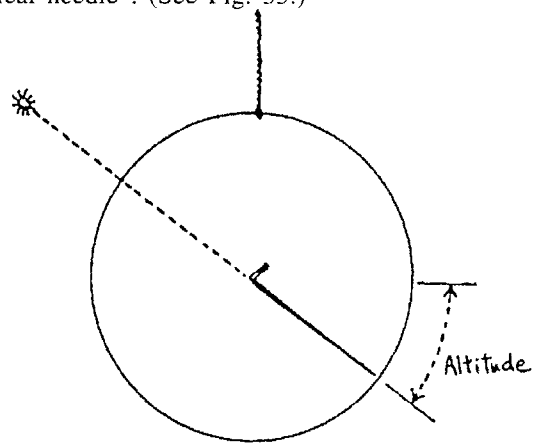


Fig. 35. Lalla, Śrīpati and Bhāskara II's cakra (reconstructed)

This point will become more clear in the description of *the dhanur-yantra* of Lalla, which we shall see later.

According to K.S. Shukla,[11)] Lalla must have used the following formula in order to deduce the *nāḍīs* elapsed since sunrise from the sun's altitude.

$$nāḍīs \text{ elapsed} = \frac{\text{semi-duration of day} \times \text{Sun's altitude}}{\text{Sun's meridian altitude.}}$$

Śrīpati also described the *cakra-yantra,* which is similar to that of Lalla, in his *Siddhānta-śekhara* (IX.12-13 (a-c)) as follows.[12)]

कृत्वा सुवृत्तं फलकं हि षष्ट्या
    चक्रांशकैश्चांकितमत्र मध्ये ।
लम्बस्तदग्रात् सुषिरेण यद्वत्
    केन्द्रे ऽर्करश्मिः पततीति दध्यात् ।।12।।

लम्बेन मुक्ता रविभागतो ऽंशा–
    स्तत्रोदितास्ते घटिकास्तु याताः
चक्राख्यमेतद् ....

"After making a well-rounded board which is marked with 60 [*ghaṭīs*] and degrees, a perpendicular rod is inserted through a hole at its centre from its tip. Then one should hold [the board] in such a way that the sun-light falls centrally [in its plane.]. The degrees left behind by [the shadow of] the perpendicular rod according to the position of the sun are those which the sun has ascended [above the horizon]. These indicate elapsed *ghaṭikās*. This is called *cakra.*"

It is seen that Śrīpati's *cakra-yantra* is similar to that of Lalla, and consists of a circular board and a central perpendicular rod. (See Fig.35) The time obtained by this instrument is rough approximation.

Bhāskara II also described the *cakra-yantra* in his *Siddhānta-śiromaṇi* (*Gola*, XI. 10-15). Its shape is as follows. (XI. 10-11(i)).[13]

चक्रं चक्रांशाङ्कं परिधौ श्लथशृङ्खलादिकाधारम् ।
धात्री त्रिभ आधारात् कल्प्या भार्धे ऽत्र खार्धं च ।।10।।

तन्मध्ये सूक्ष्माक्षं क्षिप्त्वा ऽर्काभिमुखनेमिकं धार्यम् ।

"The *cakra* is graduated with 360° on its circumference, and suspended by a loose chain etc. The horizon *(dhātrī)* is considered to be at the distance of 90° from the suspended point, and the point at the distance of 180° is the zenith.

Putting a thin stick *(sūkṣma-akṣa)* to its centre, one should hold it [vertically] in such a way that its circumference faces the sun."

It is seen that Bhāskara II's *cakra* is practically the same as that of Lalla and Śrīpati. (See Fig. 35.) Bhāskara II tells to obtain the sun's altitude and zenith distance by this instrument. He also gives the following formula of "some former astronomers" to obtain time.

$$\text{time} = \frac{(\text{semi-duration of day}) \times (\text{sun's altitude})}{(\text{sun's meridian altitude})}$$

Bhāskara II rightly ponted out that this is rough approximation.

Bhāskra II further tells to keep the circle in the plane of the ecliptic at night in such a way that fixed stars which have zero latitude appear to touch the circumference of the circle. Then the longitude of a planet is obtained by the observation of the angular distance between the planet and a fixed star whose longitude is known.

When the *cakra* is used in the last case, the *cakra* is kept in the plane of the ecliptic. Otherwise, the *cakra* is kept vertically in all cases, and the sun's altitude and zenith distance are obtained.

We can summarize the *cakra-yantra* of several authors as follows. The *cakra* of Āryabhaṭa has two holes, and that of Varāhamihira has a hole on its circumference, and is held in such a way that the sun-light enters centrally through the hole. The *cakra* of Lalla, Śrīpati, and Bhāskara II has a central rod whose shadow indicates the sun's altitude and zenith distance. The circle of Varāhamihira and Brahmagupta has a plumb hung from the centre. The *cakra* of Bhāskara II is hung by a chain which indicates the direction of zenith and nadir, and probably that of Lalla and Śrīpati is similar.

iii) The dhanur-yantra

A fragment of the *Āryabhaṭa-siddhānta* of Āryabhaṭa, quoted by Rāmakṛṣṇa Ārādhya in his commentary on the *Sūrya-siddhānta,* has a description of the *dhanur-yantra* (semi-circle instrument) as follows.[1)]

वृत्तव्यासो धनुर्ज्या स्याद् व्यासार्धं धनुषः शरः ।।6।।

शङ्कुच्छाया धनुर्ज्यायां दिङ्मध्यत्त्विष्टभा सदा।
प्रागग्रं धनुषो वृत्ते भ्रामयेदर्कदिङ्मुखम्।।7।।

चापाग्रोदयमध्यांशाः षड्भिर्भाज्या दिने गता।

"The chord of the *dhanuryantra* is equal to the diameter of the circle (i.e.the perfect circle), and its arrow is equal to the radius. (It is mounted on the circle vertically with the two ends of its arc coinciding with the east and west points.)

The eastern end of the *dhanuryantra* should be moved along (the circumference of) the circle until the *dhanuryantra* is towards the Sun. The shadow of the gnomon will then fall along the chord of the *dhanuryantra,* and (the shadow-end being at the centre of the circle) the distance of the gnomon as measured from the centre of the circle, will always be equal to the shadow for the desired time.

The degrees intervening between the (eastern) end of the *dhanuryantra* and the rising point of the sun divided by six give the *ghaṭīs* elapsed in the day.'' (Translated by K.S. Shukla)[2)]

Āryabhaṭa's *dhanur-yantra* is firstly held vertically in such a way that its two ends coincide with the east and west points of the graduated level circle, and then rotated along the level circle in such a way that the plane of the instrument coincides with the direction of the sun. Thus, the sun's azimuth is obtained. (See Fig. 36.) Then a small stick on the arc of the *dhanur-yantra* is moved along the arc in such a way that its shadow falls at the centre of the level circle. Thus the sun's altitude is obtained.Here, the "gnomon"is represented by a plumb line (CD in Fig. 36) dropped from the small stick (c), and its distance (OD) from the centre is the "shadow for the desired time", that is the R.sine of the sun's zenith distance. Āryabhaṭa says that the azimuthal difference between the sun and its rising point divided by 6 gives the *ghaṭīs* since sunrise. However, it is rough approximation.

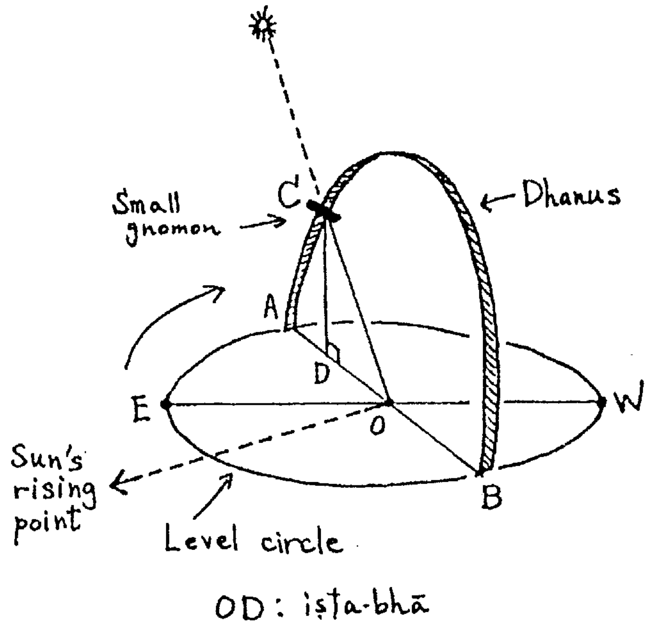


Fig. 36. Āryabhaṭa's dhanus (reconstructed)

Tamma Yajvan also briefly explained the *dhanur-yantra* of Āryabhaṭa in his commentary on the *Sūrya-siddhānta* as follows[3)]

धनुः त्रिज्यांशाङ्गुलकल्पितवृत्तस्यार्धं षष्ट्यंशाभ्य–
धिकशतत्रयांकितमर्धवृत्तं ससंज्ञं सशरं ज्ञातव्यम्।

"The *Dhanuryantra* (i.e. the Semi-circle) is a semi-circle whose radius has as many digits as there are degrees in a radian, and whose circumference is graduated with the marks of degrees, and which is furnished with the chord and the arrow." (Translated by K.S. Shukla)[4)]

The above is the descriptions of the *dhanus* of Āryabhaṭa. Now, let us proceed to the discussion of the *dhanur-yantra* of Brahmagupta.

Brahmagupta described the *dhanur-yantra* in detail in his *Brāhma-sphuṭa-siddhānta* (XXII. 8-16).[5)] Let us see the text one by one. The verse (XXII. 8) reads as follows.

धार्यं धनुस्तथा ऽन्यच्छायासाम्यं यथोन्नता भागाः।
दिनगतशेषा घटिकाः स्वलम्बभुक्ता धनुर्मध्यात्।।8।।

"The *dhanus* should be held in such a way that the shadow of its one end coincides with the other end. The degrees of the altitude, which are equal to the distance of the plumb from the middle of the *dhanus*, give the *ghaṭikās* elapsed or remaining".

In this case, the *dhanur-yantra* is kept vertically, and its chord is directed towards the sun. (See Fig. 37.) Here, the sun's altitude is represented by the arc between the middle of the arc (B in Fig. 37) and the plumb (E) which is hung from the middle of the chord (D).

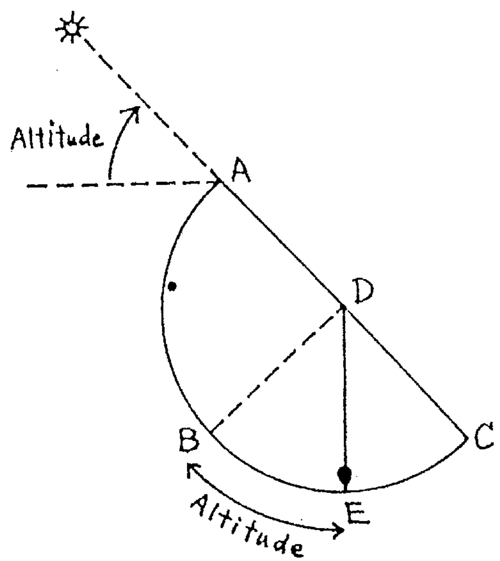


Fig. 37. Brahmagupta's dhanus (1st type) (reconstructed)

Brahmagupta explained another mounting of the *dhanur-yantra* as follows (XXII. 9).

धार्यं समं तथा वा ज्याछाया मध्यगा यथा भवति।
अग्रादिष्टा घटिका ज्यामध्यच्छायया भुक्ताः।।9।।

"Alternatively, it is held evenly in such a way that the shadow of the chord is cast centrally. The desired *ghaṭikās* are obtained by the distance of the shadow of [the rod at] the middle of the chord from the end [of the arc]."

In this case also, the *dhanur-yantra* is kept vertically, but its chord is kept horizontally. (See Fig.38.) Here, the sun's altitude is represented by the arc between the end of the arc (C in Fig.38), which repressents the horizon, and the position of the shadow (F) of a small rod (D) at the middle of the chord.

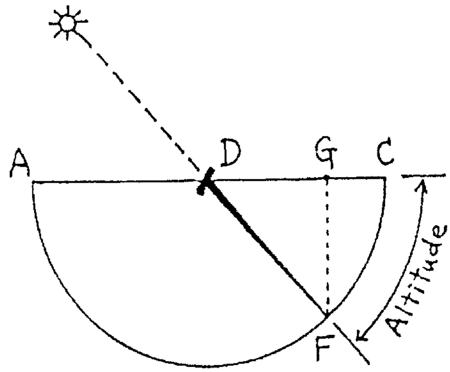


Fig. 38. Brahmagupta's dhanus (2nd type) (reconstructed)

Brahmagupta continues to explain this mounting as follows (XXII.10)

घटिका स्वशङ्कुभागैः पृथग्गतैर्लम्बभूसमज्यार्धात् ।
साशीतिशतांशाङ्कं चक्रस्यार्धं धनुर्यन्त्रम् ।।10।।

"The *ghaṭikās* are obtained , using the degrees of the sun's altitude *(sva-śaṅku-bhāga)* which have been marked severally [on the arc], from [the shadow of] the perpendicular rod at the centre of the horizontal chord.

The *dhanur-yantra* is a half of the *cakra,* and graduated with 180 degrees."

Here also, the *dhanur-yantra* is kept vertically, and its chord is kept horizontally. (See Fig. 38.) On its arc, marks corresponding to the sun's altitude are marked from the end of the arc. The time can be calculated from the sun's altitude.

Now, Brahmagupta criticizes a theory of some former astronomer as follows (XXII. 11).

मध्यदिवसोन्नतांशैर्दिनार्धनाडीर्वदन्ति तुल्या ये ।
ते मूर्खास्तच्छाया इष्टच्छायासमा न यतः ।।11।।

"Those who consider that the degrees of the sun's midday altitude exactly indicates the *nāḍīs* of semi-duration of day are fools, because that shadow is not the same as the desired shadow."

Probably, Brahmagupta is criticizing the following formula in this verse.

$$\textit{nāḍīs}\text{ elapsed} = \frac{\text{(semi-duration of day)} \times \text{(sun's altitude)}}{\text{(sun's midday altitude)}}$$

Brahmagupta is right in saying that this formula is not exact.

Brahmagupta requests to calculate the time properly as follows (XXII. 12).

जीवां स्वाहोरात्रे परिकल्प्याग्रान्नतोन्नतत्रिज्याः।
अनुपातात् कार्यास्तुर्यगोलके चक्रके चैवम्।।12।।

"After finding the chord [corresponding to the sun's ascension] along the diurnal circle, one should mark the R.sine of the zenith distance, R.sine of the altitude, and Radius, which are calculated by the proportion, from the end [of the arc]. This may be applied to the *turya-golaka* (quadrant) and *cakraka* (circle instrument) also."

This calculation is done according to the method explained in the *Tripraśnādhyāya,* as we have discussed in the section of the gnomon. After calculating the R.sine of the zenith distance etc. corresponding to the time, the arc of the *dhanur-yantra* can be graduated with time.

Brahmagupta continues as follows (XXII.13).

दिनघटिकांकितयष्टे (पृष्ठे?) व्यस्तनतज्याग्रमुन्नतज्यां च।
दिङ्मध्ये च शलाका तच्छायाग्रान्नता नाड्यः।।13।।

"On the surface *(pṛṣṭha)* which is marked with *ghaṭikās,* the R.sine of the zenith distance and the R.sine of the altitude are also marked reversely (from the end of the arc). A rod is at the middle of the chord. From the tip of its shadow, the zenith distance and *nāḍīs* [are known]."

This verse can easily be understood as a request to graduate the surface of the *dhanur-yantra* which is mounted like Fig. 38.

Brahmagupta explains an alternative method of observation as follows (XXII. 14).

धनुषः पृष्ठे द्रष्ट्रा वेध्या ज्यामध्यसंस्थया दृष्ट्या।
इष्टान्तरं नतज्या धनुषि च्छायोन्नतज्यायाः।।14।।

"One should observe [a heavenly body] from the back of the *dhanus* looking through the middle of the chord. The desired distance on the *dhanus* is the R.sine of the zenith distance. From the R.sine of the altitude [and R.sine of the zenith distance thus obtained], the shadow [of the gnomon can be calculated]."

This is a method to observe a heavenly body by putting one's eye on the back of the *dhanus* (F in Fig.38), and looking through the middle of the chord (D). Then the segment DG is the R.sine of the zenith distance, and GF the R.sine of the altitude.

The shadow of the 12-*aṅgula* gnomon can be calculated as :

$$\text{shadow} = \frac{12 \times (\text{R.sine of zenith distance})}{(\text{R.sine of altitude})}$$

Brahmagupta continues as follows. (XXII. 15).

ज्यार्धं दृष्टेर्दृग्ज्यां नतजीवांशं कुमुन्नतज्यां च।
धनुषि प्रकल्प्य योज्यं यद्युक्तं नाडिकाद्यं च।।15।।

"[The segment of] the half-chord [obtained] from the eye-point is the R.sine of the zenith distance. The degrees of the zenith distance and the R.sine of the altitude, which is measured [from the eye-point] upto the ground (i.e. the chord of the instrument), are also [known]. After settling them on the *dhanus*, the *nāḍikā* etc. are obtained as was explained.'

This is a continuation of the previous verse. The segment of the half-chord is the segment DG, the eye-point is F, and the ground is AC in the Fig.38. The method of calculation has been explained in the *Tripraśnādhyāya* as we have seen in the section of the gnomon.

Lastly, Brahmagupta explains the method to determine the height of a terrestrial object by the *dhanur-yantra* as follows (XXII. 16).

अवलम्बनं शलाकां ज्यार्धं यष्टिं प्रकल्प्य वा धनुषि।
भूम्युच्छ्रायाल्लम्बो यष्ट्युक्तैरानयेत् करणैः।।16।।

"Alternatively, the vertical line may be considered to be the needle *(śalākā)*, and the half-chord the *yaṣṭi*. One should calculate the height of a terrestrial object by the method explained in connection with the *yaṣṭi*"

In this case, the segment GF is considered to be the needle, and the half-chord DC, or rather the segment DG, is considered to be the *yaṣṭi*. Then, the method explained in the *Brāhma-sphuṭa-siddhānta* (XXII. 32), which we already have discussed under the section of the staff, can be applied, and the height of a terrestrial object can be obtained.

The above is the description of the *dhanur-yantra* by Brahmagupta. It is seen that the sine and cosine are graphically represented on this semi circle instrument.

Now, let us proceed to the desription of the *dhanur-yantra* by Lalla and Śrīpati.

Lalla described the *dhanur-yantra* in his *Śiṣysdhī-vṛddhida-tantra* (XXI. 22-23) as follows.[6]

चक्राख्यं यन्त्रमिदं दलं धनुर्यन्त्रमाहुरस्यैव।
ज्याकार्मुकभृच्छिद्रप्रविष्टदिनकरकरं धार्यम्।।22।।

मध्यस्थलम्बमुक्ताः कोटेरारभ्य नाडिका द्युगताः।
उदिताश्च दिनकरांशादारभ्य भवन्ति गृहभागाः।।23।।

"The above is the so-called *Cakra-yantra* ('The Circle'). Half of it is called *Dhanuryantra* ('The Semi-circle'). This latter instrument should be held (with its chord horizontal) in such a way that a ray of the Sun passing through the hole in the middle of the chord may fall on the arc.

Then the *nāḍīs* left behind by the shadow of the central needle, as measured from the arc-end, denote the *nāḍīs* elapsed during the day (since sunrise), and the signs and degrees left behind denote the signs and degrees by which the Sun has (ascended above the horizon)." (Translated by K.S.Shukla)[7]

This *dhanur-yantra* of Lalla has a central perpendicular needle *(lamba)* at the middle of the chord, and probably it is fixed to the hole there. (See Fig. 39). There is no doubt that the word *"lamba"* in the text means the central small perpendicular needle, and not a plumb, because the arc between its shadow (F in Fig, 39) and end of the arc (C) indicates the sun's altitude.

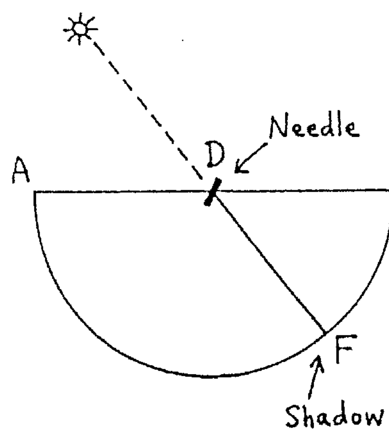


Fig. 39. Lalla and Śrīpati's dhanus (reconstructed)

Śrīpati also described the *dhanur-yantra* in his *Siddhānta-śekhara* (XIX. 13 (ii)) as follows.[8]

चक्राख्यमेतद्दलमस्य चापं
ज्यामध्यरन्ध्रस्थितलम्बमेतत्।।13।।

"...This is called *cakra* (circle instrument). A half of it is the *cāpa* (semi-circle) . It has a perpendicular needle which is fixed to a hole at the middle of the chord."

It is seen that this *cāpa* (= *dhanur-yantra)* of Śrīpati is similar to that of Lalla.

Bhāskara II mentioned the *cāpa* by its name only in his *Siddhānta-śiromaṇi* (*Gola*, XI. 15).[9]

iv) The turya-golaka-yantra

The *turya-golaka-yantra* (or *turya-yantra*) is the quadrant. Brahmagupta described this instrument in his *Brāhma-sphuṭa-siddhānta* (XXII.17) as follows.[1]

अंकितमंशनवत्या धनुषो ऽर्धं तुर्यगोलकं यन्त्रम्।
घटिकानतोन्नतांशग्रहान्तराद्यं धनुर्वदिह।।17।।

"The *turya-golaka-yantra* is a half of the *dhanus* (semi-circle), and is marked with 90 degrees. Here, the amount of the *ghaṭikās,* the degrees of the altitude, and the distance between [two] planets etc. are [measured] just like the *dhanus.*"

According to this description, the *turya-golaka-yantra* must have been a simple quadrantal board, and the angular distance was measured just like the semi-circle instrument.

Bhāskara II mentioned the *turya-gola* by its name only in his *Siddhānta-śiromaṇi* (*Gola*,XI.15).[2]

Bhāskara II did not describe the construction of the *turya-gola,* but probably it was also a simple quadrantal board, and special description of its construction was not necessary.

There is a more detailed description of the quadrant in the *Vṛddhavasiṣṭha-siddhānta* (III. 53-58), whose date is unfortunately unknown. The *Vṛddha-vasiṣṭha-siddhānta* (III. 53-55) reads as follows.[3]

यन्त्रं चक्रदलार्धं धातुजमथवा सुदारुजं श्लक्ष्णम्।
प्रकारद्वययुक्तं तत्र च रन्ध्रे भखेटवेधार्थम्।।53।।

तत्रेशानगकेन्द्राद् वृत्तं तत्रोन्नतांशका नवतिः।
स्थाप्याः समान्तरस्थाः केन्द्रगसूत्रावलम्बितो लम्बः।।54।।

विध्वा खगं लम्बगतोन्नतांश–
ज्या त्रिज्यका–1000 ध्वनी च विभाजिता च।

परोन्नतांशज्यकया ऽऽप्तचापं
दिनार्धनिघ्नं खनवा–90 प्तनाड्यः ||55 ||

"The instrument is a half of a semi-circle, which is smooth, made of metal or good timber, and has two sides. There are two slits in order to observe a heavenly body.

There, the light enters from the centre towards the arc. There, 90 equal divisions of the altitude should be marked. A plumb is hung by a string from the centre.

The sun is observed, and the R.sine of the altitude, which is known from the plumb, is multiplied by the Radius 1000, and divided by the R. sine of the maximum altitude (i.e. midday altitude), and its corresponding arc is multiplied by semi-duration of day, and divided by 90. The result is *nāḍīs*."

By this instrument, the R.sine of the altitude is obtained by the observation through a couple of slits.(Sec Fig.40.) Then, the time is calculated by the well known approximate formula, which we already have seen in connection with the *cakra* and *dhanus*.

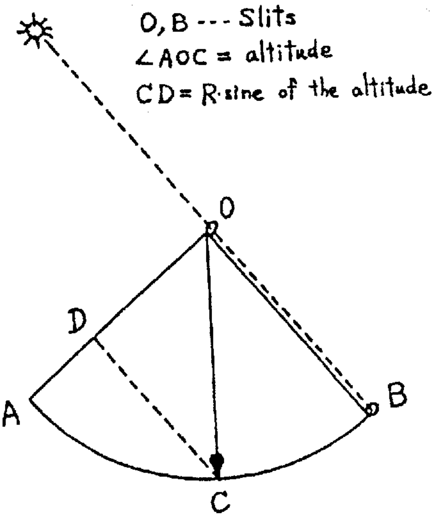


Fig. 40. Vṛddha-vasiṣṭha-siddhānta's quadrant (reconstructed); O,B.....Slits, ∠AOC = altitude, CD = R.sine of the altitude

In later period, from about the 15th century, the quadrant develops into a more complex instrument,which has 30 parallel lines which enable graphic calculation immediately. This type of developd quadrant was described by Padmanābha (ca.AD 1400), Jñānarāja (ca.AD 1503), Cakradhara etc.[4)]

It may be that the quadrant described in the *Vṛddha-vasiṣṭha-siddhānta* is a forerunner of those later developed quardants.

v) The bhagaṇa or nāḍīvalaya-yantra

The *bhagaṇa-yantra* of Lalla and the *nāḍīvalaya-yantra* of Bhāskara II are practically the same, and consist of an equatorial circle and a central perpendicular needle. So, this is a kind of the equatorial sundial.

Lalla described the *bhagaṇa-yantra* in his *Śiṣyadhī-vṛddhida-tantra* (XXI. 27-29).[1] The *bhagaṇa-yantra* consists of a circular plate, where its rim is marked with segments corresponding to the oblique ascensions of the zodiacal signs, and also with degrees, and a perpendicular needle at its centre. The segments of signs should be marked in reverse order, although Lalla does not explicitly say so. The board is kept in the plane of the equator, with its needle pointing to the north pole, in such a way that the shadow of the needle at sunrise points the sun's position (sign and degree) at the graduation of the rim. Then the shadow of the needle at desired time indicates the rising sign *(lagna)* at the time, and the angular distance between the shadow at desired time and at sunrise indicates the degrees corresponding to the time elapsed since sunrise. (See Fig. 41.) In the figure, the angle AOB is the angle corresponding to the time elapsed since susnrise. As the oblique ascension of a sign is the angle corresponding to the time required to ascend for the sign, the angle AOB indicates the signs already have ascended since sunrise. Therefore, the position of the shadow (B) indicates the sign on the horizon, i.e. *lagna*. (We should recall that the signs are marked in reverse order.)

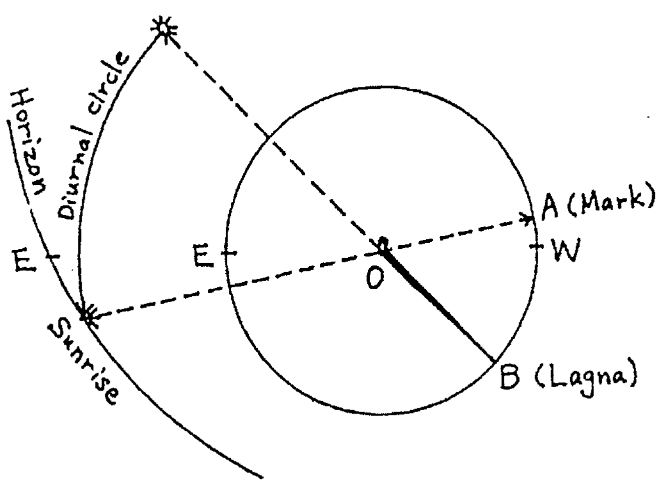


Fig. 41. Bhagaṇa-yantra or nāḍīvalaya-yantra (reconstructed)

Bhāskara II described the *nāḍīvalaya-yantra*, which is the same as the *bhagaṇa-yantra* of Lalla in principle, in his *Siddhānta-śiromaṇi* (*Gola* ,XI. 5-7).[2] Bhāskara II's *nāḍīvalaya-yantra* is graduated with *ghaṭikās* also.

vi) The kartarī-yantra

Brahmagupta described the *karttari-yantra* ("scissors instrument") in his *Brāhma-sphuṭa-siddhānta* (XXII. 43 (ii)-44) as follows.[1)]

कर्त्तरियन्त्रं स्थूलं कृतं यतो ऽन्यैर्वदामि ततः ।।43।।

दिक्स्थितफलकद्वियुतिस्तले तदग्रस्थसूत्रयोर्मध्ये ।
कीलस्तच्छायाग्रात् कर्त्तर्यां नाडिकाः स्थूलाः ।।44।।

"The *karttari-yantra,* which is rough, was made by others, so I shall explain it. Two boards, kept in proper directions, are joined at the base. At the centre where the lines from the ends [of the board] join lies the gnomon (*kīla*). From the tip of its shadow, gross *nāḍikās* are obtained on the *karttarī.*"

Brahmagupta does not write the mounting of this instrument explicitly, but probably it is a combination of a semi-circular board kept in the plane of the equator with its chord in the direction of east-west line, and another semi-circular board kept in the plane of the meridian with its chord in the direction of north and south poles. The chord of the latter is then considered to be gnomon. (See Fig.42.) Here, Brahmagupta says to obtain "gross *nāḍikās*", but actually the exact time can be obtained by this instrument, if the ascensional difference is added or substracted properly.

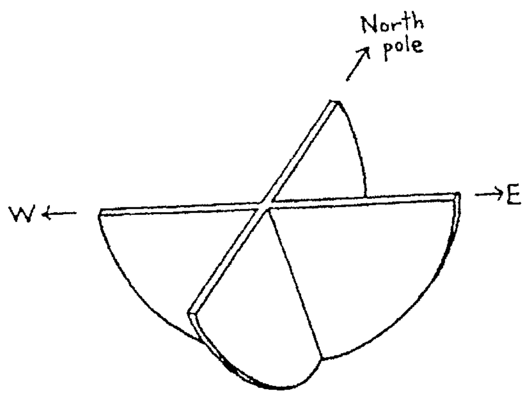


Fig. 42. Brahmagupta's kartarī (reconstructed)

Lalla and Śrīpati described a simplified version of the *kartarī-yantra*. Lalla wrote in his *Śiṣyadhī-vṛddhida-tantra* (XXI. 24) as follows.[2)]

समपूर्वापरमेतत् स्थिरं स्थितं भवति कर्तरीयन्त्रम् ।
ज्यामध्यस्थिततिर्यक्कीलच्छायोज्झिता घटिकाः ।।24।।

"When (the *Dhanur-yantra*) is permanently fixed in the plane of the equator (forming the lower half of the equator) (and a needle pointing towards the north pole is fixed at the middle of the chord), it is known as *Kartarī-yantra.* In this case, the *ghaṭīs* left behind by the shadow of the needle fixed at the middle of the chord denote the *ghaṭīs* elapsed since sunrise."(Translated by K.S. Shukla)[3)]

This *kartarī-yantra* is a kind of the equatorial sundial, just like the *kartarī-yantra* of Brahmagupta. However, the shape of Lalla's *kartarī-yantra* is not like the scissors (*kartarī*). (See Fig. 43.)

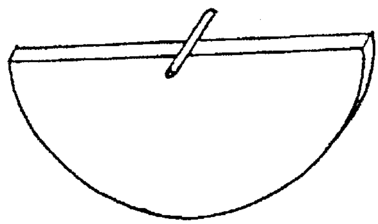


Fig. 43. Lalla and Śrīpati's kartarī (reconstructed)

Śrīpati also described the *karttarī* in his *Siddhānta-śekhara* (XIX.14) as follows.[4)]

ज्यामध्यतिर्यक्स्थितकीलमेतत्
पूर्वापरस्थं स्थिरकर्त्तरी स्यात्।
प्रत्यग्धनुः कोटिमुखात् द्युनाड्यः
समुज्झिताः कीलरुचा भवन्ति।।14।।

"When this *(dhanur-yantra)* , having a perpendicular needle at the middle of the chord, is fixed in the east-west direction, it is called *karttarī.* The western arc left behind by the shadow of the needle from the end of the arc indicates the *nāḍīs* elapsed in the day".

From the above quotations, it is seen that the *kartarī-yantra* of Lalla and Śrīpati is the same, and consists of the semi-circular board in the plane of the equator and a perpendicular needle at the middle of its chord. (See Fig. 43.) Its shape is not like the scissors (*kartarī*), but it can be said to be a simplified version of Brahmagupata's *kartarī-yantra,* which is indeed like the scissors. Both types of the *kartarī-yantra* give the same result.

vii) The kapāla-yantra

The name "*kapāla-yantra*" ("bowl instrument") has been used for three different instruments as follows.

(1) The hemispherical clepsydra, described in the *Āryabhaṭa-siddhānta,* and the modern *Sūrya-siddhānta.* (We shall discuss this instrument later under the section of the

clepsydra.)

(2) The hemispherical sundial, described in the *Pañca-siddhāntikā,* and the *Brāhma-sphuṭa-siddhānta.*

(3) A simplified version of the 2nd case, which consists of a horizontal semi-circular board and a vertical gnomon,described in the *Śiṣyadhī-vṛddhida-tantra,* and the *Siddhānta-śekhara.*

Let us see the hemispherical sundial first. The *Pañca-siddhāntikā* (XIV.19-20) of Varāhamihira reads as follows.[1)]

छेद्यवदर्धकपालं सचिह्नमक्षोन्नतं सदिक्चक्रम्।
सुसमावनिविन्यस्तं कुर्याद् व्यस्तं सनाभ्यङ्कम्।।19।।

सूत्रद्वयसम्पातच्छायाभुक्तांशका रवौ देयाः।
स भवति उदयो राशिर्दिनस्य नाड्यश्च ता याताः ।।20।।

"One should make a concave *kapāla* ("bowl") whose radius is the same as the level circle *(chedya),* having marks [of degrees and signs], [marks of solar] altitude at the [observer's] latitude (?), circles of directions, and a mark of its centre, and being fixed on a well-levelled ground.

The degrees that have been passed by the shadow of the intersection of two strings [passing through the cardical points at the rim] are to be added to [the longitude of] the sun. The result is the rising sign (i.e. *lagna*). The *nāḍīs* elapsed in the day are also obtaned [by dividing the degrees by six]."

David Pingree translated the expression "*akṣonnatam*" as "tilt it by (the amount of) the terrestrial latitude".[2)] but this translation is not acceptable in this context. In order to obtain the time elapsed since sunrise and the longitude of the rising point of the ecliptic (*lagna*) (neglecting the ascensional differeces), the rim of the hemispherical sundial should be horizontal. (See Fig. 44.)

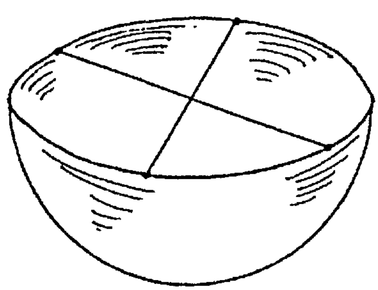


Fig. 44. Varāhamihira and Brahmagupta's kapāla (reconstructed)

Now, let us see the *Brāhma-sphuṭa-siddhānta* (XXII. 42-43 (i)) of Brahmagupta, which reads as follows.[3)]

मध्याद्यस्वनतांशैः कपालकं दिक्स्थसूत्रमध्याग्रात्।
व्यस्तोन्नतांशविवरे सूत्रैक्यापाततो नाड्यः।।42।।

अथवा कपालके नाडिकादि सर्वं यथा धनुष्युक्तम्।

"The *kapālaka* [is marked] with zenith distance starting from the centre. Using the intersection and ends of [two] strings passing through the cardinal points, the *nāḍīs* [are known] as the distance indicating the reverse altitude (on the concave hemisphere) from the shadow of the intersection of the strings.

Otherwise, everything, *nāḍīs* etc., [may be marked] just like the case of the *dhanus* (semi-circle instrument)."

It appears that this Brahmagupta's *kapālaka* is also the hemispherical sundial with horizontal rim, just like the *kapāla* of Varāhamihira. (See Fig. 44)

Now, let us see the simplified version of the *kapāla*, described by Lalla and Śrīpati.

Lalla wrote in his *Śiṣyadhī-vṛddhida-tantra* (XXI. 25 (i)) as follows.[4)]

इदमेवोर्ध्वशलाकं भुवि स्थितं स्यात् कपालकं यन्त्रम्।

"This very (*Kartarī-yantra*), with its dial set horizontally on the ground and its needle vertical, is called *Kapāla-yantra*." (Translated by K.S. Shukla)[5)]

From this quotation, it appears that Lalla's *Kapāla-yantra* is a horizontal semi-circular board with a vertical needle. (See Fig. 45.)

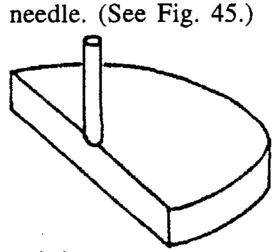


Fig. 45. Lalla and Śrīpati's kapāla (reconstructed)

Śrīpati wrote in his *Siddhānta-śekhara* (XIX.15 (i)) as follows.[6)]

इदं भवेदूर्ध्वशलाकमुर्व्यां
स्थितं कपालं द्युतिदिक् च चापम्।

"When this (*Karttarī-yantra*) is kept on the ground, with its needle directed upwards and the semi-circle in the direction of the shadow, it is called *kapāla*."

This *kapāla* of Śrīpati is evidently the same as the *kapāla* of Lalla. (See Fig. 45.) This instrument can be used to obtain azimuth only. So, the name *kapāla* ("bowl") seems to be misnomer, and it seems that Lalla and Śrīpati simplified the *kapāla* too much.

viii) The pīṭha-yantra

The *pīṭha-yantra* ("seat instrument') is a horizontal circle instrument. Brahmagupta wrote in his *Brāhma-sphuṭa-siddhānta* (XXII.45) as follows.[1)]

दृष्ट्यौच्च्यं समपीठं यष्टिव्यासार्धमन्तिकं परिधौ।
दिग्भगणांशैर्मूर्धन्यग्राघटिका दिभिश्चाङ्क्यम्।।45।।

"The *pīṭha*, which is flat, is kept at the eye-level, and its radius is approximately the length of the *yaṣṭi*. Its circumference is marked with cardinal points and degrees. Its surface is also marked with *agrā* (sun's amplitude) and *ghaṭikās* etc."

This *pīṭha-yantra* is a horizontal circle kept at the eye-level. Brahmagupta did not write whether it had a central needle or not.

Lalla described the *pīṭha-yantra* in his *Śiṣyadhī-vṛddhida-tantra* (XXI. 25 (ii)) as follows.[2)]

चक्रे (चक्रं) चोध्र्वशलाकं वदन्ति पीठं सुसिद्धाशम्।।25।।

"...and the *Cakra-yantra*, with its dial having the directions marked in its rim, and set on the gound with its axis vertical, is called *Pīṭha-yantra*." (Translated by K.S.Shukla)[3)]

Lalla further writes (XXI.26) that the *ghaṭīs* elapsed, which is denoted by the arc left behind by the shadow of the needle, and the degrees of the sun's amplitude *(agrā)*, represented by the arc between the sun's rising point and the east point,are obtained by this instrument (See Fig. 46.)

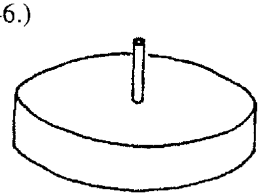


Fig. 46. Pīṭha-yantra (reconstructed)

Śrīpati described the *pīṭha-yantra* in his *Siddhānta-śekhara* (XIX. 15(ii)-17) as follows.[4)]

संसाधिताशं खलु चक्रयन्त्रं
पीठं भवत्यूर्ध्वशलाकमेव ।।15।।

मध्यस्थकीलप्रभया विमुक्ताः
प्रत्यग्गतास्ता घटिका निरुक्ताः ।
पीठे तु सूर्योदयबिम्बवेधा–
द्भुक्तांशजीवा स्फुटमग्रका स्यात् ।।16।।

या यन्त्रसिद्धा द्युगतास्तु नाड्य–
स्ताः स्वद्युमानेन हता विभक्ताः ।
नभोगुणैः 30 स्पष्टतरा भवन्ति
नाड्यो ऽन्यथा स्थूलतरा निरुक्ताः ।।17।।

"The *cakra-yantra* (circle instrument) [which is settled horizontally] with a vertical needle and marks of cardianl points (and degrees) is the *pīṭha* ("seat").

[The arc] left behind in the west [of the rim of the circle] by the shadow of the needle standing at the centre is considered to be the *ghaṭikās*. The R. sine of the angle [between the rising sun and the east cardinal point], obtained from the observation of the disc of the sun at sunrise on the *pīṭha*, is exactly the R.sine of the sun's amplitude (*agrakā*).

The *nāḍīs* elapsed in the day which are obtained by the instrument should be multiplied by the length of daytime and divided by 30. They are more correct *nāḍīs*. Otherwise, they are considered to be more rough''.

This *pīṭha-yantra* of Śrīpati is the same as the *pīṭha-yantra* of Lalla, and is a horizontal circular board with a central vertical needle. (See Fig. 46.) According to the above text, the azimuthal difference betwen the east (or west) point and the sun is observed, and its corresponding time is corrected as follows.

$$\text{corrected time} = \frac{\text{(observed time)} \times \text{(length of daytime)}}{30}$$

Even this correction is applied, correct time cannot be obtained from the sun's azimuth.

ix) The earthen platform

The earthen platform *(sthalī)* is used to observe the sun's amplitude. One of the earliest descriptions of the earthen platform is the *Mahābhāskarīya* (III. 56-60(i)) of Bhāskara I.[1)]

Bhāskara I says to erect a circular platform, as high as one's neck, with its floor in the same level, and to graduate its circumference with the divisions of signs, degrees etc., and cardinal points. Then, one should stand on its western side, and observe the rising sun in such a way that it appears clinging to the circumference, and the line of sight passes through the centre of the circular platform. The arc of the circumference between the east cardinal point and the point where the sun is observed is the arc of the sun's amplitude. (See Fig. 47)

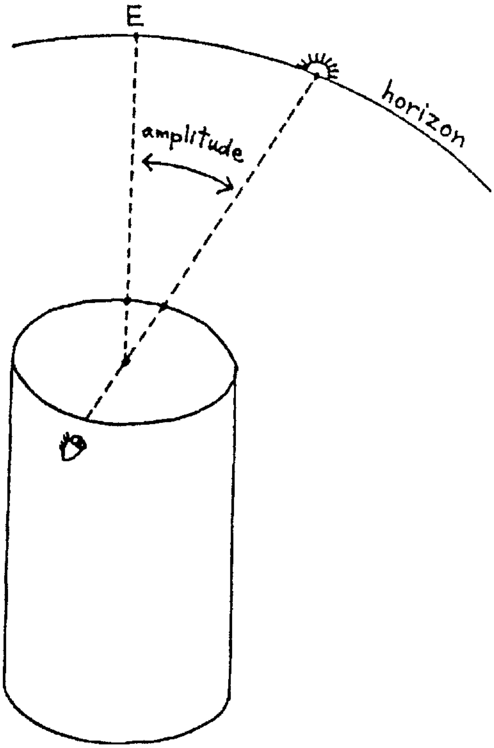


Fig. 47.

This earthen platform is practically the same as the *pīṭha-yantra.*

x) Consulsion

From the above discussions, it appears that Āryabhaṭa's *cakra* and *dhanus* are different from later instruments in some respects. It may be that the time of Brahamgupta was the period of transition, and he recorded some ancient type of the instruments. For example, the *kartarī* of Brahmagupta and the *kapāla* of Varāhamihira and Brahmagupta must be the original type of these instruments. At the time of Lalla, some instruments such as *kartarī* and *kapāla* were simplified, and their shape was changed. Śrīpati seems to have followed Lalla, and described almost the same instruments. Bhāskara II also described similarly, but he only described some important instruments, and omitted the *kartarī, kapāla* etc. A very insteresting description of the quardrant is found in the *Vṛddha-vasiṣṭha-siddhānta*, which might be one of the forerunners of the later developed quadrant. We have also noted in the previous section that the horizontal gnomon has been described in this text, which is not found in othr early Siddhāntas. From the above discussions, we can see the evidence of the development of astronomical instruments in Classical Siddhānta period.

The variations of the circle instrument can be classified into three groups, i.e. vertical, equinoctial, and horizontal variations. The altitude (or zenith distance), the hour angle (or its complementary angle), and the azimuth of a heavenly body can be obtained by respective instruments. The time can be exactly determined by the equinoctial type, if the sun's ascensional difference is corrected. The time is roughly obtained from the sun's altitude or azimuth in the case of vertical or horizontal type.

7. The Celestial Globe and the Armillary Sphere

i) Introduction

Both the solid celestial globe and the armillary sphere which consists of hoops are called *gola-yantra.*. The celestial globe was first described by Āryabhaṭa in his *Āryabhaṭīya* .This is uniformly rotated with the help of mercury, oil, and water. Varāhamihira also described the celestial globe. As regards the methods to rotate the globe, we shall discuss in the section of the self-rotating globe.

The armillary sphere was described by Brahmagupta, Lalla, Śrīpati, and Bhāskara II. They wrote a special section or a chapter on the armillary sphere entitled *Golabandha*. Lalla, Śrīpati, and Bhāskara II described the armillary sphere in the *Yantrādhyāya* also. And also the armillary sphere has been described in the modern *Sūrya-siddhānta*.

It may be mentioned here that the armillary sphere has been described in the *Soma-siddhānta (Golādhyāya)*[1] and also in the *Gola-dīpikā* I (AD 1443)[2] of Parameśvara.

ii) The celestial globe

Ārybhaṭa described a rotating model of the celestial globe in his *Āryabhaṭīya* (IV.22) as follows.[1]

काष्ठमयं समवृत्तं समन्ततः समगुरुं लघुं गोलम्।
पारदतैलजलैस्तं भ्रमयेत् स्वधिया च कालसमम्।।22।।

"The Sphere *(Gola-yantra)* which is made of wood, perfectly spherical, uniformly dense all round but light (in weight) should be made to rotate keeping pace with time with the help of mercury, oil and water by the application of one's own intelligence." (Translated by K.S. Shukla)[2]

Āryabhaṭa did not write how to rotate the globe, but later authors made several devices to rotate the globe, some of which are possible to make, while some of which are practically impossible to make. (See below, under the section of the self-rotating globe)

A commentator Someśvara commented on the above verse that the globe should be rotated by a string whose one end is attached to a nail on the globe and the other side is attached to a gourd. The gourd is floating on water in a cylindrical container which has a hole near its bottom. As water flows out from the hole, the gourd pulls the string. Mercury is contained by the gourd in order to make its weight suitable.[3] (See Fig. 48.)

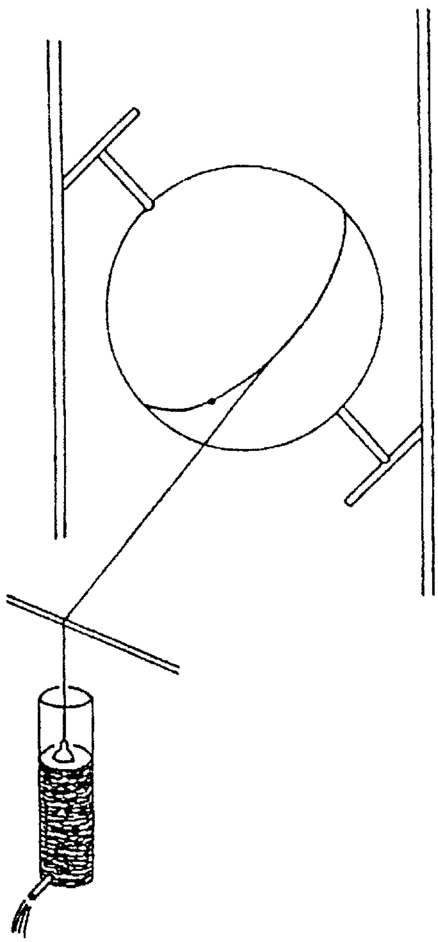


Fig. 48. Rotating model of celestial globe (reconstructed)

Probably the above method is what Āryabhaṭa himseld was also thinking, because the device of the water instruments in his *Āryabhaṭa-siddhānta*[4] is the same as above.

Varāhamihira also described the celestial globe in his *Pañca-siddhāntikā* (XIV.23-25). His globe is not a self-rotating globe, but an instrument to represent time. The *Pañca-siddhāntikā* (XIV.23-25) reads as follows.[5)]

समवृत्तपृष्ठमानं सूक्ष्मं गोलं प्रसाध्य दारुमयम्* ।
स्थगितार्कसमांकितकालभोगरेखाद्वये परिधौ ।।23 ।।

(Note:) * ..."*dhātumayam*" in another manuscript.

याम्योदग्रेखाया झषाजसन्ध्युभयतो न्यसेद्वेधात् ।
अयनांशकाङ्कतुल्यांस्तिर्यग्वेधप्रकाशकरान् ।।24 ।।

अक्षोत्क्षिप्तस्योदक् तिर्यग्वेधप्रकाशहरिजस्थाः ।
या नाड्यस्ता याताः षडंशकसमन्विता मध्ये ।।25 ।।

"Make a perfectly round fine wooden (or metal, acording to another manuscript), globe *(gola),* [and draw] two circles representing the equator *(kāla-rekhā)* and the ecliptic *(bhoga-rekhā)* which is marked with the sun's position when it stops (i.e. two solstices), on the surface.

By the observation of [the sun's altitude] on the meridian, [obtain the sun's declination corresponding to the sun's longitude, and] make the graduation as the marks for the oblique observation [of the sunlight] with the figures of [the sun's] declination [on the ecliptic] on the both sides of the junction of Pisces and Aries (i.e the vernal equinox).

Raising the axis [of the globe] towards the north by the amount of the observer's latitude, [the degrees] between the horizon and the mark for the oblique observation [directed towards the sun] divided by six correspond to the *nāḍīs* elapsed [since sunrise]."

It appears from the above description that this celestial globe is a model of the celestial sphere in order to explain the relationship between the position of the sun and the time, rather than an instrument for observation.

It should be noted that Āryabhaṭa and Varāhamihira described the celestial globe, but did not describe the armillary sphere which consists of hoops and used for actual observations.

iii) The armillary sphere in the Golabandha

Some Siddhāntas have a special section or a chapter on the armillary sphere entitled *Gola-bandha,* namely the *Brāhmasphuṭa-siddhānta* (XXI. 49-69), the *Śiṣyadhī-vṛddhida-tantra* (whole of chapter XV), the *Siddhānta-śekhara* (XVI. 29-39), and the

Siddhānta-śiromaṇi (Gola, whole of chapter VI). Let us read the full text of the *Brāhma-sphuṭa-siddhānta* (XXI.49-69) first, as it is the earlest *Golabandha,* although it is rather a presentation of sphrics than the description of an instrument. The description of the actual armillary sphere is more clearly given in the *Yantrādhyāya* of Lalla etc., which we shall see later.

a) Brahmagupta's Golabandha

Let us see the texts of the *Golabandha-adhikāra* of the *Brāhma-sphuṭa-siddhānta* (XXI. 49-69)[1] of Brahmagupta one by one.

प्राच्यपरं सममण्डलमन्यद्याम्योत्तरं क्षितिजमन्यत् ।
परिकरवत् तन्मध्ये भूगोलस्तत्स्थितद्रष्टुः ।।49।।

"49. [There is] the prime vertical *(sama-maṇḍala)* in the east-west direction, and another [circle called] meridian *(yāmya-uttara)*, and also another [circle called] horizon *(kṣitija)* which is like a girdle. At its centre is the earth *(bhū-gola)* on which the observer stands."

पूर्वापरयोर्लग्नं याम्योत्तरयोर्नतोन्नतं क्षितिजात् ।
स्वाक्षांशैरुन्मण्डलमहर्निशोर्हानिवृद्धिकरम् ।।50।।

"50. The six o'clock circle *(un-maṇḍala)*, which causes the increment and decrement of daytime and nighttime, has two intersections with the horizon in the east and the west, and is inclined to the horizon in the south and north by the degrees of latitude of the observer *(sva-akṣa-aṁśa)*."

विषुवन्मण्डलमूर्ध्वं सममण्डलतः स्थितं स्वकाक्षांशैः ।
याम्येनोत्तरतो ऽधः क्षितिजे प्राच्यपरयोर्लग्नम् ।।51।।

"51. The equator *(viṣuva-maṇḍala)* is inclined to the prime vertical towards the south at the top, and towards the north at the bottom, by the degrees of latitude of the observer, and has two intersections with the horizon in the east and the west."

विषुवन्मण्डललग्नं मेषतुलादावुदक् कुलीरादौ ।
जिनभागैर्याम्येन मृगादावपमण्डलमिहार्कः ।।52।।

पाताश्चन्द्रादीनां भ्रमन्ति भार्धे रवेश्च भूछाया ।

"52-53(i). The ecliptic *(apa-maṇḍala)* has intersection with the equator at the first points of Aries and Libra, and is inclined [to the ecliptic] towards the north at the first point of Cancer and towards the south at the first point of Capricorn by 24 degrees. The sun, the nodes *(pāta)* of the moon etc. (i.e. moon and five planets), and the earth's shadow at the diagonal direction to the sun revolve here."

पातादपमण्डलवद् विमण्डलानि स्वविक्षेपैः।।53।।

सौम्यं विमण्डलार्धं प्रथमं याम्यं द्वितीयमेतेषु।
चन्द्रकुजजीवमन्दा भ्रमन्ति शीघ्रेण बुधशुक्रौ।।54।।

"53(ii)-54. The orbits *(vi-maṇḍala)* [of the moon and five planets] are inclined to the ecliptic from the nodes according to their own latitudes *(sva-vikṣepa).* The first half of the orbit is the north [to the ecliptic], and the second half is the south. Along them, the moon, Mars, Jupiter and Saturn, and also Mercury and Venus according to their *śīghra* (epicyclic motion) revolve."

(Notes:) In the case of the moon, Mars, Jupiter and Saturn, the *vi-maṇḍala* can be considered to be their deferents. In the case of Mercurry and Venus, their *śīghra-uccas* (an imaginary point whose direction from the earth is the same as the direction of the mean Mercury or Venus from the sun) revolve along their *vi-maṇḍalas.*

दृग्मण्डलार्धमूर्ध्वं यत् तत्परिधिस्थितं ग्रहं द्रष्टा।
पश्यति यतः क्षितिस्थस्तद् भ्रमति ततो ग्रहाभिमुखम्।।55।।

"55. A half of the *dṛg-maṇḍala,* which is above [the horizon], revolves in such a way that it faces the planet, and that the observer standing on the ground looks at the planet situated on its circumference."

(Note:) The *dṛg-maṇḍala* is a great circle which passes through the zenith and the planet.

क्षितिजापमण्डलयुतिर्लग्नं लग्नाग्रया दिशा लग्नम्।
दृक्क्षेपमण्डलं दक्षिणोत्तरं वित्रिभविलग्ने।।56।।

"56. The intersection of the horizon and ecliptic is the *lagna* (rising point of the ecliptic). The *[dṛk-kṣepa-maṇḍala's] lagna* (intersection with the horizon) [is shifted from the north or south point] towards the same direction and by the same amplitude as the [ecliptic's] *lagna.* The *dṛk-kṣepa-maṇḍala* runs towards the south and north, passing through the central ecliptic point *(vitribha-vilagna).*"

(Note:) The *dṛk-kṣepla-maṇḍala* is a great circle which passes through the pole of ecliptic and the zenith. So, it intersects with the eclipctic at the mid-point of the visible half of the ecliptic (i.e. *vitribha-vilagana)* at right angle.

विषुवदुदग्बध्नीयात् क्रान्त्यंशसमान्तरेष्वजादीनाम्।
वृत्तत्रितयं व्यस्तं कर्क्यादीनां तुलादीनाम् ।।57।।

विषुवद्दक्षिणतो ऽन्यन्मकरादीनां तदेव विपरीतम् ।
स्वाहोरात्राण्येषां व्यासाः पृथगेवमिष्टमपि ।।58।।

"57-58. One should bind three [diurnal] circles beginning with Aries towrds the north from the equator at the distance of their declination, [other three diurnal circles] beginning with Cancer reversely (form the north towards the equator), others beginning with Libra towards the south, and others beginning with Capricorn reversely. Their own diurnal circles (*sva-ahorātra)* and their diameters [are represented here] severally. Like this, [the diurnal circle of] desired star [may be made] also."

लङ्कासमपश्चिमगं प्राणेन कलां भूमण्डलं भ्रमति।
अपमण्डलस्य राशिर्द्वादशभागः क्षितिजलग्नाः।।59।।

यान्त्युदयं मेषाद्या यतस्तदुदया न कालसमाः।
क्रान्तिवशाल्लङ्कायां तदूनताधिक्यमक्षवशात्।।60।।

"59-60. The [diurnal] circle of a star *(bha-maṇḍala)* revolves towards the right west at Laṅkā (on the terrestrial equator) at the velocity of 1 minute (of arc) per 1 *prāṇa* (4 seconds). One 12th of the ecliptic is a sign *(rāśi).* The *lagna* (rising point of the ecliptic) on the horizon ascends, beginning with Aries, but their ascensions are not equal even at Laṅkā, because of the effect of their declination (i.e. effect of the reduction to the equator). Further, [the oblique ascension is] increased or decreased according to he effect of the observer's latitude."

क्षितिजोन्मण्डलयोर्यत् स्वाहोरात्रान्तरं चरदलं तत्।
क्षितिजे ऽग्रा प्राच्यपरस्वाहोरात्रान्तरांशज्या।।61।।

"61. The arc of the [sun's]diurnal circle intervened by the horizon and the six o'clock circle corresponds to the [sun's] ascensional difference *(cara-dala).* The R.sine of the degrees between the east or west point and the diurnal circle on the horizon is the *agrā* (R.sine of the sun's amplitude)."

स्वाहोरात्रे क्षितिजाद्दिनगतशेषोच्चता रवेः शङ्कुः।
तस्माद्दिनगतशेषं शङ्कुकुमध्यान्तरं दृग्ज्या।।62।।

"62. The height, in the forenoon or afternoon, of the sun on the diurnal circle from the horizontal plane is the *śaṅku* (R sine of the altitude). The distance, in the forenoon or afternoon, between [the foot of] the *śaṅku* and the centre of the ground is the *dṛg-jyā* (R.sine of the zenith distance)."

(Note:) We have discussed these relations in the section of the staff.

दृग्मण्डले नतांशज्या दृग्ज्या शङ्कुरुन्नतांशज्या।
अर्कोदयास्तसूत्राद्दिनशङ्कोर्दक्षिणेन तलम्।।63।।

"63. On the *drg-maṇḍala* (see vs.55), the R sine of the zenith distance *(natāṁśa-jyā)* is the *drg-jyā*, and the R sine of the altitude *(unnatāṁśa-jyā)* is the *śaṅku.* Towards the south from the sun's rising-setting line is the *śaṅku-tala* of the day."

(Note:) The *śaṅku-tala* is the distance between the foot of the *śaṅku* and the sun's rising-setting line, as we have discussed in the section of the staff.

दृश्यादृश्यं दृग्गोलार्धं भूव्यासदलविहीनयुतम्।
द्रष्टा भूगोलोपरि यतस्ततो लम्बनावनती।।64।।

"64. The visible and unvisible spheres [of planets] are smaller and larger [respectively] than a half of the sphere of planets *(drg-gola)* by the radius of the earth. As the observer stands on the earth, the *lambana* (parallax in celestial longitude) and the *avanati* (parallax is celestial latitude) [are produced]."

क्षितिजे भूदललिप्ताः कक्षायां दृङ्नतिर्नभोमध्यात्।
अवनतिलिप्ता याम्योत्तरा रविग्रहवदन्यत्र।।65।।

"65. On the horizon, [the parallax is maximum which is equal to] the minutes (of arc) corresponding to the radius of the earth on the orbit [of the planet]. The *drg-nati* is from the zenith [towards the secondary to the ecliptic passing through the planet]. The minutes (of arc) of the *avanati* is in the north-south direction (along the secondary to the ecliptic). Others are the same as the case of solar eclipse."

(Note:) The *drg-nati (= drg-gati)* is a quantity which is used for the calculation of parallax, and is the arc dropped from the zenith perpendicularly to the secondary to the ecliptic passing through the plannet. The theory of parallax is explained in the section of solar eclipse of Siddhāntas.

सत्रिगृहक्रान्तिरुदग्दक्षिणयोस्त्रिज्यया हृतं वलनम्।
विक्षेपगुणमृणधनं ग्रहे ऽन्यदृक्कर्म चरदलवत्।।66।।

"66. The [*ayana-*] *valana* is the declination, towards the north or south, of the point on the ecliptic whose longitude is of the star increased by 90°. The [R.sine of the *ayana-*] *valana* multiplied by the [R.sine of the star's] latitude and divided by the Radius is the [R.sine of the *ayana-*] *drk-karman*, which is negative or positive.

The other [*akṣa-*] *drk-karman* [is obtained] just like the ascensional difference."

(Note:) The *ayana-valana* is the angle between the secondaries to the equator and to the ecliptic at the star. It can be calculated as in the text approximately.

The *ayana-dṛk-karman* is the arc of the ecliptic interred by the secondaries to the equator and to the ecliptic, both passing through the star. This is the difference between the star's celestial longitude and polar longitude.

The *akṣa-dṛk-karman* is the arc of the ecliptic intervened by the horizon and the hour circle passing through the star when it is rising. This is the polar logitudinal difference between the lagna and the rising star.

कक्षामण्डलतुल्यं प्राच्यपरं दक्षिणोत्तरं क्षितिजम्।
उन्मण्डलविषुवन्मण्डले स्थिराणि ग्रहर्क्षाणाम्।।67।।

"67. The prime vertical, meridian, horizon, six o'clock circle, and equator, whose radius is equal to the radius of the deferent *(kakṣā-maṇḍala)* of planet, are fixed for each planet."

मन्दोच्चानां सप्तोच्चनीचवृत्तानि पञ्च शीघ्राणाम्।
प्रतिमण्डलानि चैवं प्रत्येकं भास्करादीनाम्।।68।।

दृग्मण्डलवि (दृक्) क्षेपापमण्डलानि क्षपाकरादीनाम्।
षट्कं विमण्डलानां चलवृत्तान्येकपञ्चाशत्।।69।।

"68-69. There are 7 *manda*-epicycles *(manda-ucca-nīca-vṛtta)* and 5 *śīghra*-epicycles *(śīghra-ucca-nīca-vṛtta)*. There are eccentric circles *(prati-maṇḍala)* also for each, the sun etc.

There are *dṛg-maṇḍalas* (see vs.55), *dṛk-kṣepa-maṇḍalas*(see vs.56), and *apa-maṇḍalas* ("ecliptic" for each radius) for the moon etc. There are 6 *vi-maṇḍalas* (orbits). [Thus,] the movable circles are 51 in number."

(Note:) Fifty one movable circles are as follows.

	number	hevenly bodies concerned
manda-epicycle	7	S,M,P.
śīghra-epicycle	5	P.
manda-eccentric circle	7	S,M,P
śīghra-eccentric circle	5	P.
dṛg-maṇḍala	7	S,M,P.
dṛk-kṣepa-maṇḍala	7	S,M,P.
apa-maṇḍala	7	S,M,P.
vi-maṇḍala	6	M,P.
total	51	

(Abbreviations: S=sun, M=moon, P=planets.)

It will be better to explain Hindu planetary theory just briefly. Two corrections are applied to mean *(madhya)* planets in order to obtain true *(sphuṭa)* planets. One correction is the *manda*-correction *(manda-karman)*, which corresponds to our equation of centre. The other correction is the *śīghra*-correction *(śīghra-karman)*, which corresponds to annual parallax in the case of outer planets, and planet's own revolution in the case of inner planets. Both corrections can be explained by epicycle model as well as eccentric model. Firstly, the *manda*-correction is applied to mean planet, which corresponds to the planet's own mean revolution in the case of outer planets, and mean sun in the case of inner planets, and "*manda-sphuṭa*-planet" is obtained. So, the "*manda-sphuṭa*-planet" is mean planet corrected by the equation of centre only. Then, the *śīghra*-correction is applied to the "*manda-sphuṭa*-planet", and the true *(sphuṭa)* planet is obtained. (In the actual calculation, some special technique, such as successive approximation, is used, but we shall not discuss about it here.)[2]

Now let us see the eccentric model, which can be used for both *manda*-correction and *śīghra*-correction. (See Fig.49.). In the figure, E is the earth, U the "*ucca*", P the uncorrected planet, and P' the corrected planet. The "*ucca*" is the apogee (*manda-ucca*) in the case of *manda*-correction, the point coresonds to the mean sun in the case of *śīghra*-correction for outer planets, and the point corresponds to the revolution of mean planet (in helio centric model) in the case of *śīghra*-correction for inner planets. In *śīghra*-correction, the *ucca* is called *śīghra-ucca*. The *ucca* revolves along the *ucca-nīca-vṛtta* around the earth. The corrected planet P' revolves along the eccentric circle (*prati-maṇḍala*) around the *ucca*, in such a way that its direction from the *ucca* is parallel to the direction of the uncorrected planet P from the earth. The direction of P' from the earth is the directioin of the corrected planet.

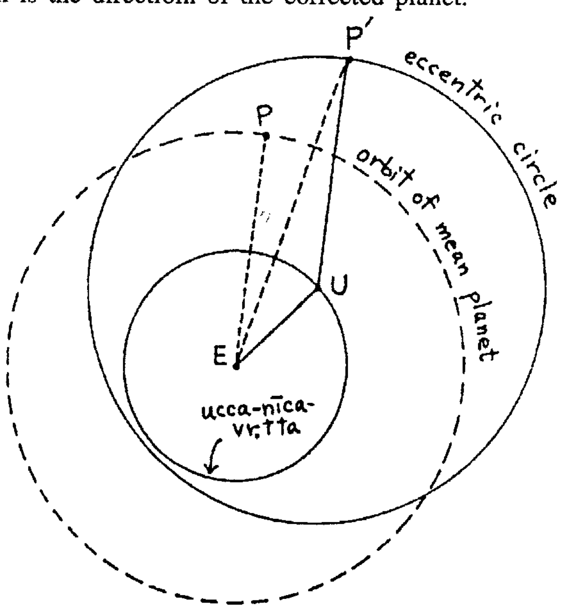


Fig. 49.

The same correction can be done by epicyclic model. (See Fig.50). Here also, E is the earth, U the *ucca*, P the uncorrected planet, and P' the corrected planet. The uncorrected planet revolves along the deferent (*kakṣā-vṛtta*) around the earth. The corrected planet P'revolves along the epcycle (*ucca-nīca-vṛtta*) around the uncorrected planet P, in such a way that its direction from the uncorrected planet P is parallel to the direction of the *ucca* form the earth. (The *ucca* also revolves along the deferent.) The direction of P' from the earth is the direction of the corrected planet. Evidently, this method gives the same result as the eccentric model.

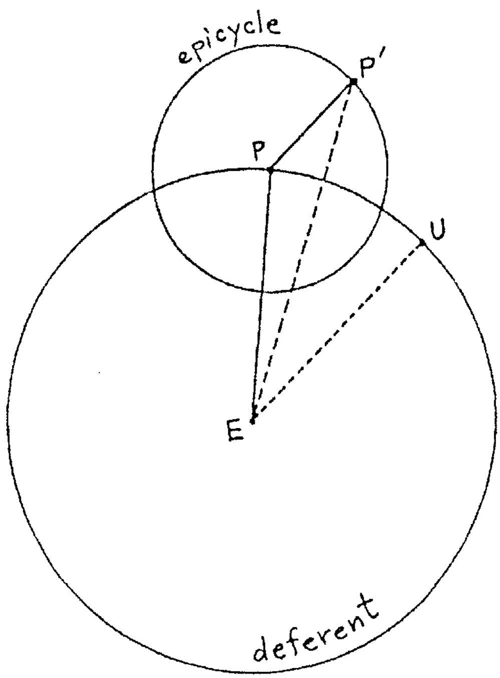


Fig. 50.

In the *manda*-correction, the angular distance between the mean planet and the *manda-sphuṭa*-planet is called *manda-phala*. In the *śīghra*-correction, the angular distance between the *manda-sphuṭa*-planet and the true planet is called *śīghra-phala*.

b) Lalla, Śrīpati, and Bhāskara II's Golabandha

Lalla devoted a whole chapter of his *Śiṣyadhī-vṛddhida-tantra* (chapter XV) to the armillary sphere as the *Golabandha-adhikāra.*[3)] Lalla classified the circles of the armillary sphere into three groups, i.e. the *kha-gola*, *bha-gola*, and *graha-gola*. The *kha-gola* (sphere of sky) represents horizontal system of coordinates, and consists of the prime vertical, meridian, two *koṇa*-circles (great circles passing throuth the zenith and the south-east and north-west point, and south-west and north-east points), horizon, and six o'clock circle. The *bha-gola* (sphere of stars) represents equatorial and ecliptic coordinates, and consists of the equator, solstitial colure, equinoctial colure, ecliptic, diurnal circles, and also the axis passing through the celestial poles with a central model of the earth. The *graha-gol*a is the sphere for each planet.

Śrīpati explained the armillary sphere in the *Golabandha* in his *Siddhānta-śekhara* (XVI.29-39).[4] Śrīpati wrote to construct a model of the armillary sphere at the beginning of the *Golabandha* (XVI.29(i)) as follows.

श्रीपर्ण्यादिससारदारुघटितैः श्लक्ष्णैः समैर्मण्डलै–
र्गोलज्ञो दृढसन्धिबन्धरुचिरं गोलं विनिर्मापयेत्।

"The knower of spherics may construct an armillary sphere (*gola*) with thin smooth circles made of hard timber such as *śrīparṇi* (Gmelina) etc., which are beautifully joined to each other firmly."

This is the erliest instruction of the practical method of the construction of the actual model of the armillary sphere. Otherwise, Śrīpati's description is similar to that of Brahmagupta etc.

Bhāskara II devoted a whole chapter for the description of the armillary sphere in his *Siddhānta-śiromaṇi* (*Gola*, chapter VI) 's *Golabandha-adhikāra*.[5] Bhāskara II instructed to make the armillary sphere *(gola)* with circles made of bambo (*vaṁśa*). Besides the *kha-gola* and *bha-gola*, he added the *dṛg-gola* outside the *kha-gola.* It is a mixture of the *kha-gola* and *bha-gola,* and it is put there only to exhibit inner circles. The *kha-gola* and *dṛg-gola* are not movable, and the *bha-gola* can be rotated around the axis. In the *kha-gola*, he added a fixed equator and a movable azimuth circle which can be rotated around the vertical line.

iv) The armillary sphere in the Yantrādhyāya

The armillary sphere has been described in the *Yantrādhyāya* of some Siddhāntas, namely, the *Śiṣyadhī-vṛddhida-tantra* (XXI.3-7) of Lalla, the modern *Sūrya-siddhānta* (XIII.3-15), the *Siddhānta-śekhara* (XIX.3-6) of Śrīpati, and the *Siddhānta-śiromaṇi* (*Gola*, XI.3-4) of Bhāskara II. Among them, the description in the *Sūrya-siddhānta* is relatively detailed, but others are brief, because the subject has been treated in detail in the *Golabandha.*

Lalla[1] wrote to construct the armillary sphere in order to determine the *lagna* and time. It has the *bha-gola* and the *kha-gola.* At the time of the observation , one should first fix a pin from the point of the equator which rises at sunrise towards the point of the ecliptic which corresponds to the sun's longitude. Then the *bha-gola*, which includes the equator and ecliptic, is rotated in such a way that the shadow of the pin passes through the centre of the sphere. Then the arc of the ecliptic intervened by the pin and the horizon indicates the degrees which have risen since sunrise, and the arc of the eguator intervened by the pin and the horizon indicates the time elapsed since sunrise. (See Fig. 51.)

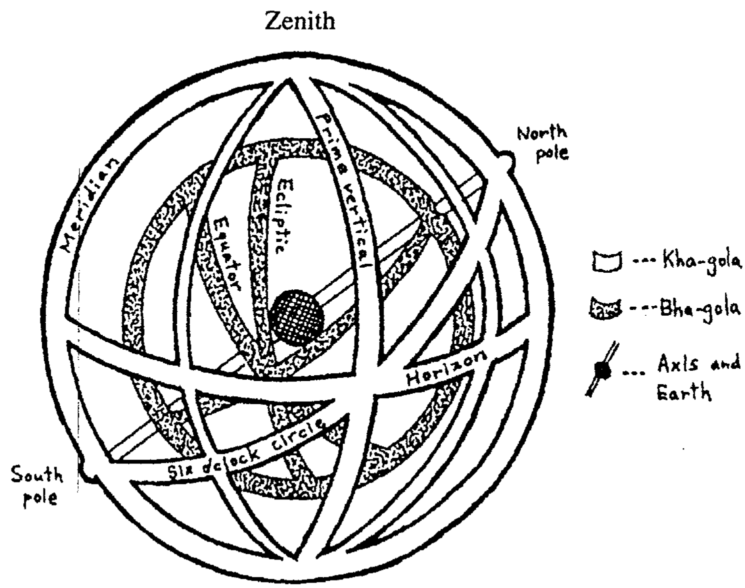


Fig. 51. Armillary sphere (reconstructed).

The *Sūrya-siddhānta*[2)] says to construct the armillary sphere with a wooden terrestrial globe of desired size at the middle of its axis, which has the solstitial and equinoctial colures as the supporting hoops, the equator, diurnal circles, and the ecliptic. Here, the method of the observation has not been given. In the subsequent verses, the self-rotating globe is mentioned, which we shall discuss later.

Śrīpati also, like Lalla, explained the method of the observation by the armillary sphere in his *Siddhānta-śekhara* (XIX.3-6).[3)] Let us see its full text as an example.

चक्रांशाङ्कं क्रान्तिवृत्तं विदध्यात्
उर्वीवृत्तं याम्यवृत्तं च तद्वत्
नाडीवृत्तं षष्टिभागांकितं हि
याम्योदक्स्था यष्टिरुर्वीजमध्ये ।।3।।

कार्यं खगोलस्य दृढस्य मध्ये
भगोलमेतत् परितस्तथा च।
यत्रांशके तिग्मकरो ऽपवृत्ते
क्षिपेच्छलाकामिह तत्र भागे ।।4।।

तान्नाडिकावृत्तगतां विधाय
समुद्गमात्सूर्यवशेन भूजात्।

तदीयभा केन्द्रगता यथा स्यात्
स खम्बुनाड्या? भ्रमयेत्तथैव॥5॥

पातङ्गचिह्नक्षितिजान्तरस्थाः
समुद्गतांशा गणकैर्निरुक्ताः।
नाड्यः शलाकाकुजयोस्तु मध्ये
समुन्नतास्ता नियतं भवन्ति॥6॥

"The ecliptic (*krānti-vṛtta*) should be graduated with 360 degrees. The horizon (*urvī-vṛtta*) and the meridian (*yāmya-vṛtta*) should also be graduated similarly. However, the equator (*nāḍī-vṛtta*) should be graduated with 60 divisions (i.e. *nāḍīs*). The axis (*yaṣṭi*) passing through the centre of the horizontal plane is attached to the north and south (celestial poles).

Inside the *kha-gola*, which is fixed, the *bha-gola* is put properly. A stick (*śalākā*) should be attached to the point of certain degrees on the ecliptic where the sun is situated.

The stick is settled towards the equator from the rising point of the sun [on the ecliptic] along the horizon. The equatorial ring(?) should be rotated in such a way that the shadow of the stick falls on the centre [of the sphere].

The distance between the sun and the horizon is called *samudgatāṃśa* by astronomers. The number of *nāḍīs* between the stick and the horizon are certainly the *nāḍīs* elapsed since sunrise."

From this description, it is clear that the time and the *lagna* can be determined with the help of the equator-ring and ecliptic-ring respectively of the armillar sphere.

Bhāskara II[4] also wrote to determine the time with the help of the equatorial ring of the armillary sphere.

v) Additional remarks

The first Greek celestial globe is ascribed to Eudoxus of Cnidus (409 BC-356 BC) by some, while George Sarton does not accept celestial globe earlier than the time of Hipparchus (2nd century BC). It is supposed that Hipparchus used the armillary sphere for his observation of stars.[1]

In the *Almagest* of Ptolemy (fl.ca.AD 150), both the celestial globe (VIII-3)[2] and the armillary sphere (V-1)[3] have been described. Ptolemy also described simple ring instruments, one is in the plane of the meridian with an alidade in order to know the sun's midday zenith distance in solstitial days, hence the obliquity of the ecliptic, (I-12)[4], and the other in the plane of the equator in order to know the day of equinox

(III-1)[5]. In connection with the meridian ring, he described the mural quadrant also.

Ptolemy's armillary sphere consists of a fixed meridian, movable ecliptic with solstitial colure which can be rotated around the poles of the equator, and an outer and an inner movable latitude circles which can be rotated around the poles of the ecliptic. The inner latitude circle is made double, and the innet one has a pair of sight holes and can be slided along the latitude circle.

H.T. Colebrooke[6] compared Hindu armillary sphere with Greek armillary sphere, and pointed out that Hindus have not copied the armillary sphere of Ptolemy, because the construction differs considerably. After examining the description in the *Sūrya-siddhānta* and *Siddhānta-śiromaṇi*, Colebroke remarked as follows.

"This, though not a complete description of Bhāskara's armillary sphere, will convey a sufficient notion of the instrument for the purpose of the present comparison; and will justfy the remark, that its construction differs greatly from that of the instrument specified by Ptolemy.

In the description of the armillary sphere cited from the *Sūrya siddhānta*, mention is made of several stars not included in the asterisms which mark the divisions of the ecliptic. ..." (H.T. Colebrooke)[7]

As Colebrooke has pointed out, Hindu armillary sphere is much different from Ptolemy's armillary sphere. The most important difference is that the main purpose of Hindu armillary sphere is to represent equatorial coordinates which rotates around the celestial poles, while the main purpose of Ptolemy's armillary sphere is to represent ecliptic coordinates.

In this respect, it is more interesting to compare with Chinese armillary sphere, which is basically based on equatorial coordinates. In China,[8] the first armillary sphere which is explicitly mentioned in historical records was made by Luo-xia Hong (late 2nd century BC) who was one of the compilers of *Taichu*-calendar (104 BC) which is the earliest Chinese calender whose mathematical principle is clearly known to us. Unlike Ptolemy's armillary sphere, Chinese armillary sphere originally had equatorial coordinates only. The ecliptic ring was officially added in AD 103, but the ecliptic ring had been used by amatuer astronomers earlier. A celebrated astronomer Zhang Heng (AD 78-139) made an armillary sphere and a water-driven celestial globe. Both the rotating celestial globe and the armillary sphere were highly developed in China afterwards. For example, an armillary sphere made by Li Chun-feng, the royal astronomer of Tang dynasty, in AD 633 had three parts. Its outer part consisted of the meridian, horizon, and equator, its intermediate part consisted of the equator, ecliptic, lunar path, and solstitial colure, and the innermost part consisted of the latitude ring and the alidade. And also, it is well known that Su Song described the water-driven celestial globe and armillary sphere in great detail in his *Xin-yi-xiang-fa-yao* (the end of the 11th century).[9]

From the above discussions, it appears that Hindu armillary sphere is more close to Chinese armillary spheres than Greek armillary sphere. Both Hindu and Chinese armillary spheres are basically based on equatorial coordinates. It is not clear whether there was any occasion to exchange the idea of the armillary sphere between China and India. We shall discuss a related topic in the section of the self-rotating globe later.

As we have seen above, the Hindu armillary sphere seems to have been used since the time of Brahmagupta (7th century AD), although Brahmagupta did not describe the method to construct the actual armillary sphere as an instrument clearly. That the armillary sphere was actually used is also suggested by the fact that the polar-longitude and the polar-latitude of the junction stars of *nakṣatras* are given in the *Brāhma-sphuṭa-siddhānta* (X.1-9), and also the celestial longitude and the celestial latitude of the junction stars are given in the *Mahā-bhāskarīya* (III. 63-71(i)) and the *Laghu-bhāskarīya* (VIII. 1-9) of Bhāskara I (7th century AD). They may have been obtained by the observation using the armillary sphere. On the contrary, Varāhamihira (6th century AD) gives the latitude of some of the junction stars in terms of *hastas* ("hands", a kind of linear measure) and not degrees in his *Pañca-siddhāntikā* (XII. 33-38). It may be that Varāhamihira did not use the armillary sphere, and obtained these values by naked-eye observation by comparing the position of the star and moon. The use of the earmillary sphere cannot be traced to the writings of Āryabhaṭa and Varāhamihira.

8. The Clepsydra and Water Instruments

i) Introduction

There are three types of the clepsydra, namely the outflow type, floating bowl type, and inflow type. (See Fig. 52.) The clepsydra used in Vedāṅga period seems to be the outflow type.[1] The inflow type was used in China, but is not found in India.

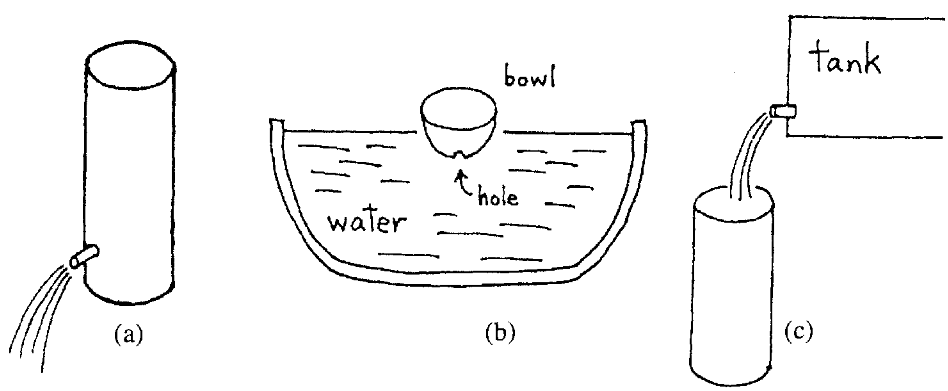


Fig. 52. **(a)** Outflow type, **(b)** Floating bowl type, **(c)** Inflow type.

The outflow type was used in Classical Siddhāntas for water instruments etc. The clepsydra proper in the Classical Siddhāntas is usually the floating bowl type. Besides, the self-rotating globe was described by some authors, some of which are similar to the water instruments, and some of which are devices of perpetual motion which are actually impossible to make.

The floating bowl type of the clepsydra was probably the most popular astronomical instrument in India until recently, and there are several historical records on the actual use of the clepsydra.

ii) The clepsydra in Siddhāntas

The *Āryabhaṭa-siddhānta* of Āryabhaṭa, quoted in Rāmakṛṣṇa Ārādhya's commentary on the *Sūrya-siddhānta,* has the description of two variations of the floating bowl type of the clepsydra, i.e. the *ghaṭikā-yantra* and the *kapāla-yantra.*

The description of the *ghaṭikā-yantra* by Āryabhaṭa is as follows.[1)]

वृत्तं ताम्रमयं पात्रं कारयेद्दशभिः पलैः।
षडङ्गुलं तदुत्सेधो विस्तारो द्वादशानने ॥29॥

तस्याधः कारयेच्छिद्रं पलेनाष्टाङ्गुलेन तु।
इत्येतद्घटिकासंज्ञं पलषष्ट्यम्बुपूरणात्॥30॥

"One should get a hemispherical bowl manufactured of copper, 10 *palas* in weight, six *aṅgulas* in height, and twelve *aṅgulas* in diameter at the top. At the bottom thereof, let a hole be made by a needle eight *aṅgulas* in length and one *pala* in weight.

This is the *ghaṭikā-(yantra),* (so named) because it is filled up by water in a period of 60 *palas* (i.e. one *ghaṭī*)." (Translated by K.S. Shukla)[2)]

The *kapāla-yantra* of Āryabhaṭa is as follows.[3)]

स्वेष्टं वा ऽन्यदहोरात्रे षष्ट्या ऽम्भसि निमज्जति।
ताम्रपात्रमधश्छिद्रमम्बुयंत्रं कपालकम्॥31॥

"Any other copper vessel made according to one's liking, with a hole in the bottom, which sinks into water 60 times in a day and night, is the water instrument called *kapāla.*" (Translated by K.S. Shukla)[4)]

From the above quotations, it appears that the *ghaṭikā-yantra* and the *kapāla-yantra* are basically the same, and only the size of the *ghaṭikā-yantra* has been specified.

Varāhamihira described the clepsydra in his *Pañca-siddhāntikā* (XIV. 31-32) as follows.[5)]

द्युनिशि विनिःसृततोयादिष्टच्छिद्रेण षष्टिभागो यः।
सा नाडी स्वमतो वा श्वासाशीतिः शतं पुंसः।।31।।

कुम्भार्धाकारं ताम्रं पात्रं कार्यं मूले छिद्रं
स्वच्छे तोये कुण्डे न्यस्तं तस्मिन् पूर्णे नाडी स्यात्।
मूलाल्पत्वाद्वेधो वा षष्टिर्योज्या चाह्ना रात्र्या
वर्णाः षष्टिर्वक्राः श्लोको यत्तत्षष्ट्या वा सा स्यात्।।32।।

"One sixtieth of the water escapes from any hole [of the clepsydra] during a day and night is a *nāḍī,* that is 180 breaths of a healthy man.

A copper vessel, whose shape is like a half of a pot, with a hole at its bottom is to be made. It is put in clear water in a basin. When it is filled [with water], it is a *nāḍī*. Observing [the clepsydra which depends] on the smallness of the hole at the bottom, there counted 60 [*nāḍīs*] during a day and night. Or, [a *nāḍī* is measured] by reciting 60 verses each of which consists of 60 long syllables."

The expression *"viniḥsṛta"* (escaped from, gone out) in the first verse suggests that the outflow type of the clepsydra is meant there. The copper vessel mentioned in the second verse is evidently the floating bowl type of the clepsydra. Its size is not specified there. As Thibaut and Dvivedin pointed out, the vs.32 itself consists of 60 long syllables.

Brahmagupta also described the copper *ghaṭikā-yantra* in his *Brāhma-sphuṭa-siddhānta* (XXII. 41) as follows.[6)]

घटिका कलसार्धाकृति ताम्रं पात्रं तले ऽपृथु च्छिद्रम्।
मध्ये तज्जलमज्जनषष्ट्या द्युनिशं यथा भवति ।।41।।

"The *ghaṭikā* (clepsydra) is a copper cup, half a pot in shape, and has a small hole at its bottom. It is made in such a way that it sinks into water 60 times in a day and night."

Here, Brahmagupta did not specify its size.

Lalla described the *ghaṭī-yantra* in detail in his *Śiṣyadhī-vṛddhida-tantra* (XXI. 34-37) as follow.[7)]

दशभिः शुल्बस्य पलैः पात्रं कलशार्धसन्निभं घटितम्।
हस्तार्धमुखव्यासं समघटवृत्तं दलोच्छ्रायम्।।34।।

सत्र्यंशमाषकत्रयकृतनलया समसवृत्तया हेम्नः।
चतुरङ्गुलया विद्धं मज्जति विमले जले नाड्या।।35।।

अथवा स्वोच्छाघटितं घटीप्रमाभिः प्रसाधितं भूयः।
त्रैराशिकसिद्धं वाङ्गुलवद् गुरु विपुलरन्ध्रं यत्।।36।।

इष्टदिनार्धघटीभिः सममथवापं निमज्जति घटी सा।
षष्टैः शतैस्त्रिभिर्वा विंशतिलघ्वक्षरासूनाम्।।37।।

"34-35. The *Ghaṭikā* vessel, looking like one-half of a (spherical water-pot called) *Kalaśa,* made of ten *palas* of copper, half a hand in diameter at the top and half as high, and having a hole bored (at the bottom) by a uniformly circular needle of 4 *aṅgulas* in length, made of $3^1/_3$ *māṣas* of gold, sinks into limpid water in one *nāḍī* exactly.

36-37. Or, it is a vessel manufactured according to one's liking (with the hole in the bottom) later adjusted by the measure of a *ghaṭī.*

Or, it is a vessel having a finger-wide hole (in the bottom) and of size determined by proportion in such a way that it may sink (in water) as many times in a day as there are *ghaṭīs* in that day.

A *ghaṭī* is also equal to 360 *asus,* each *asu* (being equal to the time of pronunciation) of 20 short syllables (or 10 long syllables)." (Translated by K.S. Shukla)[8)]

Here, Lalla described three types of the clepsydra, one of which has been given definite size. All of them are used to measure one *ghaṭī.*

The *Sūrya-siddhānta* (XIII.23)[9)] mentions the copper clepsydra called *kapālaka,* which has a small hole at its bottom, and is placed on water in order to measure one *ghaṭī.* Its size is not specified there.

Śrīpati described the *ghaṭī-yantra* in his *Siddhānta-śekhara* (XIX. 19-20) as follows.[10)]

शुल्बस्य दिग्भिर्विहितं पलैर्यत्
शडङ्गुलोच्चं द्विगुणायतास्यम्।
तदम्भसा षष्टिपलैः प्रपूर्य
पात्रं घटार्धप्रमितं घटी स्यात्।।19।।

सत्र्यंशमाषत्रयनिर्मिता या
हेम्नः शलाका चतुरङ्गुला स्यात्।

विद्धं तया प्राक्तनमत्र पात्रं
प्रपूर्यते नाडिकया ऽम्बुना तत्।।20।।

"The *ghaṭī* (clepsydra) is made of 10 *palas* of copper, 6 *aṅgulas* in height, its double (12 *aṅgulas*) in width, and is a vessel shaped just like a half of a water-pot and filled with water in 60 *palas* (i.e. one *ghaṭī*).

The aforesaid vessel should be bored [a hole at its bottom] by a needle of 4 *aṅgulas* in length, which is made of 3$^1/_3$ *māṣas* of gold, so as to be filled with water in one *nāḍikā*."

Evidently, this Śrīpati's clepsydra is the same as the first type of Lalla's clepsydra.

Bhāskara II described the copper *ghaṭikā* in his *Siddhānta-śiromaṇi* (*Gola,* XI.8).[11] This is a copper vessel with a large hole at its bottom, and its size is not specified. The text (XI. 8) reads as follows.[12]

घटदलरूपा घटिता घटिका ताम्री तले पृथुच्छिद्रा
द्युनिशनिमज्जनमित्या भक्तं द्युनिशं घटीमानम्।।8।।

"The *ghaṭikā* (clepsydra) is made of copper, half of a water-pot in shape, and has a large hole at its bottom. The duration of a day and night is divided by the number of its sinking [into water] in a day and night, and this is the measure of the *ghaṭī* (clepsydra)."

Here, the duration of time measured by one sinking of the clepsydra is not specified, and is determined by experiment for each clepsydra. So, it may be said that this is a practical instruction for common use of the clepsydra.

iii) Indian clepsydra in historical accounts

The clepsydra was probably the most popular astronomical instrument in India until recently, and there are several historical records of this instrument.

There is an inscription on the "Manikiala Stone" found at Manikiala in the Rawal Pindi District.[1] It is written in Kharoṣṭhi script, in Prakrit language. F.E. Pargiter conjectured that this inscription indicates that Lalaṇa, the President of the people and the scion of the Kuṣāṇa race of Kaniṣka, established some kind of water-clock in the market-place of the Satrap Vespaśi.[2] This inscription appears to be very difficult to read, and different people interpreted it in different way. So, it is difficult to say whether Pargiter's conjecture is correct or not.

There is an earthernware fragment of Guhasena of Valabhi. Its inscription reads : ".......[200] 40 7 *Śrīguhasenaḥ ghaṭā*......."[3] This date corresponds to AD 566/567. J.F.

Fleet conjectured that this water-jar was used for measuring time.[4]

S.R. Sarma[5] pointed out that the outflow type of the clepsydra is mentioned in the *Kādambarī* of Bāṇa (7th century AD). The expression *"anavarata-galan-nāḍikā"* (continuously dropping *nāḍikā*) is found in the *Kādambarī*,[6] and this shows that the *nāḍikā* is the outflow type of the clepsydra.

A Chinese Buddhist traveller Yi-jing (AD 635-713)[7] recorded that the clepsydra had been widely used in India. In his *Nanhai-jigui-neifa-zhuan* (Record of Buddhism sent home from the South Seas),[8] Yi-jing described the clepsydra in Nālandā temple as follows.

"And also, all the temples in the western country (i.e. India) have the clepsydra, which have been donated by successional kings. There a time-keeper appointed, who announces time for people.

Below is a copper basin filled with water, and above is a copper bowl floating on it. The bowl is thin and smooth, and two *sheng* (one *sheng* was about 0.59 liter) in capacity. A hole has been bored in its bottom, from which water flows in. It is thin like a needle. It is the standard for measuring time. When the bowl is filled with water, it sinks, and a drum is beaten [by the time-keeper]."[9]

After this description, Yi-jing wrote that the clepsydra sank four times in one watch (i.e. *prahara*), and the daytime as well as the nighttime was divided into four watches. So, this clepsydra is for 1.875 *ghaṭīs* in equinoctial days. The measurement of the clepsydra was changed by adding or subtracting water in the clepsydra according to the change of the length of daytime and nighttime.

Yi-jing added that in the temples of Mahābodhi and Kuśinagara, the clepsydra sank 16 times between sunrise and midday.

Al-Bīrūnī (AD 973-ca.1050) quoted from a book called *Srūdhava* of Utpala the Kashmirian in his *India* as follows.

"If you bore in a piece of wood a cylindrical hole of twelve fingers' diameter and six fingers' height, it contains three *manā* water. If you bore in the bottom of this hole another hole as large as six plaited hairs of the hair of a young woman, not of an old one nor of a child, the three *manā* of water will flow out through this hole in one *ghaṭī*." (Translated by E.C. Sachau)[10]

This is the outflow type of the clepsydra, and different from the floating type of the clepsydra described in Classical Siddhāntas.

The earliest Indo-Persian source which mentions the clepsydra is the *Tārīkh-i Fīrūz Shāhī* of Shams Sirāj 'Afīf (14th century AD).[11]

Description of the clepsydra is also found in the *Bābur-nāmā* of the first Mughal Emperor Bābur (reigned 1526-1530 AD),[12)] and the *Ā'īn-i Akbarī* of Abū'l-Fazl (1551-1602 AD).[13)]

The clepsydra is found in some Mughal Miniatures also, as pointed out by S.R. Sarma.[14)]

There are several accounts and studies of Indian clepsydra by European people, namely, John Gilchrist,[15)] Francis Buchanan,[16)] Hermann von Schlagintweit-Sakünlünski,[17)] J.F. Fleet,[18)] F.E. Pargiter,[19)] and Hermann Jacobi.[20)]

iv) Water instruments

Water instruments are a kind of mechanical clock driven by water-flow. The outflow type of the clepsydra is connected to a stature of a man or animal etc., which indicates time by its movement etc.

Āryabhaṭa wrote to construct water instruments in his *Āryabhata-siddhānta,* quoted in Rāmakṛṣṇa Ārādhya's commentary on the *Sūrya-siddhānta.*[1)] Āryabhaṭa tells to make a cylinder, which is actually the same as the outflow type of the clepsydra. The height of the cylinder is divided into *aṅgulas* which correspond to the water-flow in each *ghaṭī.* A gourd containing mercury is placed on water in it, and tied to the end of a cord. Another end of the cord is connected to a model of a man or a figure of a peacock or monkey, whose movement shows time as water flows out through the hole at the bottom of the cylinder.

Varāhamihira also mentioned water instruments in his *Pañca-siddhāntikā* (XIV. 27-28) as follows.[2)]

गुणसलिलपांशुभिर्योजितानि बीजानि सर्वयन्त्राणाम्।
तैः फलके कूर्ममानवयथेष्टरूपाणि कार्याणि।।27।।

गुरुरचपलाय दद्याच्छिष्यायैतान्यवाप्य शिष्यो ऽपि।
पुत्रेणाप्यज्ञातं बीजं संयोजयेद्यन्त्रे।।28।।

"The seed (*bījas*) for all [water] instruments (*yantras*) are string, water, and sand. By them, [water instruments with the models of] tortoise, man, or any desired form are to be made on the board.

The teacher should give them only to a steadfast pupil. Receiving the seed, which is unknown even to his son, the pupil should apply it to the instrument."

Neugebauer and Pingree misunderstood the above verses, and translated the word *yantra* as "magical diagram". However, it cannot be supposable that the magical diagram, which is described in Tantra literature, is mentioned in purely astronomical literature. The *yantra* in the above verses is evidently the water instrument, and the figures of tortoise, man etc. are used to indicate time. As water instruments are described by Āryabhaṭa, it is clear that they were known at the time of Varāhamihira.

Brahmagupta explained water instruments in his *Brāhma-sphuṭa-siddhānta* (XXII. 46-52).[3] Let us see the text one by one.

नलको मूले विद्धस्तत्स्रुतिघटिकोद्धृतः समुच्छ्रायः।
लब्धाङ्गुलैस्तु तैर्नाडिकाक्रियायन्त्रसिद्धिरतः।।46।।

"A cylinder [is made], which has a hole at its bottom. Its height is divided by the number of *ghaṭikās* during which water flows out. By the *aṅgulas* obtained, the *nāḍikā* (= *ghaṭikā*) is graduated. Thus the instrument is settled."

घटिकाङ्गुलान्तरस्थैश्चीरिर्गुटकैर्घटीधृतैरङ्क्या।
उपरि नरो ऽधः सुषिरस्तिर्यक् कीलो ऽस्य मुखमध्ये।।47।।

"A strip should be marked with beads for measuring *ghaṭīs,* which are attached at intervals of *aṅgulas* corresponding to *ghaṭikās.* A figure of a man is above, and the cylinder is below. A horizontal rod is kept at the middle of the mouth [of the figure of a man]."

कीलोपरि गामिन्यां चीर्यां धृतपारदमलाबु तस्मिन्।
स्रवति जले क्षिपति नरो गुटिकां कूर्मादयश्चैवम्।।48।।

"A bottle-gourd containing mercury is attached to the strip which is coiled round the rod. The [figure of] man throws out the bead on the flowing water. In the same manner, the [figure of] tortoise etc. [may be made]."

जलपूर्णकृतघटीभिः स्तनास्यकर्णादिभिर्जलं क्षिपति।
पुरुषो ऽन्यस्याऽऽसक्ते(क्तं?) वक्त्रं पुरुषस्य कृतमुपरि।।49।।

"The [figure of] man pours out water from his teat, mouth, or ear, which is connected with the clepsydra filled with water, to the fixed mouth of another man directed upwards."

एवं वधूवरं नाडिकाङ्गुलैः संयुता वरे योज्या।
युद्धानि मल्लगजमहिषमेषविविधायुधभृतां च।।50।।

"In the same manner, a [figure of] bride, which is connected with [a strip marked with] *aṅgulas* corresponding to *nāḍikās,* should be joined with a bridegroom in a room. And also, [models of] fightings of various fighters, such as boxer, elephant, buffalo, and sheep, [may be made]."

निगिरति गिरति घटिकाङ्गुलाङ्कितैः खण्डकैर्मयूरो ऽहिम्।
चीर्यामेवं गुटिकोपरिस्थितैर्ब्रह्मचार्याद्यैः।।51।।

"The [figure of] peacock eats or throws out serpent by segments according to *aṅgulas* corresponding to *ghaṭikās.*

By the [stature of] students etc., who are above the beads on the strip, ...(to be continued)"

कीलोत्क्षेपाभिहतः पटहः शब्दं करोति घण्टा वा।
एवं यन्त्रसहस्राण्यनेन बीजेन कार्याणि।।52।।

".....a drum or bell is struck by a strick, and makes sound.

Thus thousands of instruments may be made with the help of this essential requirement (*bīja;* i.e. water, mercury, oil, strip etc.)."

The water instruments described by Brahmagupta as above are several devices of the water-driven clock. Their constructions are easily under-stood from the text.

Lalla also described similar water instruments in his *Śiṣyadhī-vṛddhida-tantra* (XXI.12-17).[4)]

The *Sūrya-siddhānta* (XIII. 21(ii)-22)[5)] mentions water instruments, but the method of their construction has not been explicitly given. The text says that it is "difficult".

Śrīpati described water instruments in his *Siddhānta-śekhara* (XIX. 9-11). Let us see the text one by one.[6)]

चीरीं प्रकुर्याद् घटिकाङ्गुलाङ्का–
मेतेन मुक्त्वा वदनेन धार्या।
तां निक्षिपेत् काष्ठनरोदरे तु
तदाऽस्य तिर्यक्स्थितकीललग्नम्।।9।।

"A strip, which is marked with *aṅgulas* corresponding to *ghaṭikās,* is set free, and should be supported by the mouth [of a figure of man]. It is put inside the wooden figure of man, and then it is coiled round the horizontal rod [in the figure]."

चीरीसूत्रं क्रोडकाधोगतं स्यात्
तस्मिंस्तुम्बं पूर्ववद्बद्धमुच्चैः।

पात्रे ऽधो ऽधस्तद्व्रजेत् कर्णयन्त्रा–
न्नाडी भुक्तामुत्सृजत्येष नाड्याः ।।10।।

"The strip goes downwards from the chest, and a bottle-gourd should be tied there from above as before. It proceeds downwards in the vessel from the instrument with ears (man's figure?). Water flows out, and it indicates *nāḍīs*."

इत्थं स्वबुद्ध्या गणकः प्रकुर्या–
न्मेषादियुद्धं गजयन्त्रमत्र ।
यत्र स्वयंवाहकनाभिमध्याद्
बीजं दशाङ्केन हि कर्मणा यः? ।।11।।

"In this manner, by his own intelligence, an astronomer may construct fighting-sheep-instrument or elephant-instrument, where it is connected with the action with ten marks (marked string?) from the centre of the self-rotating machine to the *bīja* (i.e. water?)."

There are some obscure words in the above text of Śrīpati, but it is clear that his water instruments are similar to those of Brahmagupta and Lalla.

v) The self-rotating globe and related topics

We already have seen that Āryabhaṭa mentioned the self-rotating celestial globe in his *Āryabhaṭīya* (IV. 22). He himself did not write how to rotate it but probably it was rotated by water-flow just like other water instruments, as was suggested by a commentator Someśvara.

Brahmagupta also described the self-rotating instrument in his *Brāhamsphuṭa-siddhānta* (XXII. 53-57(i)), but he explained a method to rotate it using mercury, which is practically impossible to practice. Let us see the text one by one.[1)]

लघुदारुमयं चक्रं समसुषिरारान्तरं पृथगराणाम् ।
अर्धे रसेन पूर्णे परिधौ संश्लिष्टकृतसन्धिः ।।53।।

"A light wooden wheel, which has spokes of smooth hollow tube inside, [should be made]. The spokes are filled with mercury by half, and are firmly connected to the circumference."

तिर्यक्कीलो मध्ये द्व्याधारस्थो ऽस्य पारदो भ्रमति ।
छिद्राण्यूर्ध्वमधो ऽतश्चक्रमजस्रं स्वयं भ्रमति ।।54।।

"A horizontal axis at its centre is placed on two supporters. Its (tube's) mercury roundly moves in the tubes upwards and downwards. Therefore, the wheel rotates by itself for ever."

छिद्रे स्वधिया क्षिप्त्वा समं यथा पारदं भ्रमति।
कालसममिष्टमानैश्चक्रसमुत्तनमूर्ध्वं वा।।55।।

"One should, by his own intelligence, pour mercury in the tube in such a way that it rotates in the same speed as time (i.e. diurnal rotation of the celestial sphere) by desired size, [keeping the wheel] either horizontally or vertically."

The above is the description of a device of perpetual motion, which is practically impossible to make. Evidently, the wheel does not rotate, even if mercury is poured in a hollow spoke.

Brahamgupta continues to explain another practical method as follows (XXII. 56-57(i)).

कीलस्योपरिगामिनि तत्पर्ययसूत्रके धृतमलाबु।
प्राग्वन्नलके प्रक्षिप्य नाडिका स्रवति पानीये।।56।।

करणैर्ज्याक्षिप्रचलनमेवं शरमोक्षणं खशब्दाश्च।

"A bottle-gourd is tied to the main string coiled round the rod, and is placed in a cylinder as before. *Nāḍikā* [is indicated as it] flows on water.

Similarly, by these devices, the quick movement of bow-string, throwing of arrow, and sound of cloud [may be made]."

This is the same as the water instruments, and is, of course, possible to make.

Lalla and Śrīpati described the method to rotate the globe, which is quite practical, more clearly. Lalla says to rotate the armillary sphere by the water-flow in his *Śiṣyadhī-vṛddhida-tantra* (XXI. 8-11).[2] However, Lalla also mentions a "wheel" with spokes, which are half-filled with mercury, just like Brahmagupta's instrument. Lalla says that it rotates by itself (XXI. 18), and instructs to rotate a globe by the "wheel" (XXI. 19).[3] This method is practically impossible.

The *Sūrya-siddhānta* (XIII. 16(ii)-19(i)),[4] also instructs to rotate a sphere-instrument combined with quicksilver, but says that it is a "mystery". The method of its construction is not explained there.

Śrīpati is perhaps the most rational and practical in this topic. He described the self-rotating globe in his *Siddhānta-śekhara* (XIX. 7-8) as follows.[5]

नीरस्रुत्या चिह्निते नाडिकाद्यै–
मूलच्छिद्रे वारिपूर्णे च पात्रे।

गोलं तुम्बं पारताढ्यं गुणेन
बद्धे केन प्रक्षिपेत्तत्र युक्ते।।7।।

यथा यथा ऽम्बु स्रवति क्रमेण
तथा तथा ऽधोव्रजदत्र तुम्बम्।
गोलं परिभ्रामयति स्वयं तत्
सूर्यांशभूजान्तरगास्तु नाड्यः।।8।।

"A vessel which has a hole at the bottom and is filled with water should be marked with divisions of *nāḍikā* etc. according to water-flow. A bottle-gourd which is possessing mercury and combined with the globe by a string should be thrown there (vessel) which is filled with water.

As water flows gradually, the bottle-gourd moves downwards. Thus the globe rotates automatically, and the distance between the position of the sun and the horizon [on the globe] indicates *nāḍīs* [elapsed since sunrise]."

The principle of this method is the same as the water instruments, and of course it is possible to make this device. This is a kind of water-driven mechanical clock.

Bhaskara II, however, explained some devices of perpetual motion which are practically impossible to make. He states that he describes them simply because they have been mentioned by former astronomers. It may be noted that Bhāskara II criticized the device of the self-rotating instrument rotated by water-flow as "vulgar" (*grāmya*) on account of its being dependent (*sāpekṣatva*). (*Siddhānta-śiromaṇi* (*Gola*, XI. 50-58).)[6)]

In the section of the armillary sphere, we have discussed that the water-driven celestial globe was made in China, by Zhang Heng (AD 78-139) etc., and there are some detailed records of its construction, notably the *Xin-yi-xiang-fa-yao* (the end of the 11th century) of Su Song. Joseph Needham suggested Chinese influence on Indian self-rotating globe. After discussing the self-rotating instrument described in the *Siddhānta-śiromaṇi* and the *Sūrya-siddhānta,* Needham wrote as follows.

"At this date, then, India shows a development closely allied to that taking place in China and Islam. Here the curious association between perpetual motion and armillary spheres may possibly give an important clue. Perhaps some observer, seeing the Chinese clocks imperfectly from the outside and marvelling at them innocently invented that myth of perpetual motion which was to dog the science of mechanics for so long. An invention of rationality produced the compromice of 'perpetual motion' wheels powered by scoops fitted to the rim and receiving a stream of water to help things along. Perhaps the perpetual motion fable first originated from an unprecise account of a Chinese clock like Su Sung's. At all events, from the middle of the twelfth century onwards, there is considerable likelihood of the transmission of such Chinese ideas to India and Islam and thence (after 1200 and 1300) to Europe." (Joseph Needham et al.)[7]

This is an interesting suggestion, but Needham's argument cannot be accepted as it is, because Āryabhaṭa already described the self-rotating celestial globe in his *Āryabhaṭīya* (AD 499), and it is before the time of "transmission" suggested by Needham. It is not clear whether there was an occasion of Chinese influence in earlier age. The self-rotating celestial globe was made in China already in the 2nd century AD, but we do not have any historical evidence which shows actual Chinese influence on Indian self-rotating globe. The origin of the Indian perpetual-motion theory is still not clear.

vi) The sand-clock

The currently popular editions of the modern *Sūrya-siddhānta* are usually based on Raṅganātha's version (AD 1603). In this version of the *Sūrya-siddhānta,* the sand-clock has been mentioned. The *Sūrya-siddhānta* (XIII. 21), according to Raṅganātha's version, reads as follows.[1]

तोययन्त्रकपालाद्यैर्मयूरनरवानरैः ।
ससूत्ररेणुगर्भैश्च सम्यक् कालं प्रसाधयेत् ।।21 ।।

E.Burgess and Whitney translated this text as follows.

"By water-instruments, the vessel (*kapāla*), etc., by the peacock, man, monkey, and by stringed sand-receptacles, one may determine time accurately." (Translated by Burgess and Whitney)[2]

The original expression for the "sand-receptacles" is *"reṇu-garbha"*. The world *"reṇu"* means dust, sand etc. As the commentator Raṅganātha used the words *"dhūli"* (dust, powder), and *"mṛd-ghaṭikā"* (soil-clock) in his commentary, there is no doubt that the instrument described in this version is a kind of the sand-clock.

However, Raṅganātha's version (AD 1603) is rather late, and there are several early versions of the *Sūrya-siddhānta*. Parameśvara's version (AD 1432), published by K.S. Shukla, gives a different reading as follows (above text of Raṅganātha's version corresponds to XIII. 21(ii)-22(i) of Parameśvara's version).[3]

तोययन्त्रैः कपालाख्यैर्मयूरनरवानरैः ।।21 ।।
सूत्रैश्च वेणुगर्भस्थैः सम्यक्कालं प्रसाधयेत् ।

Here, the word *"reṇu"* does not appear, but the expression *"veṇu-garbha"* (bamboo-container) appears. So, this is not the sand-clock, but must have been the usual water instrument. According to a footnote of K.S. Shukla's edition, the versions of Rāmakṛṣṇa Ārādhya (AD 1472) and of Yallaya (AD 1472) read *"sa-sūtra-veṇu-garbha-sthaiḥ"* etc.[4] Here also the word "*veṇu*" (bamboo).

I consulted some other manuscripts of early versions of the *Sūrya-siddhānta,* and confirmed that the following manuscripts have the expression "*veṇu-garbha*", and not "*reṇu-garbha*". A manuscript of Caṇḍeśvara's version (AS Calcutta G-10758)[5], a manuscript of Madanapāla's version (Lucknow 47051)[6], a manuscript of Bhūdhara's version (Lucknow 45760),[7] and a manuscript of Tamma Yajvan's version (Lucknow 46145)[8]. All of them are earlier than Raṅganātha. (Caṇḍeśvara : AD 1185, Madanapāla : late 14th century, Bhūdhara : AD 1572, and Tamma Yajvan : AD 1599.)

From the above facts, it is likely that the original reading of the *Sūrya-siddhānta* was "*veṇu*", and not "*reṇu*". So, the sand-clock was not described there originally.

In later period, the sand-clock appears in the *Yantra-prakāśa* (AD 1428) of Rāmacandra and in the *Sundara-siddhānta* (AD 1503) of Jñānarāja.[9] As Raṅganātha is later than both of them, it is not surprising if he was familiar with the sand-clock.

At present, we can say that we do not have evidence which shows the existence of the sand-clock in Classical Siddhānta period.

9. THE PHALAKA-YANTRA

The *phalaka-yantra* ("board-instrument") is Bhāskara II's invention, and is an instrument to determine time. Firstly, the sun's altitude is observed by this instrument, and then the time is calculated graphically. (*Siddhānta-śiromaṇi* (*Gola,* XI. 16-27).)[1]

The *phalaka-yantra* is a rectangular board whose height is 90 *aṅgulas* and breadth is 180 *aṅgulas.* Horizontal lines are drawn at every *aṅgula,* and a hole is made at the middle of the 30th line from the top in order to place a pin. A circle with the radius of 30 *aṅgulas* is drawn with the hole as centre. Its circumference is graduated with *ghaṭīs* and degrees. An index arm is suspended by the pin in such a way that it can be rotated around the pin. (See Fig. 53.)

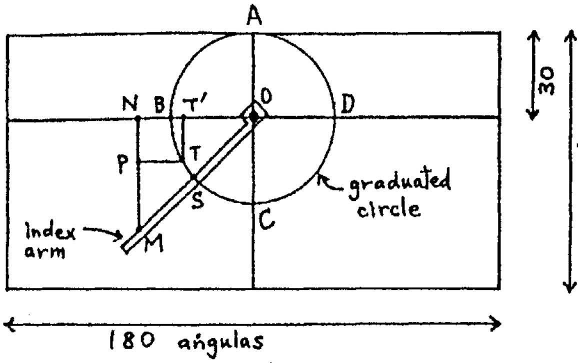


Fig. 53. Phalaka-yantra (after Sastri and Wilkinson)

The method to use this instrument is as follows. Firstly, the board is held in such a way that its side faces the sun, keeping the side vertical. Then the index arm (OM in Fig. 53) is fixed along the shadow of the central pin (0). So, the angle NOM is equal to the sun's altitude. Previously, the index arm has been marked with a point M, of which distance (OM) from the pin is equal to the amount called *yaṣṭi.* (For the definition of the *yaṣṭi,* see below.) Then, a vertical line (NM) passing through the point M is drawn, and a point P above or below (according to the sun's declinatin which is north or south) at the distance (MP) equal to the R.sine of the sun's ascensional difference is marked. Here, the Radius is the radius of the circle on the board, i.e. 30 *aṅgulas.* Then, a horizontal line (PT) from the point P is drawn, and its cross point (T) with the circle indicates the time. In the figure, the arc CT indicates the time until midday or since midday.

The rationale fo this method is as follows. Let *a* be the sun's altitude, δ the sun's declination, ∅ the observer's latitude, and T the sun's hour angle (i.e. angle corresponding to the time until midday or since midday). (See Fig. 54.) In Fig. 54 (a), which is the orthographic projection of the celestial sphere onto the plane of the meridian, the segment OR is the R.sine of the sun's altitude, where the point 0 is the projection of the sun. The segment OR can be divided into the two segments OK and KR. Form Fig. 54(b), which is the orthographic projection of the celestial sphere onto the plane of the equator, we have

$$BO = CO'.\cos T = r.\cos T = \frac{r}{R} R.\cos T.$$

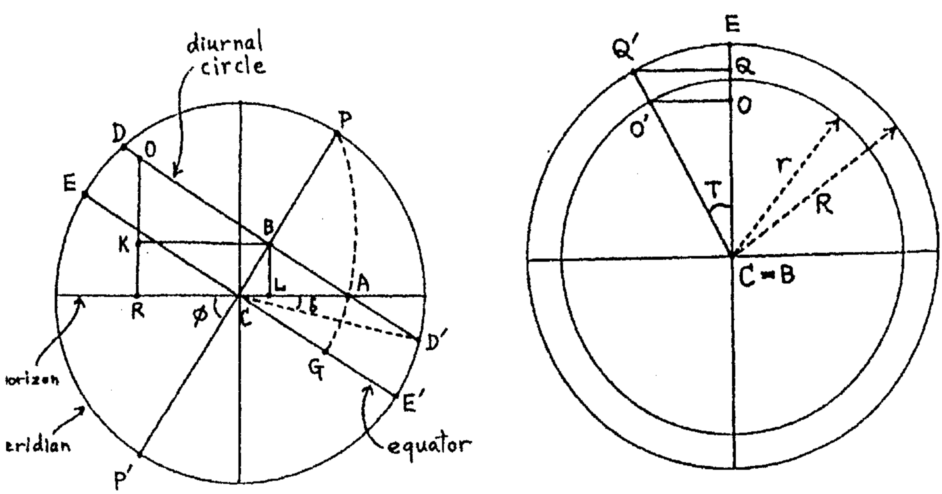


Fig. 54. (a) Orthographic projection onto the plane of the meridian, (b) Orthographic projection onto the plane of the equator.

Since $\frac{r}{R} = \cos\delta$, we have

$$BO = R.\cos\delta.\ \cos T.$$

So, from Fig. 54(a), we have

$$OK = BO.\cos\varnothing = R.\cos\varnothing.\ \cos\delta.\ \cos T, \qquad \text{------(1)}$$

because the angle OBK is (90°-∅).

Now, in Fig. 54(a),

$$BC = R.\sin\delta, \text{ and}$$

$$BL = BC.\sin\varnothing.$$

So, we have

$$KR = BL = R.\sin\varnothing.\ \sin\delta. \qquad \text{-----(2)}$$

From the equations (1) and (2), we have

$$R.\sin a = R.\sin\varnothing.\ \sin\delta + R.\cos\varnothing.\ \cos\delta.\ \cos T, \qquad \text{-----(3)}$$

because R.sin a is equal to the sum of the segments OK and KR.

From the equation (3), we obtain the following equation.

$$R.\cos T = \frac{R.\sin a}{\cos\varnothing\ .\ \cos\delta} \mp R.\tan\varnothing.\ \tan\delta$$

$$= \frac{R^2 \times R.\sin a}{R.\cos\varnothing.\ R.\cos\delta} \mp \frac{R.\tan\varnothing.\ R.\tan\delta}{R}, \qquad \text{-----(4)}$$

where the value of δ is taken as positive, and the last term is subtracted when δ is north, and added when δ is south.

Nextly, let ω be the ascensional difference. In Fig. 54(a), the segment CG is equal to the R.sine of the ascensional difference (R.sin ω).

As $CG = \frac{R}{r}$ AB, we can calculate the amount of the R.sine of the ascensional difference as follows. Firstly, we have

$$AB = BC.\tan \varnothing = R.\sin \delta.\ \tan \varnothing.$$

Therefore,

$$R.\sin \omega = CG = \frac{R}{r} AB = \frac{1}{\cos \delta} AB = R.\tan \delta.\ \tan \varnothing. \qquad \text{-----(5)}$$

Now, we define the *yaṣṭi* as follows.

$$yaṣṭi = \frac{R}{\cos \varnothing.\ \cos \delta}\ . \qquad \text{-----(6)}$$

Let *y* be the *yaṣṭi*. Then, from the equations (4), (5), and (6), we can calculate the R.cosine of the sun's hour angle as follows.

$$R.\cos T = y.\sin a \mp R.\sin \omega. \qquad \text{-----(7)}$$

The graphical calculation on the *phalaka-yantra* exactly corresponds to this equation. (See Fig. 53 again.) The segement OM is the *yaṣṭi*, MN is y.sin a, MP is R.sin ω, and PN or TT' is R.cos T. Therefore, the arc TB corresponds to the complementary angle of T, and the arc TC corresponds to T.

The method to calculate the *yaṣṭi* is as follows. Let *k* be the equinoctial midday hypotenuse (*akṣa-karṇa*) of 12-*aṅgula* gnomon at the observer's latitude. Then

$$R.\cos \varnothing = \frac{12.R}{k}.$$

Therefore,

$$yaṣṭi = \frac{R}{\cos \varnothing.\ \cos \delta}$$

$$= \frac{R.k}{12} \times \frac{1}{\cos \delta}$$

$$= \frac{R.k}{12} \left(\frac{\cos \delta + \text{vers}\ \delta}{\cos \delta}\right)$$

$$= \frac{R}{12} \left(k + \frac{k}{12} \times \frac{12\ R.\text{vers}\ \delta}{R.\cos \delta}\right). \qquad \text{-----(8)}$$

Bhāskara II gives the value of $\frac{12\ R.\text{vers}\ \delta}{R.\cos \delta}$ as 4/60, 15/60, 32/60, 50/60, 63/60, and 68/60, for the *bhuja* (longitudinal distance from the equinoctial points upto 90°)

of the sun 15°, 30°, 45°, 60°, 75°, and 90°. (Actually, Bhāskara II gives the differences of the above values in the text.) Using these values, the *yaṣṭi* is calculated by the equation (8), putting R = 30 for this instrument.

From the above discussion, it is clear that the *phalaka-yantra* is a ingenious instrument to determine time, and the time can be determined exactly.

10. METHODS OF OBSERVATION

i) Introduction

Astronomical observations can be divided into two kinds. One type of astronomical observations is an application of astronomy for civil life, such as the determination of directions, time, etc. Another type of astronomical observations is for the determination and improvement of astronomical constants.

We already have seen the first type of observations which can be summarized as follows.

(1) The cardinal directions can be determined by the Gnomon (*śaṅku*) with a level circle. The Staff (*yaṣṭi*) can also be used for the similar observation, according to Brahmagupta.

(2) The time can be ditermined by the Clepsydra (*ghaṭī-yantra*), and several water instruments. The time can also be determined by the observation of the sun's hour angle (or its complementary angle), which is obtained by the hemispherical *Kapāla* (of Varāhamihira and Brahmagupta), the *Kartari-yantra*, the *Bhagaṇa* or *Nāḍīvalaya-yantra*, and the *Gola-yantra* (armillary sphere). In the case of this observation, the sun's ascensional difference should be corrected in order to know the time since sunrise. The *lagna* (rising point of the ecliptic) can also be determined by the hemispherical *Kapāla*, the *Bhagaṇa* or *Nāḍīvalaya*, and *Gola*. The time can be calculated from the Rsin of the sun's altitude determined by the Gnomon. The time can be graphically calculated from the sun's altitude by the *Phalaka-yantra* of Bhāskara II. The time can roughly be determined from the sun's altitude determined by the Circle (*cakra*), the Semi-circle (*dhanus*), and the Quardrant (*turīya*), or from the sun's azimuth determined by the *Pīṭha*, and the flat *Kapāla* (of Lalla and Śrīpati).

(3) The angular distance can be determined by the *Śalākā*, and the *Śakaṭa*. The *agrā* (the sun's amplitude) can be measured by the *Śalākā*, and also by the *Pīṭha*, and the earthern platform.

(4) The distance, height etc. of terrestial objects can be measured by the Staff or the *Dhī-yantra* of Bhāskara II which is actually the staff which is supposed to be in the rectangular coordinates.

Now let us see the observations for the determination and improvement of astronomical constants. Bhāskara II explained this type of observations in his auto-commentary *Vāsanābhāṣya* (hereafter V-SŚ) on his *Siddhānta-śiromaṇi*, *Graha-gaṇita* (I.ii.1-6)[1], and in his commentary *Vivaraṇa* (hereafter V-ŚDV) on Lalla's *Śiṣyadhīvṛddhida-tantra* (I. 1-8)[2] and (V.2)[3]. And also, Someśvara explained this type of observation in his commentary on the *Āryabhaṭīya* (IV.48)[4].

ii) The sun's revolution

Firstly, let us see Bhāskara II's method of the determination of the sun's revolution (V-SŚ, I.ii.1-6). The revolution number in a *kalpa* is called *bhagaṇa*. In the case of the sun, Venus, and Mercury, the *bhagaṇa* is 4320000000. In the case of Venus and Mercury, the *bhagaṇa* is measured by their mean motion, which is apparently the same as the sun's mean motion. Bhāskara II tells to determine the length of one revolution of the sun, that is the length of one year, by a circular platform. He writes as follows (V-SŚ, I.ii.1-6).

> अथ समायां भूमावभीष्टकर्कटकेन त्रिज्यामिताङ्कैरंकितेन वृत्तं दिगंकितं भगणांशैश्चांकितं कृत्वा तत्र प्राचीचिह्नाद्दक्षिणतो नातिदूरे प्रदेश उत्तरे ऽयने वृत्तमध्यस्थितेन कीलेन रवेरुदयो वेध्यः। ततो ऽनन्तरं वर्षमेकं रव्युदया गणनीयाः। ते च पञ्चषष्ट्यधिकशतत्रय ३६५ तुल्या भवन्ति। तत्रान्तिमोदयः पूर्वोदयस्थानादासन्नो दक्षिणत एव भवति। तयोरन्तरं विगणय्य ग्राह्यम्। ततो ऽन्यस्मिन् दिने पुनरुदयो वेध्यः। स तु पूर्वचिह्नादुत्तरत एव भवति। तदप्युत्तारमन्तरं ग्राह्यम्। ततो ऽनुपातः। यद्यन्तरद्वितयकलाभिरेकीकृताभिः षष्टि ६० घटिका लभ्यन्ते तदा दक्षिणेनान्तरेण किमिति।

"Now, on a flat ground, a circle should be drawn by a pair of compasses of desired measure, which is marked with the measurements of the Radius, and the circle should be graduated with directions and degrees. During *uttarāyaṇa*, [when the sunrise is seen] towards a point which is south from the eastern cardinal point but not so far, the sunrise should be observed with a pin at the centre of the circle. Then, the number of sunrise should be counted for one year. It is equal to 365. At this sunrise, the point of the sunrise is a little south from the previous (i.e. the last year's) point. The distance of these two points should be measured. Then, on the next day, the sunrise should be observed again. It is a little north from the previous (the last year's) mark. The distance of these two points should also be measured. Then the ration-proportion. If the sum of these two differences is 60 *ghaṭikās*, what is the amount of the southern difference?"

The length of a year obtained by this method is evidently a tropical year, because it is based on the movement of the sun, and different from sidereal year which is necessary for Hindu astronomy. It is quite strange that Bhāskara II does not mention this difference, and immediately gives the length of a year as 365 days, 15 *ghaṭikās*, 30 *palas*, 22½ *vipalas*, which is apparently the length of a sidereal year.

Bhāskara II wrote in his V-ŚDV (I.1-8) to observe the rising time of the star Aśvinī and of the sun. Next day, the star will rise about 10 *palas* (= 4 minutes) earlier. He tells to determine the difference between the sun's rising and the star Aśvinī's rising during the sun's revolution, and obtain the sun's mean daily motion. By this method, the length of a sidereal year can be obtained.

Someśvara wrote to observe the sunrise by a circular platform, and count the number of sunrise from the sun's entrance to the first point of Aries to the sun's next entrance to the first point of Aries. Someśvara followed Āryabhaṭa's original text in the *Āryabhaṭīya* (IV.48)[1)] which reads as follows.

क्षितिरवियोगाद् दिनकृद् रवीन्दुयोगात् प्रसाधितश्चेन्दुः।
शशिताराग्रहयोगात् तथैव ताराग्रहाः सर्वे।।48।।

"The Sun has been determined from the conjunction of the Earth and the Sun, the Moon from the conjunction of the Sun and the Moon, and all the other planets from the conjunction of the planets and the Moon." (Translated by K.S. Shukla)[2)]

Let the number of the conjunction (i.e. the number of civil days) be C, and the number of the rotation of the earth during the same period be E. Then the number of the sun's revolution is E minus C. So, if the rotation of the earth is known, the number of the sun's rotation (i.e. the number of sidereal years) can be calculated.

iii) The moon's revolution

Bhāskara II explained in his V-SŚ (I.ii.1-6) that the revolution of the moon should be determined by the observation with an armillary sphere (*Gola-yantra*). He writes as follows (V-SŚ, I.ii.1-6).

ततस्तद्गोलयन्त्रं सम्यग्ध्रुवाभिमुखयष्टिकं जलसमक्षितिजवलयं च यथा भवति तथा स्थिरं कृत्वा रात्रौ गोलमध्यचिह्नगतया दृष्ट्या रेवतीतारां विलोक्य क्रान्तिवृत्ते यो मीनान्तस्तं रेवतीतारायां निवेश्य मध्यगयैव दृष्ट्या चन्द्रं विलोक्य तद्वेधवलयं चन्द्रोपरि निवेश्यम्। एवं कृते सति वेधवृत्तस्य क्रान्तिवृत्तस्य च यः संपातस्तस्य मीनान्तस्य च यावदन्तरं तस्मिन् काले तावान् स्फुटचन्द्रो वेदितव्यः। क्रान्तिवृत्तस्य चन्द्रबिम्बमध्यस्य च वेधवृत्ते यावदन्तरं तावांस्तस्य विक्षेपः। ततो यावतीषु रात्रिगतघटिकासु वेधः कृतस्तावतीष्वेव पुनर्द्वितीयदिने कर्तव्यः। एवं द्वितीयदिने स्फुटं चन्द्रं ज्ञात्वा

तयोर्यदन्तरं सा तद्दिने स्फुटा गतिः। अथ तौ चन्द्रौ "स्फुटग्रहं मध्यखगं प्रकल्प्य" – इत्यादिना मध्यमौ कृत्वा तयोरन्तरं सा मध्यमा चन्द्रगतिः। तया ऽनुपातः। यद्येकेन दिनेनैतावती चन्द्रगतिस्तदा कुदिनैः किमित्येवं चन्द्रभगणा उत्पद्यन्ते।

"Thus, this *Gola-yantra*, which has an axis rightly pointed to the celestial poles and has a horizontal circle which is adjusted to the water level, should be firmly fixed. At night, the star Revatī should be looked by seeing through the centre of the sphere, and the end point of Pisces on the ecliptic circle should be adjusted in such a way that it touches the star Revatī. Then the moon should be looked by seeing through the centre, and the 'observational circle' (a circle which is perpendicular to the equator, and can be rotated around the celestial poles) should be adjusted in such a way that it touches the moon. After doing like this, the distance between the cross point of these two circles and the end point of Pisces is known as the true moon at this time. The distance between the ecliptic and the centre of the moon's disc along the 'observational circle' is the polar latitude (*vikṣepa*). Then on the next day, at the same *ghatikās* elapsed at night, the observation should be made again. Like this, after knowing the true moon on the next day, the difference between them is obtained as the true motion on this day. Then, these true values should be converted into the mean position by the rule started as '*Sphuṭagrahaṃ madhyakhagaṃ prakalpya*' etc., and the difference between them is obtained as the mean motion of the moon. From this, the ration-proportion. What is the lunar motion during the number of civil days [in a *kalpa*] when the lunar daily motion is such? Thus, the moon's *bhagaṇa* is obtained."

The rule quoted above is from the *Siddhānta-śiromaṇi, Grahagaṇita* (II.45)[1], whose text and an English rendering by Arkasomayaji are as follows.

स्फुटग्रहं मध्यखगं प्रकल्प्य
कृत्वा फले मन्दचले यथोक्ते।
ताभ्यां मुहुर्व्यस्तधनर्णकाभ्यां
सुसंस्कृतो मध्यखगो भवेत् सः।।45।।

"Assume the true planet to be the mean; compute the *manda* and *śīghra-phalas* and applying them inversely, we have an approximation of the mean planets. Treating these as the mean planets, again obtaining the *manda* and *śīghra-phalas* and again applying them inversely and repeating the process till constant values are obtained, we have by this method of successive approximation the mean planets required." (Translated by D. Arkasomayaji)[2]

Bhāskara II wrote in his V-ŚDV (I.1-8) to count the number of conjunctions of the sun and moon during the sun's revolution, and calculate the moon's mean motion. The number of the conjunction plus the number of the sun's revolution (one, in the above case) is the number of the moon's revolution. This method had already been suggested by Āryabhaṭa.

Someśvara also wrote to determine the moon's mean motion from the number of the conjunction of the sun and moon. Someśvara wrote that the moon's mean motion can also be determined by counting the number of moonrise during a year. The number of sidereal days in a year minus the number of moonrise in a year is the number of the moon's revolution in a year.

iv) The moon's apogee

Bhāskara II tells in his V-SŚ (I.ii.1-6) to observe the true motion of the moon every day, with the armillary sphere just like the observation for the moon's revolution, and find out the day when the true motion is smallest. On this day, the position of the moon is its apogee (*ucca*), and the true moon and the mean moon coincides. After a revolution of the moon, the day when the true motion is smallest should be determined again. After knowing the difference of these two points, i.e. the motion of the apogee during an anomalistic month, and the length of an anomalistic month, the *bhagaṇa* of the lunar apogee is obtained by ratio-proportion.

Bhāskara II wrote in his V-ŚDV (I.1-8) to determine the moon's true motion by using the *Yaṣṭi-yantra* in successive day at just the same time. The moon's true motion can be determined by the angular distance of the tip of the V-shaped two *Yaṣṭis* fixed in successive two days.

Someśvara wrote to determine the time from sunset to moonrise during *kṛṣṇa-pakṣa* (waning fortnight), by the *Ghaṭī-yantra*, and calculate the moon's true position from it, and obtain the moon's true motion by successive observations.

v) The moon's node

Bhāskara II tells in his V-SŚ (I.ii.1-6) to observe the moon every day, just like the observation for the moon's apogee, when its southern polar latitude is decreasing, and find out the day when the polar latitude is zero. Then, one should mark the position of the moon on the ecliptic circle of the armillary sphere. It is the position of the node (*pāta*). After a revolution, the position of the node should be determined again. It is west of the previous position, that is the motion of the lunar node is restrograde. After knowing these points, i.e. the motion of the node during a nodical month, and the length of a nodical month, the *bhagaṇa* of the lunar node is obtained by ratio-proportion.

Bhāskara II wrote in his V-ŚDV (I.1.8) that one should firstly determine the position of the moon by the *Yaṣṭi-yantra* when the moon's polar latitude is zero near the star Aśvinī, and again observe the *nakṣatra* where the moon comes to its node next time. Then the movement of the lunar node is known.

Someśvara wrote a different method to determine the lunar node as follows.

चन्द्रग्रहणे [स्पर्शकालात्] मध्यग्रहणं यावत् स्थित्यर्धघटिकाः चन्द्रग्रहणकालोत्पन्नाः ताः स्फुटसूर्यशशिभुक्त्यन्तरेण गुणयेत्, षष्ट्या विभजेत्, स्थित्यर्धलिप्ताः स्युः। तद्वर्गं सम्पर्कार्धवर्गाद् विशोध्य शेषस्य मूलं चन्द्रविक्षेपः। स त्रिज्याहतः खगाक्षिभक्तः [270] काष्ठितो (कोष्ठतो?) भुजचापम्।

"At the time of lunar eclipse, the *ghaṭikās* of the half duration of the lunar eclipse, from the time of commencement to the middle of the eclipse, [should be determined, and] it should be multiplied by the difference of the true daily motion of the sun and moon, and divided by 60. It is the minutes corresponding to the half duration [of the eclipse]. Its square should be subtracted from the square of the half of *samparka* (the sum of the apparent diameter of the eclipsed and eclipsing bodies, i.e. the moon and the earth's shadow), then the square root of the result is the polar latitude (*vikṣepa*) of the moon. It should be multiplied by the Radius (3438') and divided by 270, and the corresponding arc of *bhuja* (the moon's angular distance from the node, upto 90°) should be obtained by using a table (sine-table)."

This method is clear, because the polar latitude is zero at the centre of the earth's shadow, and the moon's polar latitude at the middle of the eclipse, which is equal to the angular distance of the moon and the centre of the earth's shadow, can be calculated applying Pythagorean theorem.

The moon's maximum polar latitude is assumed to be 270', i.e. 4°30'. The Rsin of the moon's *bhuja* is proportional to the Rsin of the moon's polar latitude, and it is roughly considered to be proportional to the moon's polar latitude itself, because Rsin $\theta = \theta$ when θ is small.

Someśvara further tells to determine the half duration from the middle of the eclipse to the end of the eclipse. If the first half is larger, the moon is in the odd quadrant, otherwise it is in the even quadrant. So, if the polar latitude is north and the moon is in the odd quadrant, the *bhuja* itself is the polar longitude, and if the moon is in the odd quadrant, the *bhuja* should be subtracted from 180°. If the polar latitude is south and the moon is in the even quadrant, the *bhuja* should be added to 180°, and if the moon is in the even quadrant, the *bhuja* should be subtracted from 360°. Someśvara tells to obtain the moon's longitudinal distance from the ascending node by this method. He further tells to make observation similarly at the next lunar eclipse, and determine the movement of the moon's node.

Someśvara wrote an alternative method that the moon's declination can be calculated by the shadow of the gnomon at the moon's transit, hence the moon's polar latitude can be calculated.

vi) The sun's apogee

The sun's apogee can be found out by the observation of the sun's smallest true motion. Bhāskara II wrote in his V-SŚ (I.ii.1-6) as follows.

मिथुनस्थे रवौ कस्मिंश्चिद्दिने रेवतीतारकोदयाद्यावतीभिर्घटिकाभी रविरुदितस्तावतीभिर्मीनान्ताल्लग्नं साध्यम्। यल्लग्नं स तदा स्फुटो रविर्ज्ञेयः। एवमन्यस्मिन् दिने ऽपि तयोः स्फुटार्कयोरन्तरं स्फुटा गतिः। एवं प्रत्यहं स्फुटगतयो ज्ञातव्याः। यस्मिन् दिने गतेः परमाल्पत्वं तद्दिने यावान् रविस्तावदेव रवेरुच्चं भवति। तस्योच्चस्य चलनं वर्षशतेनापि नोपलक्ष्यते। किन्त्वाचार्यैश्चन्द्रमन्दोच्चवदनुमानात् कल्पिता गतिः। स चैवम्। यैर्भगणैः सांप्रताहर्गणाद्वर्षगणाद्वा, एतावदुच्चं भवति ते भगणा युक्त्या कुट्टकेन वा कल्पिताः।

"On the day when the sun is in Gemini, the *ghaṭikās* between the rising of the star Revatī and the sunrise, that is the *lagna* from the end point of Pisces, should be determined. The *lagna* is the true position of the sun. On the next day, again [the observation should be made], and the distance between these positions of the true sun is the true motion. Like this, the true motion should be known every day. On the day when the motion is smallest, the sun is at the apogee (*ucca* or *tuṅga*). The movement of the apogee cannot be observed even after a hundred years. However, its motion has been conjectured, just like the lunar apogee, by the savants. This is also like that. As after *ahargaṇas* (number of days since the epoque) or years, the apogee is at such and such position, the *bhagaṇa* in a *kalpa* should be determined skilfully, or by *kuṭṭaka* (a kind of indefinite equation) method."

Someśvara wrote to calculate the sun's true position from the sun's midday shadow, and determine the difference from the mean sun. From successive observations, the position of the sun's apogee can be determined.

vii) The outer planets' epicyclic motion

As regards the epicyclic motion of the outer planets, Bhāskara II tells in his V-SŚ (I.ii.1-6) that the planets are drawn by the *śīghra-ucca* and departs from the mean motion. The *bhagaṇa* of the *śīghra-ucca* is the same as that of the sun. He wrote as follows.

शनेर्जीवात् कुजाद्वा यदा रविरग्रे वर्तते तदा मध्यग्रहात् स्फुटग्रहो ऽग्रतो दृश्यते। यदा तु पृष्ठगतो ऽर्कस्तदा मध्यात् स्फुटग्रहः पृष्ठतो दृश्यते। अतस्तेषां त्रयाणां रविसमानं शीघ्रोच्चं धीरैः कल्पितम्। अतो रविभगणतुल्याः शीघ्रोच्चभगणा इत्युपपन्नम्।

"If the sun proceeds in front of Saturn, Jupiter, and Mars, then the true planet is seen in front of the mean planet. When the sun is at the back [of the planets], the true planet is seen behind the mean planet. Thus, it is inferred by intelligent men that the *śīghra-ucca* of those three (Saturn, Jupiter, and Mars) is the same as the sun.

Therefore it is proved that the *bhagaṇa* of the sun is equal to the *bhagaṇa* of the *śīghra-ucca*."

viii) The planets' apogees

This is the method to find out the planet's apogee (*manda-ucca*). In this method, it is assumed that the *śīghra-ucca* is already known.

Bhāskara II wrote in his V-SŚ (I.ii.1-6) to obtain the *manda-sphuṭa* from the observed true planet. The *manda-sphuṭa* is the position of the mean planet corrected by *manda*-correction (equation of the centre) only, without the *śīghra*-correction. He says to obtain the *manda-sphuṭa* by deducting the *śīghra-phala* from the observed true planet by successive approximation. It should be obtained every day, and the day when the *manda-phala* is zero should be found out. On this day, the position of the planet is the apogee (*manda-ucca*) or perigee (*manda-nīca*). Its *bhagaṇa* is obtained just like the case of the sun's apogee.

ix) The inner planets' epicyclic motion

This is the method to find out the inner planet's *śīghra-ucca*. In this method, it is assumed that the *manda-ucca* is already known.

Bhāskara II tells to observe the angular distance between the sun and Venus by the circle instrument (*cakra-yantra*) when they are seen in the east. Then, the position of true Venus should be calculated from this angular distance and the position of the true sun. After that, the *manda-phala* is deducted from the true Venus. (Let this point be A.) And also, the mean sun should be obtained. (Let this point be B. This point is the same as the mean Venus.) From these values, the *śīghra-phala* is obtained as the angular distance between the points A and B. When the *śīghra-phala* is maximum, the *śīghra-ucca* is assumed to be pulling perpendicularly to the direction of true Venus. So, the *śīghra-ucca* is at the distance of 90° from true Venus. The position of the *śīghra-ucca* should again be determined at the next greatest elongation at the same side, and the *bhagaṇa* of the *śīghra-ucca* is calculated from these angular distance and the time span.

The *śīghra-ucca* of Mercury is also determined similarly.

x) The planets' nodes

Bhāskara II further wrote to determine the nodes (*pātas*) of the planets by the observation of the polar latitude just like the node of the moon.

In the case of outer planets, the node is the point where the *manda-sphuṭa* coincides with the ecliptic.

In the case of inner planets, the node is the point where the *śīghra-ucca* coincides with the ecliptic.

xi) The apparent diameter of the sun and moon

Bhāskara II wrote in his V-SŚ (V.6) to observe the apparent diameters of the sun and moon by the V-shaped staffs. He wrote as follows.

यस्मिन् दिने ऽर्कस्य मध्यतुल्यैव स्फुटा गतिः स्यात् तस्मिन् दिन
उदयकाले चक्रकलाव्यासार्धमितेन यष्टिद्वतयेन मूलमिलितेन
तत्रस्थदृष्ट्या तदग्राभ्यां बिम्बप्रान्तौ विध्येत्।

"On the day when the mean motion and the true motion of the sun are the same, the edges of the disc at sunrise should be observed by the two tips of the double staff of which two roots are combined (i.e. V-shaped staffs), and whose length is equal to the number of minutes in a radian."

When the mean motion and the true motion of the sun are the same, the sun is at the mean distance, So, the result is the mean apparent diameter of the sun. Bhāskara II gives the values 32'31"33"'. He also tells to observe the full moon, when the true motion is equal to the mean motion, and gives the value 32'0"9"'.

Bhāskara II mentioned another method in his V-ŚDV (V.2). He wrote to determine the time during which the disc of the moon rises from the horizon. From the time thus obtained and the moon's mean motion, the apparent diameter of the moon can be calculated by proportion.

xii) Conclusion

From the above discussions, it is seen that the methods of observations explained by Bhāskara II is theoretically correct, although some of the observations, such as the determination of the sun's apparent diameter by the V-shaped staffs, appear to be very difficult to practice. As we have discussed in the Introduction of the present chapter, the possibility of the actual observation in Classical Siddhānta period has been suggested by Roger Billard etc. The correctness of the methods of observation as we have seen above will also support this possibility.

Table of Astronomical Instruments

Vedāṅga Period	Āryabhaṭa	Varāhamihira	Brahmagupta	Lalla	Śrīpati	Bhāskara II
śaṅku (gnomon) →	*śaṅku* →	*śaṅku* →	*śaṅku* →	*śaṅku* →	*śaṅku* →	*śaṅku*
	chāyā →			shadow instrument →	shadow instrument	
	chatra					*phalaka*
		graduated level circle				
				nalaka →	*nalaka* →	*nalaka*
	yaṣṭi (staff) →		*yaṣṭi* →	*yaṣṭi* →	*yaṣṭi* →	*yaṣṭi*, *dhī*
		V-shaped staffs →	V-shaped staffs →	*śakaṭa* →	*gantrī* (=śakaṭa)	
				śalākā		
	cakra (circle) →	*cakra* →	*cakra* →	*cakra* →	*cakra* →	*cakra*
				bhagaṇa (equatorial circle) →		*nāḍī-valaya*
			pīṭha (horizontal circle) →	*pīṭha* →	*pīṭha*	
		kapāla (hemispherical sun-dial) →	*kapālaka* →	*kapāla* (horizontal semi-circle) →	*kapāla*	
	dhanus (semi-circle) →		*dhanus* →	*dhanus* →	*kārmuka* (=cāpa) →	*cāpa*
			kartarī →	*kartarī* (equatorial semi-circle) →	*kartarī*	
			turya-golaka (quadrant) →			*turya* (=*turya-gola*)
			armillayry sphere →	*gola* →	*gola* →	*gola*
	gola (celestial globe) →	*gola*	self-rotating instrument →	self-rotating globe →	self-rotating globe →	self-rotating instrument
outflow type of clepsydra →	*toya* (water instruments) →	water instruments →	water instruments →	water instruments →	water instruments	
	ghaṭikā *kapāla* (floating bowl type of cyepsydra) →	clepsydra →	*ghaṭikā* →	*ghaṭī* →	*ghaṭī* →	*ghaṭī* (=*ghaṭikā*)

11. CONCLUSION

In *Vedāṅga* period, there were only two kinds of astronomical instruments, namely the gnomon and the clepsydra. In Classical Siddhānta period, several astronomical instruments were used, and we can trace their development in astronomical literature of this period.

Āryabhaṭa (born 476 AD) already described several astronomical instruments, and many later instruments can be considered to be their descendants. The time of Brahmagupta (born 598 AD) appears to be the time of transition, and some new instruments were added. Lalla further added some new instruments and changed the construction of some instruments. Śrīpati (11th century AD) mainly followed Lalla. Bhāskara II (born 1114 AD) simplified the description of astronomical instruments, and added two instruments, namely the *dhī-yantra* and the *phalaka-yantra*. Among them, the *phalaka-yantra* is an ingenious instrument.

Besides, some interesting instruments are described in anonymous Siddhāntas. Among them, the quadrant and the horizontal gnomon described in the *Vṛddha-vasiṣṭha-siddhānta* are important, and may be a kind of forerunner of later developed instruments.

We should also note that the methods of observation were explained by Bhāskara II in detail. This paper is confined to the theoretical aspect of the instruments, and we should further investigate the condition of the actual use of these instruments.

The development of the astronomical instruments in Classical Siddhānta period can roughly be tabulated as Table of Astronomical Instruments. (I excluded anonymous works from the table, as the date of most of them is not certain.)

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27) French original reads as follows.

"*Śaṅkudviguṇamānena :* Il ne peut s'agir du rayon car ce dernier faisant alors deux coudées, le cercle serait tangent aux bords du carré nivelé et il ne restrait plus de place pour tracer les cercles complémentaires qui permettent de situer l'axe nord-sud et les directions intermédiaires."

(Dagens (ed. and French tr.), op. cit., part 1, (1970), p.68, footnote 8.)

28) We know, by spherical astronomy, that

$$\sin\delta = \sin\varphi.\cos\zeta + \cos\varphi.\sin\zeta\cos A,$$

where δ is the sun's declinatioin, φ the observer's latitude, ζ the sun's zentith distance, and A the sun's azimuth (measured from the north). And also, we know that

$$\sin\delta = \sin\varepsilon.\sin\lambda,$$

where ε is the obliquity of the eccliptic, and λ the sun's longitude.

If we assume that $\lambda = 20°$, ζ=90° – archtant (½) = 63.°4, and A = 90° (hence cos A is zero), we get $\varphi = 18°$.

If λ is 30°, we get $\varphi = 26°$. If λ is 40°, we get $\varphi = 35°$.

29) The rationale of the calculation of the actual corrction is as follows. From the first equation in the

note 28) above, we have

$$\cos A = \frac{\sin \delta - \sin \varphi . \cos \zeta}{\cos \varphi . \sin \zeta}$$

The value of ζ is (90° - arctant (½)) as before. From this equation, we can calculate the sun's azimuth A. Let θ be the difference betweent the azimuth A and 90°. Then, the chord of PW or P'E is 2r.sin (θ/2), where *r* is the radius of the circle.

30) Sudhākara Dvivedin's ed., p.207.

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36) Pingree, D.: "The Gaṇitapañcaviṃśī of Śrīdhara", *Ṛtam, Ludwik Sternbach Ferlicitaton Volume,* Lucknow (1979), 887-909; p.906.

37) D. Pingree's ed., text : vol.I, p.500; tr. : vol. II, p.189.

38) For the Indian origin of these formulae, see my previous paper (*IJHS,* **28** (1993)), 2.iii.c. (pp. 222-225)

39) Thibaut an Dvivedin (ed. and tr.), text : p.17, tr.: pp.34-35; or Neugebauer and Pingree (ed. and tr.), part 1, pp.66-67.

40) Thibaut and Dvivedin (ed. and tr.), text : p.6, tr.: p.12: or Neugebauer and Pingree (ed, and tr.), part 1, pp.36-37.

41) Raṅgācārya's ed., text : p.154; tr.: p.278.

Section 3.iv)...

1) *Jyautiṣa-siddhānta-saṅgraha,* fasc.2 (1917), p.30.

2) For the astronomical instruments of Delhi Sultanate and Mughal periods, see Ohashi, Yukio :"Sanskrit Texts on Astronomical Instruments during the Delhi Sultanate and Mughal Periods", *Studies in History of Medicine and Science,* vols. X-XI, (1986-87), p.165-181.

Section 4.ii)

1) Shukla, K.S., op.cit. (*Gaṇita,* **18** (1)(1967) 83-105), pp.92-93.

2) Ibid., pp.95-96

3) I have followed K.S. Shukla's explanation. (See ibid., p.96.)

4) This is from a manuscript Lucknow 45749. (Commentary on the *Sūrya-siddhānta* (XIII.25))

5) Bina Chatterjee (ed.), part 1, p.247.

6) Ibid., part 2, pp.288-289.

7) Babuāji Miśra's ed., part 2, pp.296-298.

Section 4.iii)

1) Shukla, K.S., op.cit. (*Gaṇita,* **18** (1)(1967) 83-105), p.93.

2) Ibid., p.99.

3) Ibid., pp93-94.

4) Ibid., pp.99-100.

5) This is from a manuscript Lucknow 46145. (Commentary on the *Sūrya-siddhānta* (XIII.20-21).)

6) Shukla, K.S., op.cit. (*IJHS*, **12** (1977) 181-186), P.185.

Section 4.iv)...

1) Thibaut, G. and Sudhākara Dvivedin (ed. and tr.), op.cit., (1889), text : pp.37-38, Sanskrit commetary : pp.72-75, English translation : pp. 75-78.

2) Neugebauer, O. and David pingree (ed. and tr.), op.cit., 2 parts, (1970-71), text and tr.: part 1, pp.122-125, and commetary : part 2, pp.85-88.

3) Since published as Ôhashi, Yukio :"Varāhamihira's Orthographic Projection .... An Interpretation of the Pañcasiddhāntikā XIV.5-11", *Journal of the Asiatic Society,* vol.xxx, (1989), pp.66-76.

4) Thibaut and Dvivedin (eds.), op.cit., pp.37-38, left column. They used two manuscrips of the text, and they printed the reading of one text which they considered to be better. The reading of the other text has been mentioned in their footnotes. I have basically followed the reading chosen by them.

5) Thibaut and Dvivedin (ed. and tr.), op.cit., p.75.

6) Bina Chatterjee's ed., part 1, p.247.

7) Ibid., part 2, pp.289-290.

8) S.R. Sarma's ed. (1966), part 1, p.29.

9) Ibid., part 2, p.94.

Section 5.ii)...

1) Shukla, K.S., op.cit. (*Gaṇita,* **18** (1) (1967) 83-105), p.93.

2) Ibid., p.98.

3) Sudhākara Dvivedin's ed., pp.386-394.

4) See Yano, Michio, op.cit. (*Indo-Iranian Journal,* **29** (1986) 17-29), pp.19-20.

5) Bina Chatterjee's ed., text: part 1, pp.247-248; K.S. Shukla's tr.: ibid. part 2,pp.289-290.

6) Babuāji Miśra's ed., part 2, pp.295-296.

7) See Burgess and Whitney's tr., op.cit., p.306.

8) See Sastri and Wilkinson's tr., op.cit., pp.218-221.

Section 5.iii)...

1) See Bina Chatterjee's tr., part 2, p.99.

2) Babuāji Miśra's ed., part 1, pp.302-303.

3) See D. Arkasomayaji's tr., pp.328-329.

4) See K.S. Shukla's tr., part 2, p.288.

Section 5.iv)...

1) See Bina Chatterjee (ed. ant tr.), part 2, pp. 287-288.

Section 5.v)...

1) Thibaut and Dvivedin's ed., pp.38-39; or Neugebauer and Pingree's ed., p.124 and 126. I have followed Neugebauer and Pigree's reading.

2) Sudhākara Dvivedin's ed., p.388.

3) See Bina Chatterjee (ed. and tr.), part 2, p.288 and 290.

4) Babuāji Miśra's ed., part 2, p.299.

Section 5.vi)...

1) Sudhākara Dvivedin's ed., pp. 390-394.

Section 5.vii)...

1) See Sastri and Wilkinson's tr., pp.211-226.

2) Ānandāśrama ed., *Gola*, part 2, p.407.

Section 6.ii)...

1) K.S. Shukla, op.cit. (*Gaṇita*, **18** (1) (1967) 83-105), p.93.

2) Ibid., p.98.

3) This is from a manuscript Lucknow 45749. (Commentary on the *Sūrya-siddhānta* (XIII.25).)

4) This is from a manuscript Lucknow 46145. (Commentary on the *Sūrya-siddhānta* (XIII.20-21).)

5) K.S. Shukla, op.cit. (*IJHS*, **12** (1977) 181-186), p.185.

6) Thibaut and Dvivedin's ed., p.40; or Neugebauer and Pingree's ed., part 1, p.128. I have followed Thibaut and Dvivedin's reading.

7) According to Sudhākara Dvivedin's Sanskrit commetary, the word "*anyākṣa*" means "*madhya-natāṁśa*" (midday zenith distance). (Thibaut and Dvivedin, op.cit., p.77)

8) Sudhākara Dvivedin's ed., p.386.

9) Bina Chatterjee's ed., part 1, p.244.

10) Ibid., part 2, pp.283-284.

11) Ibid., footnote.

12) Babuaji Misra's ed., part 2, p.287.

13) Ānandaāśama ed., *Gola*, part 2, pp.367-368. Also see Sastri and Wilkinson's tr. p.212.

Section 6.iii)...

1) Shukla, K.S., op.cit. (*Gaṇita*, **18**(1) (1967) 83-105), p.93.

2) Ibid., p.97

3) This is from a manuscript Lucknow 46145. (commetary on the *Sūrya-siddhānta* (XIII.20-21).)

4) K.S. Shukla, op.cit. (*IJHS*, **12** (1977) 181-186), p185.

5) Sudhākara Dvivedin's ed., pp.382-385.

6) Bina Chatterjee's ed., part 1, p.244.

7) Ibid., part 2, p.284.

8) Babuāji Miśra's ed., part 2, p.287.

9) See Sastri and Wilkinson's tr., p.212.

Section 6.iv)...

1) Sudhākara Dvivedin's ed., p.385.

2) See Sastri and Wilkinson's tr., p.212.

3) *Jyautiṣa-siddhānta-saṅgraha*, fasc. 2 p.29.

4) For the astronomical instruments of Delhi Sultanate and Mughal periods, see Ohashi, Yukio : "Sanskrit Texts on Astronomical Instruments during the Delhi Sultanate and Mughal Periods", *Studies in History of Medicine and Science*, vols. X-XI, (1986-1987), pp.165-181.

Section 6.v)...

1) See Bina Chatterjee (ed. and tr.), part 2, p.286 (K.S. Shukla's tr.).

2) See Sastri and Wilkinson's tr., pp.210-211.

Section 6.vi)...

1) Sudhākara Dvivedin's ed., p.396.

2) Bina Chatterjee's ed., part 1, p.244.

3) Ibid., part 2, pp.284-5.

4) Babuāji Miśra's ed., part 2, p.289.

Section 6.vii)...

1) Thibaut and Dvivedin's ed., pp.39-40; or Neugebauer and Pingree's ed., part 1, p.128.

I have followed Thibaut and Dvivedin's reading.

2) Neugebauer and Pingree (ed. and tr.), part 1, p.129.

3) Sudhākara Dvivedin's ed., pp.395-396.

4) Bina Chatterjee's ed., part 1, p.244.

5) Ibid., part 2, p.285.

6) Babuāji Miśra's ed., part 2, p.290.

Section 6.viii)...

1) Sudhākara Dvivedin's ed., pp.396-397.

2) Bina Chatterjee's ed., part 1, p.244.

3) Ibid., part 2, p.285.

4) Babuāji Miśra's ed., part 2, pp.290-291.

Section 6.ix)...

1) See K.S. Shukla's tr., pp.94-95.

Section 7.i)...

1) *Jyautiṣa-siddhānta-saṅgraha*, fasc.1, p.34.

2) K.V. Sarma's ed. and tr., op.cit. (*Brahmavidyā*, **20**(1-2) (1956) 119-186, and **21** (1-2) (1957) 87-144).

Section 7.ii)...

1) Shukla, K.S. and K.V. Sarma (ed. and tr.), (Pt.1), (1976), p.129.

2) Ibid.

3) See K.S. Shukla's ed. of Someśvara's commentary on the *Āryabhaṭīya*, (1976), p.268. Also see K.S. Shukla and K.V. Sarma's ed. and tr. of the *Āryabhaṭīya*, (Pt.1), pp.129-1360.

4) See K.S. Shukla (*Gaṇita*, **18**(1) (1967) 83-105), pp.100-101.

5) Thibaut and Dvivedin's ed., p.40; or Neugebauer and Pingree's ed., p.128 and 130. I have followed Neugebauer and Pingree's reading.

Section 7.iii)...

1) Sudhākara Dvivedin's ed., pp.375-380.

2) In the actual calculation of five planets in the *Brāhma-sphuṭa-siddhānta* (and also in the *Siddhānta-śiromaṇi*), the *manda-phala* is a function of the true anomaly of the planet, and not of the mean anomaly of the planet. The process of the calculation is as follows. Firstly, the *manda-phala* is calculated from the mean planet, and is applied to the mean planet. The result is the *manda-sphuṭa*. Secondly, the *śīnghra-phala* is calculated from the *manda-sphuṭa*, and is applied to the *manda-sphuṭa*. The result is the true planet after the first approximation. From the result, the *manda-phala* is calculated and applied to the original mean planet. From this result, the *śīghra-phala* is calculated and applied. The result is the true planet after the second approximation. This process is repeated unitl a constant value is obtained. (In the case of Mars, this method is not used. Firstly, a half of the *manda-phala* and a half of the *śīghra-phala* are applied, and the once corrected Mars is obtained. From the result, the *manda-phala* is calculated, and its whole amount is applied to the original mean Mars. From this result, the *śīghra-phala* is calculated and applied. The result is the true Mars.)

3) See Bina Chatterjee's tr., part 2, pp.232-37.

4) Babuāji Miśra's ed., part 2, pp.207-217.

5) See Sastri and Wilkinson's tr., pp.151-160.

Section 7.iv)...

1) See Chatterjee, Bina (ed. and tr.), part 2, pp.280-281.

2) See Burgess and Whitney's tr., pp.298-304.

3) Babuāji Miśra's ed., part 2, pp.279-280.

4) See Sastri and Wilkinson's tr., p.210.

Secxtion 7.v)...

1) Needham, Josheph : *Science and Civilisation in China*, vol.3, Cambridge (1959), p.340 and 382.

2) "The Almagest of Ptolemy", translated by R. Catesby Taliaferro, in *Great Books of the Western World*, vol.16, Chicago (1952), pp.261-263.

3) Ibid., pp.143-144.

4) Ibid., p.25.

6) Ibid., p.78.

6) Colebrooke, H.T. : "On the Indian and Arabian Divisions of the Zodiac", originally in the *Asiatic Researches*, **9** (1807), pp.323-376; reprinted in his *Miscellaneous Essays*, vol. II, (1837), reprinted New Delhi (1977) 321-373; especially p.345 ff.

7) Ibid., p.351.

8) See, for ecsample Needham, op.cit. (1959), pp.339-390; and Bo Shuren: "Astrometry and Astrometric Instruments", in Institue of the History of Natural Sciences (ed.): *Ancient China's Technology and Science*, Beijing (1983), pp.15-32.

9) Needahm, Joseph, Wang Ling, and Derek J. Price : *Heavenly clockwork*, Cambridge (1960) is a detailed study of this work.

Section 8.i)...

1) See my previous paper (*IJHS*, **28** (1993)), 2.iv. (pp. 225-233)

Section 8.ii)...

1) K.S, Shukla, op.cit. (*Gaṇita*, **1̃8**(1) (1967) 83-105), p.95.

2) Ibid., p.101.

3) Ibid., p.95.

4) Ibid., p.101.

5) Thibaut and Dvivedin's ed., p.41; or Neugebauer and Pingree's ed., part 1, p.132. I have followed Thibaut and Dvivedin's reading.

6) Sudhākara Dvivedin's ed., p.395.

7) Bina Chatterjee's ed., p.246.

8) Ibid., part 2, p.287.

9) See Burgess and Whitney's tr., p.308.

10) Babuāji Miśra's ed., part 2, pp.293-294.

11) See Sastri and Wilkinson's tr., pp. 211-212.

12) Ānandāśrama ed., *Gola*, part 2, p.366.

Section 8.iii)...

1) Dowson, J. "On a newly discovered Bactrian Pali Inscription; and on other Inscriptions in the Bactrian Pali Character", *Journal of the Royal Asiatic Socity*, vol.20, (1863) pp.221-268, especilly p. 250f; Senart, M.E. "Notes d'epigraphie indienne, VI", *Journal Asiatique*, sér. 9, tome 7, (1896), pp.5-25; Lüders, H. "The Manikiala Inscription", *Journal of the Royal Asiatic Society*, (1909), pp.645-666; Pargiter , F.E, "The Inscription on the Manikiala Stone", *Journal of the Royal Asiatic Society*, (1914), pp.641-660; and Konow, Sten : *Corpus Inscriptionum Indicarumm*, vol.II, part I, *Kharoṣṭhi Inscriptions*, Calcutta (1929), pp.145-150. According to Konow, this inscription's date is AD 145. All interpretations, except that of Pargiter, suggest nothing about water clock.

2) Pargiter, op.cit. (*JRAS*, 1914, 641-660); and Pargiter, F.E, "The Telling of Time in Ancient India". *Journal of the Royal Asiatic Society*, (1915), pp.699-715, especially pp.703-704.

3) Hultzsch, E. : "An Earthenware Fragement of Guhasena of Valabhi", *Indian Antiquary*, vol. XIV, (1885), p.75.

4) Fleet, J.F. : "The Ancient Indian Water-clock", *Journal of the Royal Asiatic Society* (1915), pp.213-230, especially p.230.

5) Sarma, S.R. : "Zeitmessung im 7. Jahrhundert in Indien", unpublised paper (1989). I am grateful to Dr. S.R. Sarma who kindly sent me this paper.

6) Pant, Prācāra Mohandeva (ed.) : *Kādambarī* (Pūrva-ardha), Delhi, Motilal, (1976) p.245.

7) His name was transcribed as I-Tsing by Takakusu, Junjirō.

8) The original text is included in the *Taishō-shinshū-daizōkyō* (Taishō edition of Chinese Tripitaka), vol. 54, Tokyo (1928), (Text no. 2125). There is an English translation by Takakusu, Junjiro : *A Record of the Buddhist Religion as practiced in India and Malay Archipalego (AD 671-695)* by I-Tsing, Oxford (1896). The description of the clepsydra is in chapter 30 (Taishō ed., pp.225-226; Takakusu's tr., pp.140-146.)

9) I have translated from Taishō edition, p.225.

10) Sachau, Edward C. (tr.): *Alberuni's India,* London (1910), reprinted New Delhi (1983), vol.1, p.334.

11) See Qaisar, Ahsan Jan : *The Indian Response to European Technology and Culture (A.D. 1498-1707),* Delhi (1982), p.65; and Elliot, H.M. and John Dowson (tr. and ed.): *History of India, as told by its own historians,* vol. III, reprinted Allahabad (n.d.) p.338.

12) Beveridge, Annette Susannah (tr.): *Bābur-nāmā* (Memoirs of Bābur), (1922), reprinted New Delhi (1979), pp.516-517.

13) Jarrett, H.S. (tr.): *Ā'īn-i Akbarī* by Abū'l-Fazl 'Allāmī reviced by Jadunath Sarkar, vol.III, Calcutta (1948), reprinted New Delhi (1978), pp.17-18.

14) Sarma, Sreeramula Rajswara : "Astronomical Instruments in Mughal Miniatures", *Studien zur Indologie und Iranistik,* Band 16/17, (1992), pp.235-276, especially pp. 241-243 and plates 2 and 3.

15) Gilchrist, John : "Account of the Hindustanee Horometry", in *Asiatic Researches,* vol.5 (1797) 81-89, p.87.

16) Martin, Montgomery (ed.) : *The History, Antiquities, Topography, and Statistics of Eastern India,* vol. 5, "Rangpur and Assam", (1838), reprinted Delhi (1976), pp.506-507.

17) von Schlagintweit-Sakünlünski, Hermann : "Eine Wasseruhr und eine metallene Klangscheibe", in *Sitzungsberichte der Bayerischen Akademie der Wissenschaften, math.-phys. Classe,* **1** (1871) 128-138.

18) Fleet, J.F. : "The Ancient Indian Water-Clock", *Journal of the Royal Asiatic Society,* (1915) 213-230.

19) Pargiter, F.E. : "The Telling of Time in Ancient India", *Journal of the Royal Asiatic Society,* (1915), pp.699-715.

20) Jacobi, Hermann : "Einteilung des Tages und Zeitmessung im alten Indien", *Zeitschrift der Deutschen Morgenländischen Gesellschaft,* **74**, (1920), pp.247-263.

Section 8.iv)...

1) See K.S. Shukla, op.cit. (*Gaṇita,* **18**(1) (1967) 83-105), pp.100-101.

2) Thibaut and Dvivedin's ed., pp.40-41; or Neugebauer and Pingree's ed., part 1, p.130.

3) Sudhākara Dvivedin's ed., pp.397-399.

4) See Chatterjee, Bina (ed.), part 2, pp.282-283.

5) See Burgess and Whitney's tr., pp.306-308.

6) Babuāji Miśra's ed., part 2, pp.284-286.

Section 8.v)...

1) Sudhākara Dvivedin's ed., p.399-400.

2) See Chatterjee, Bina (ed.), part 2, pp.281-282.

3) Ibid., p.283.

4) See Burgess and Whitney's tr., p.305.

5) Babuāji Miśra's ed., part 2, pp.282-283.

6) See Sastri and Wilkinson's tr., pp.227-228.

7) Needham, Joseph, Wang Ling, and Derek J.Price : *Heavely Clock Work*, Cambridge (1960), p.192.

Section 8.vi)...

1) Hall and Sastri's ed., p.324. Actually, vs.21 has three half-verses in this edition and the half-verses which I guote here are 21(ii and iii).

2) Burgess and Whitney's tr., p.307.

3) K.S. Shukla's ed., (1957), p.136.

4) Ibid., p.136, footnote 3.

5) AS Calcutta G-10758, folio 152a. Here, the quarter verse reads "sa-sūtra-veṇu-garbha-sthe".

6) Lucknow 47051, folio 43a. (This is actually a photo-copy of AS Bombay Bhau Daji -294.) Here, the quarter verse reads "sva-sūtra-veṇu-garbhais taiḥ".

7) Lucknow 45760, folio 98 a. Here, the quarter verse reads "sa-sūtra-veṇu-garbhas taiḥ".

8) Lucknow 46145, p.336. Here, the quarter verse reads "sa-sūtra-veṇu-garbha-sthaiḥ".

9) For the astronomical instruments of Delhi Sultanate and Mughal periods, see Ohashi, Yukio : "Sanskrit Texts on Astronomical Instruments during the Delhi Sultanate and Mughal Periods". *Studies in History of Medicine and Science*, vols. X-XI, (1986-87), pp.165-181. For the sand-clock, see Sarma, S.R., op.cit. (*Studien zur Indologie und Iranistik*, Band 16/17, 1992, 235-276), p.243 f.

Section 9....

i) See Sastri and Wilkinson's tr., pp. 213-218. Also see Gurjar, L.V., op. cit. (1947), pp. 168-173; and Ray, R.N., op.cit., (1985), pp.328-332.

Section 10.i)....

1) Ānandāśrama ed., no. 110, part 1, pp.18-24.

2) Pandey, C.B. (ed.), Varanasi (1981), pp. 4-5.

3) Ibid., pp.98-99.

4) Shukla, K.S. (ed.), (Pt.2), (1976), pp.283-286.

Section 10.ii)...

1) Shukla, K.S. and K.V. Sarma (ed. and tr.), (Pt.1), (1976), p.162.

2) Ibid.

Section 10.iii)...

1) Ānandāśrama ed., no. 110, part 1, p.114.

2) D. Arkasomayaji's tr. (1980), pp.170-171.