begins simultaneously for any place of the earth, if \( y > x \), then he should know that he lies on the east of the primary meridian because his Sun-rise happens to be earlier than the Sun-rise on the primary meridian. Also the difference \( y - x \) gives the Desāntara correction in time for his place. The converse is the case if he happens to lie on the western side of the primary meridian. The time at which the eclipse takes place on the primary meridian after the Sun-rise there which is obtained by computation is called Drik-grahaṇa - Kāla; whereas the local time after Sun-rise observed by the observer is called pragrahaṇa-Kāla. Their difference is therefore the Desāntara correction in time.

If the Desāntara is to be got in yojanas, \( \frac{T \times C}{60} \) is the answer, where \( T = y - x \) and \( C \) is the rectified circumference of the earth, for, if a difference of 60 ghātis be there for \( C \) yojanas, what should be the distance in yojanas in order that the difference is \( T \)? The answer is as given above.

Hence to obtain the positions of the Sun and the Moon at the beginning of the eclipse at the locality we have to add or subtract as the case may be \( \frac{T \times \delta m}{60} \) where \( \delta m \) is the daily motion of the Sun or the Moon, and \( T \) is \( y - x \) cited above, for, "If in 60 ghātis the motion be \( \delta m \), what would it be in \( T \)?" is the rule of three for which the answer is as stated above.

Now the question is when the week-day begins for the locality. It must be noted clearly, that in Hindu Astronomy the moment of Sun-rise at the primary meridian alone is to be reckoned as the beginning of the week-day universally. This convention is adopted for convenience. Thus the astronomical week day for any locality does not begin from the Sun-rise of the locality, but may begin earlier or later. This difference is given by \( y - x \) cited above.
There is yet another subtlety in the commencement of the week-day, arising out of the latitude of the place. The former analysis pertains to the longitudinal difference. The difference arising out of latitude between the local Sun-rise and the Lanka-Sun-rise is given by what is called Chara-Kāla. Since the week-day begins at Lanka Sun-rise and the local Sun-rise differs from the Lanka Sun-rise not merely by a longitudinal difference but also by a latitudinal difference, to compute the actual beginning of the week-day before or after the local Sun-rise, we have to take into account both the differences cited above. In other words, computing the local Sun-rise and also the Lanka Sun-rise, we have to decide the beginning of the week-day before or after the local Sun-rise.

Verses 7, 8. The correction called Bijakarma for the planetary positions.

The number of years from the beginning of the Kalpa divided by 12000, the remainder, or the difference of the divisor and the remainder whichever is less is to be divided by 200. The quotient in minutes of arc, multiplied by 3, 5, 5, 15, 2 respectively is a negative correction in the positions of the Sun, Moon, Jupiter, Venus, and the lunar apogee and multiplied by 1, 52, 2 and 4 gives the positive correction in the positions of Mars, Mercury, the lunar Node and the Saturn respectively.

Comm. By the phrase 'The remainder or the difference of the remainder and the divisor', it is plain that the corrections positive or negative increase for 6000 years and decrease for the next 6000 years. Bhāskara gives no reason for these corrections, but, we have to construe these corrections on the following rational grounds. Bhāskara, however, says that the corrections were accepted by him on the basis of Āgama. This Āgama—stipulation was there in Brahma-Sphuta-Siddhānta and was later incorporated by Sripati also in his Siddhānta-Sekhara and as such was
accepted by Bhāskara also. However, in Brahma Suhuta Siddhānta both as first published as an edition of M. M. Sudhākara Dwivedi and later by the late Rāmaswarupa S'arma in 1966, the verses 59, 60 of Madhyamādhikāra suggest that the corrections are negative in the case of all the planets; whereas both Sripati and Bhāskara make them positive in the case of the latter four viz. Mars, Mercury, the lunar Node and Saturn. By this we have to construe that Sripati and Bhāskara must have had before them a text which should have read 'स्व' in the place of 'च' in the last pāda of verse 61. M. M. Sudhākara-Dwivedi did not notice this anomaly of the positiveness of the correction with respect to the latter four, but he remarked, however, that there was a prosodial lapse in the last pāda of verse 61, for which he offered a suggestion that instead of ब्रेः, we had better read ब्रेः:—This suggestion, no doubt, rectifies the prosody of the verse, but not the the anomaly cited above which was not noticed by M. M. Sudhākara Dwivedi. So, we have offered our own suggestion namely that in the place of च as mentioned above if we read स्व, we not only rectify the prosodial error but also the anomaly referred to. Rāmaswarupa S'arma noted the anomaly but did neither refer to the prosodial error nor offer a correction. It seems that Rāmaswarupa S'arma did not verify the corrections stipulated from the verses 91, 92, 93 of Madhyamādhyaśya of Siddhānta Sēkhara. In this latter work, there is another anomaly namely that in the case of Mercury, the number 62 is the multiplier and not 52. Makkibhatta, the ancient commentator had before him a text which read 62 in the place of 52, in all probability, a mistake of the scribe. M. M. Sudhākara Dwivedi is reported to have later pronounced that 52 must be the correct figure when this was brought to his notice as this number 52 was found both in Brahmagupta and Bhāskara. As reported by the editor of Siddhānta Sēkhara Pandit Babuaji Mishra, who mentions this latter pronouncement of Sudhākara Dwivedi his teacher, also says that Sudhākara Dw-
vedi suggested the reading ध्रृशार in the place of ध्रृश्य of verse 93 of Siddhānta Sekhara. The fact that Makkibhātta commented ध्रृश्यवक्कुण as ध्रप्रियवक्कुण shows that he did not consult Brahmaśphuta Siddhānta in this place; also, he must have had a manuscript before him which scribed ध्रृश्य in the place of ध्रृशार. Using श र ल, indiscretely is not uncommon in many books of North India, from a long time and the scribe of the manuscript probably having used श in the place of ल and then by an oversight a latter scribe having inverted ल as र, Makkibhātta must have commented like that.

Incidentally a remark may be made here about Makkibhātta. He was evidently a keralite because he used letters to signify numbers as was a common practice among the Kerala Astronomers, and as he also commented upon Brihad-Bhaskariya. Further, it is interesting to note that he wrote in his commentary under verse 39 of the Sādhana-dhyāya of Siddhānta Sekhara viz. "भागोविग कर्तव्यवतः सत्ताव्यथा चुदिनानि तातिवा", "भु ने पालकुची चहतिन्" etc”. This idea shows that he accepted Aryabhata’s verse "अनुभोव गति: etc” implying that the earth is rotating.

Bhāskara says that the Bija correction mentioned was purely based on Āgama and Upalabdhi (meaning authority and observation'). M. M. Sudhākara Dwivedi seems to have reiterated the same as reported by Babuaji Mishra, in a foot-note. Kamalākara, condemned this Bijakarma as it was unwarranted and had no proof.

A rational explanation as to why this Bija Karma was prescribed either by Brahmagupta himself or some authority which he seems to have accepted may be given as follows. The small differences in the numbers of sidereal revolutions or what is the same the minute differences in the accepted daily motions of the planets and the assumption of a conjunction of all the planets and planetary points at the beginning of Kalpa, which is beyond proof,
resulted in a difference between the computed planetary positions and their observed positions. So, the originator of this Bija-Samskāra, noting the differences in his own time devised a formula, which could account for those differences. But this formulation was bound to go wrong in later times as long as the daily motions are not corrected to the minutest extent possible and as long as the fundamental basis of the conjunction of all the planets and planetary points is not proved. This seems to be the reason why so many texts were written incorporating small differences in different times as reported by Gaṇeśa (1507 A.D.) in his work Brihat-Tithi-Chintāmani in the words “The calculations of planetary positions according to the methods indicated by Brahma, Vasishtha and Kasyapa Siddhantas held good in their own times, but grew obsolete later; Then Maya, the demon at the end of Krita obtained the science from the Sun God, which again grew obsolete in this Kaliyuga wherein parāśara began to hold the ground for a good length of time. Then Āryabhata rectified the methods; when even those methods grew obsolete, Durga-Simha, Varāha Mihira and others set them right. Again Brahmagupta came into the picture to rectify the methods by his own observations. Then came Kṛṣava (Gaṇeśa’s father) who rectified further. After a lapse of sixty years, his son Gaṇeśa has now to correct the Science. If this also grows obsolete (as it is bound to) in course of time, let others again rectify it by observing conjunctions of the Moon and planets with the asterisms.”

Obsoleteness arises out of two contexts, one a justifiable situation and the other based upon a wrong premise. The first is as follows. Suppose as a first approximation we take the length of an year as 365 days. We will have committed an error nearly $\frac{1}{4}$ of a day, so that the error accrues to a day in 4 years. Thus the convention of the leap year arose so that during four years we give a day more to February. Here again we have overestimated the error by nearly $\frac{1}{400}$ of a day. Hence in 400 years the
above correction leads to an error of a day. So, it is that we pronounced that out of the years 2000, 2100, 2200, 2300 A.D., the year 2000 A.D. alone is a leap year, and not the remaining, the convention being that the number of the century, here 20, must be also a multiple of four. On this back-ground, suppose we prepare a manual called a Karāṇa grantha taking the length of the year to be 365.25 days. It works alright for some time but in the course of 400 years the error will have reached to as much as one day. Thus a manual like the above works only for a short time and the approximation made gradually brings in a divergence on account of which such a manual grows obsolete. That is why one Narasimha who happened to prepare a manual in 1333 Saka year (1411 AD) opens his work with the words "तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि तत्त्विच्च यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि यद्यपि

This kind of obsolescence arising out of inevitable approximations that have to be made in the preparation of manuals is permissible. But Suppose the premise of the manuals itself is incorrect, then the rectification of the manuals is no good so long as the data given in the premise are not corrected. There are two fundamental detects in the ancient works according to a modern analysis namely (1) The Supposition that all the planets were in conjunction at the Zero-point of the Zodiac in the beginning of a Mahāyuga (2) Small variations in the constants like the daily motion of the planets and the like. According to the modern interpreters of Hindu Astronomy the
first premise was not correct. According to them, some astronomers having observed the daily motions of the planets or what is the same the sidereal periods of the planets to a sufficiently good approximation calculated back or extrapolated a date on which these planets should have been in conjunction at the Zero-point of the Zodiac. The extra-polated date was naturally wrong to some extent because the sidereal periods found could not but be correct only to a particular degree of approximation. Thus a little alteration in the number of sidereal revolutions alone or the number of days in a Mahayuga made to suit the observed positions at a particular epoch would be only a tinker- ing of the problem and not a true solution. Thus Hindu Astronomy could be saved and its methods could still be followed provided instead of trying to presume a date at which all the planets were in conjunction (No doubt in the long bosom of time, such a presumption also could not be ruled out) correctly observed positions of the planets by the help of modern instruments were taken as the basis of an epoch and thereafter using more correct values of the constants such as the sidereal revolutions, maximum equations of centre and maximum Sīghraphala, obliquity of the ecliptic etc. The second defect cited above thus being removed, and the original premise being changed, the methods of calculation still hold good and there would be no necessity to be going on with tinkerings of the problem.

The Bija-correction which we are commenting upon was rightly criticised by Kamalākara as irrational though he himself fanatically tried to uphold Sūrya Siddhānta. Even today there are a good number of the traditional Hindu Astronomers who do hold that the Sūrya Siddhānta was revealed to Maya at the end of Kritayuga in spite of the fact that scholars like M. M. Sudhākara Dwivedi pronounced that it was an extra-polated work shortly after the time of Brahma-Guptāchārya. It is interesting to note that Bhaṭṭācarya, a very rational astronomer, had before him the verse “सिध्दांतव्रो युगेमातां चं चं द्राक्षु परिक्रमये” of the
Sūrya Siddhānta (verse 9 ch. 3). He did not give it the interpretation that was later put upon it through the two subsequent lines “तद्वृत्र चित्रम्” etc., which lines were not there evidently in Bhāskara’s time. Without these two latter lines the rate of precession was too small to be accepted by Bhāskara and so he chose to follow Munjāla rather than the Sūrya Siddhānta. Our Traditional astronomers today have no reservation to accept the greatness of Bhāskara and worship him though they do not question what necessity Bhāskara had to write another treatise and that too basing it upon the Āgama accepted by Brahmagupta and not Sūrya Siddhānta, when there existed Sūrya Siddhānta before him and from which he had no objection to quote verses like “अक्षरश्रू पां कालस्य मून्यः” etc. (verse I ch. 2.)

The Bija-correction first incorporated by Brahmagupta and later followed by a good number of astronomers because Sripati and Bhāskara accepted it, will not be acceptable to modern astronomers, though it might have worked well at the time of Brahmagupta and for some years later. The reason is that it is construed only as a tinkering of the defect as explained before.

It is also to be noted that the originator of this Bija-correction did not make it secular i.e. valid for all time increasing without a limit, for, then, the respective corrections transcend all limits and render the corrections meaningless. So, he said that the corrections would be increasing for 6000 years and thereafter begin to decrease to nothing. They were Zero at the beginning of the Kali because all the yugas are multiples of 12000 years. Also the maximum correction is in the case of Mercury $\frac{3000}{2} \times 52 = 1560' = 26^\circ$. Let us see how far this is justifiable. The daily mean motion of Mercury as given by Bhāskara is $4^\circ - 5' - 32'' - 18'' - 25''$ whereas as per modern astronomy it is $4^\circ - 5' - 37\frac{5}{7}''$ approximately. So there is a positive error of $5\frac{5}{28}''$ which will accrue to $18' - 35''$ in
course of 200 years. But as per the Bija-correction it should be 52°. Hence it is a fact that there is a positive error but not so much as indicated. But it must be noted that Mercury's orbit has the highest eccentricity of as much as .2, and the observer who stipulated the correction must have observed when Mercury was near its perihelion, where the error could have been as much as indicated and even more. Similarly on close analysis it could be proved that the Bija-correction should have been as indicated, say, roughly about 3300 Kali era, which might be roughly the date of its stipulation.

Verses 9, 10. Concluding verses of the Madhyadhikāra.

If the work is made more voluminous by describing various methods which are easy and interesting to unintelligent people, learned men look down upon such a work as indulging in unnecessary verbosity. Hence the volume of a work does not add to its greatness; So I have made my work neither voluminous nor brief-worded. The reason is that both the intelligent as well as the unintelligent people are to be enlightened.

For the sake of clarity of exposition, different ingenious methods being used in such a way that the work does not exceed the normal limits of the previous works, and in incorporating as far as possible unit numerators, fractions having numerators and denominators mutually prime, using methods of interpolation and reduction, making use of different kinds of denominators and numerators in many ways, this kind of treatment must be given to a work of this nature by an intelligent man.

Comm. Easy.

Before we proceed to the next chapter, we shall add here tables of astronomical constants as given by different authorities, which will help comparison and appreciation of the work.
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SPAŚTĀDHIKĀRA — RECTIFICATION OF PLANETS

Introduction. In the Bhagānādhyāya section of the previous chapter Bhāskara gave under the Caption Bhaga-nopapatti his proofs as to how the ancient is might have obtained the number of sidereal revolutions of the planets and the planetary points called apogees or aphelia and Nodes. But in trying to give those proofs, he was aware and he confessed also in so many words that some of his proofs at least were obsessed by what is called Itarēśa-rāṣṭraya-Doṣa i.e. "answer begging the question". It is worth hearing his words in his commentary under verses 1-6 of the section cited above—"That the planets, and the planetary points perform so many revolutions in a Kalpa, is essentially conveyed by the Āgama i.e. the Sāstra (which is to be taken on faith). That Āgama, got diversified i.e. there are many versions of that Science, due to the defects of scribes, the teachers and the students and due to a long lapse of time from the originators of the Āgama. That being so, the question arises as to which of the versions is to be trusted as the right authority. If it be said so, in mathematics only an āgama which could be proved also should be taken as authority. Such a number of revolutions as are obtained by proof, is to be accepted. Even that could not be (a proof); for, a great scholar could just understand the proof and by that proof alone, it is not possible to know the exact number of revolutions (in a kalpa), for, a man's longevity is not much. In the proof that could possibly be given, the planet's position is to be observed and noted every day, during the entire course of its revolution. Thus Saturn Completes its sidereal revolution in about 30 years. The apogee of the Sun and the aphelia of the planets have their revolutions running into hundreds of years. Hence the observation of one complete revolution (of such a planetary point) is beyond the capa-
city of a mortal. Hence great astronomers accept such an āgama as would give results which accord with observations during their times, and such a one as was formerly accepted by a very intelligent astronomer. Then they produce their own works exhibiting their own Skill in the Science and refuting wrong notions of others. Their idea is 'Let the Āgama we take as an authority be whatever it would be. Let us show our own skill in the course of our work.' Just as in this work, the āgama accepted by Brahmagupta is taken on faith as the authority. Then it might be argued "Better not attempt at trying to prove how the numbers of sidereal revolutions were arrived at. Even if a proof be attempted, that proof would be obsessed by the 'Itarētarāśrayadōsa' (cited above). Nevertheless we shall give a brief proof, That 'itarētarāśrayadōsa' is apparently a dōsa i.e. an apparent defect; for, different proofs could not be adduced simultaneously. The proof will now be given".

These words indicate that even such a highly rational and supremely intelligent astronomer like Bhāskara could not set aside his faith in our āgama and attempt at a pure and rigorous proof, which would not invoke the āgama. Let us see where in his proof he does commit the so-called itarētarāśrayadōsa and where he invokes the āgama. Also we shall try to construct a proof, of course to a good extent on the lines on which Bhāskara tries to give his proof, but at the same we shall not invoke the āgama. Where he does, but try to proceed purely on a rational basis. We shall take up the proof under verse 18, in its appropriate context. We shall now proceed with the text up to that point, which gives a brief sketch of the Hindn trigonom-
Verse 1. In as much as true positions of the planets alone are required to decide auspicious moment; for journeys, marriages, celebrations pertaining to temples, astrology and the like, we shall now give the methods of rectifying the mean positions of the planets so as to accord with their observed positions.

Comm. Clear.

Verse 2-9. Obtaining the sines of the angles and tabulation of the sines.

The planet deflected to the true position from the mean lies at the end of a half-chord (which is the Hindu sine of an angle) so that many processes pertaining to a planet are carried through Sines of angles; hence the word half-chord alone is connoted in this work wherever the word Jyā meaning a chord is used.

The lengths of these half-chords (or the Hindu Sines) for angles increasing from 0° to 90° at intervals of $3\frac{1}{4}$° are as follows—225', 449', 671, 890, 1105, 1315, 1520, 1719, 1910, 2093, 2267, 2431, 2585, 2728, 2859, 2977, 3084, 3177, 3256, 3321, 3372, 3409, 3431, 3438. The ut-kramajyās or the Hindu versed-sines are respectively 7, 29, 66, 117, 182, 261, 354, 461, 579, 710, 853, 1007, 1171, 1345, 1528, 1719, 1918, 2123, 2333, 2548, 2767, 2989, 3213, 3438.

The word Tribhajyā or Trijyā is half-diameter. The word jyā khandas used by pandits connote the differences between successive sines.

Comm. There is a difference between modern trigonometrical sines and the Hindu sines as detailed below (Ref.
fig. 6 overleaf). Let (0) be a circle i.e. a circle with centre '0'. Let AB be an arc called 'Chāpa'; let BC be drawn perpendicular on 0A; then BC is half of the full chord BD (known as jyā). The half-chord Ardha-jyā is itself spoken of as jyā for convenience and is the Hindu-sine of the arc or chāpa AB. In Hindu trigonometry 'angle' is connoted by the arc corresponding to it and as such spoken of as chāpa. OC is spoken of as the Hindu-cosine or Kotijyā and CA is called the ut-kramajya or the Versed-sine. The radius 0 B is called trijyā and let us connote it by R. To differentiate between the modern terms and the Hindu terms, we use the words H. Sine, H. Cosine, H. vers-sine for the Hindu sine, the Hindu cosine and the Hindu vers-sine respectively. Also the radius R is generally taken to be 3438′ which, we know to be the approximately the minutes in a radian. To talk of a length in minutes appears rather odd but no confusion need be there, for, an arc of length R subtends 3438′ at the centre. It is called Trijyā for the reason that it is the H. sine of 3 Rasis or 90°. A Rasi is equal to 30° because the ecliptic circle of 360° is divided into 12 Rasis Mesha, Vrishabha etc. meaning
Aries, Taurus etc. The names of the Rasis in Sanskrit and the modern English words we use for them have the same meaning, which raised a suspicion in the minds of many orientalists that the Hindu Astronomy drew upon the Greek. Many scholars of India assert that the Greeks derived this knowledge from the ancient Hindus; but we shall not enter into the controversy here. It may be noted also that the Sanskrit names of week-days have the same meaning as Sunday, Monday etc.

On the basis of taking trijya equal to 3438', the other H. sines or half-chords are also expressed in minutes. Generally twenty-four H. Sines are given in a quadrant and to obtain the H. sine of an angle intermediate, a formula for interpolation also is given. Also the method of calculating the H. sines for every degree is given, as we shall see shortly. In the table of 24 H. sines, the first is H. sine 3°-45' or H. sine 225' and this is approximately taken as 225' because in fig. 6 if \( \hat{A}OB = 3°-45' \), the H. sine BC will be almost equal to the arc AB. The H. Vers-sines are also given to get the corresponding H. Cosines easily, for, H. Vers sine 3°-45 = R — H. Cos 3°-45 = 7' means H. Cosine 3°-45' = 3431 = H. sine (90°-3\(\frac{3}{4}\)) = H. 22 where we use the notation H. r to mean the rth H. Sine.

Now we propose to give here some essential formulae used by the Hindu astronomers, as given in the golādhyāya by Bhāskara under the caption jyotpatti-krama. Incidentally it may be noted that H. sine \( \theta = R \) sine \( \theta \) where sine \( \theta \) is the modern sine of the angle \( \theta \). Similarly H. Cos \( \theta = R \) \( \cos \theta \) and H vers \( \theta = R \) vers \( \theta \). Thus when we have an equation of the type. \( \sin \delta = \sin \phi \cos z + \cos \phi \sin z \sin a \) in modern astronomy arising out of the famous spherical triangle PZS where P is the Celestial Pole, Z the Zenith and S the position of the Sun or a Star, the Corresponding Hindu formula would be \( R^2 H \) \( \sin \delta = RH \sin \phi \) \( H \cos Z + H \cos \phi \) \( H \sin Z \) \( H \sin a \). Occasionally the
radius is taken to be 120, and the corresponding $H$ sines are called Laghu-\textit{jiyās} or simpler \textit{H} sines used where great accuracy is not required. Sripati took the radius to be 3270 units in addition to 120 as did Brahmagupta. Munjala took 488' and some others some other values also. Out of these 3438' alone has a right significance (Vaitūswara took 3272)

Bhāskara says under verses 1 to 5 under Jyotpati-Vāsanā in the Golādhyāya that the Hindu astronomers got the values of the main $H$ sines of 30°, 45°, 60°, 18° and 36° by inscribing regular polygons in a circle. They are called the pancha-\textit{jiyākās} or the fundamental $H$ sines. From these the others were calculated according to the methods given by Bhaskara as follows.

To start with, we have the fundamental formula

$$H \sin^2 \theta + H \cos^2 \theta = R^2 \text{ (from fig-6)} \quad \text{I}$$

In addition to this formula, Bhāskara gives another formula (verse 10, 11 Ibid) $H \sin \frac{\theta}{2}$

$$\frac{\theta}{2} + H \text{ vers}^2 \theta =$$

In the commentary under the above verses, he has given the method by which II was obtained (Ref. fig. 7) $BM = H \sin \theta$ where $A0B = \theta$; also $AM = H \text{ vers } \theta$ and $AB^2 = AM^2 + MB^2$. Let $N$ be the mid-point of $AB$.

$$= AN = H \sin \frac{\theta}{2}$$

$$H \sin \frac{\theta}{2} = \frac{1}{2} \sqrt{AM^2} + M$$

$$= \frac{1}{2} \sqrt{H \sin^2 \theta + H \text{ vers}^2 \theta}$$

which proves the first part of II. Again from the right-angled triangle $ABC$, $AB^2 = AM \cdot AC = H \text{ vers } \theta \times 2R$

$$= \frac{1}{2} \sqrt{2R H \text{ vers } \theta} = \sqrt{1}$$

which proves the second part.
In the Commentary under verses 1—25 ibid, Bhāskara tells us how formulae I and II are used to construct the table of 24 H sines. To start with, the four H sines of 30°, 45°, 60° and 90° which may be denoted by the symbol $H_r$ where $r=8, 12, 16$ and 24, are known. Now using the formula II, $H_8$ is obtained from $H_5$, $H_2$ from $H_4$ and $H_6$ from $H_3$. Similarly from $H_{16}$, $H_8$ and $H_3$ are successively obtained. Now using formula I, $H_{10}$, $H_{22}$, $H_{23}$, $H_{18}$, $H_{21}$ are obtained respectively from $H_8$, $H_2$, $H_1$, $H_6$ and $H_3$. Now again from $H_{22}$, $H_{23}$, $H_{18}$, we obtain using formula II $H_{10}$, and $H_8$, $H_{12}$, $H_6$ respectively. Formula I gives again $H_{14}$, $H_{19}$, $H_{16}$, $H_{13}$ from the above. $H_{14}$ gives $H_7$ and $H_7$ gives $H_{17}$ using formula II and I respectively. Thus the table is Completed.

Then Bhāskara poses the problem as to how a table of the H sines could be computed when a quadrant is divided into 30 equal parts. He says that formula I and II do not suffice in this behalf and shows how they do not, as follows
in the same commentary cited above. To start with, the 
$H$ sines of $18^\circ$, $30^\circ$, $36^\circ$, $45^\circ$, $54^\circ$, $60^\circ$ are known. They are 
respectively $H_6$, $H_{10}$, $H_{12}$, $H_{15}$, $H_{18}$, $H_{20}$. Also $H_{30}$ i.e. $H \sin 90^\circ = R$ is also known. Formulae I and II will help us to 
derive,

from $H_6$, $H_8$ and from $H_3$, $H_{37}$; also from $H_6$, we derive $H$

From $H_{10}$, $H_5$ and from $H_5$, $H_{25}$ and again
from $H_{18}$, $H_9$ and from $H_9$, $H_{31}$ are derived.

The remaining $H$ sines sixteen in number cannot be got 
from either of the formulae.

To meet the situation Bhāskara gives other formulae 
of his own discovery as he says "पत्तक्षेपय विदिधिपमक्षात्" 
i.e. "I shall tell something more than this. These formulae 
gives in the verses 12 to 15. They are

\[
\sin \left(90^\circ \pm x\right) = \sqrt{R^2 + R \cdot H \cdot \sin x}
\]

\[
\sin x + H \sin y)^2 + (H \cos x - H \cos y)^2 = IV
\]

\[
\sqrt{\frac{(H \cos x - H \sin x)^2}{2}} = H \sin (45 - x) = V
\]

\[
R - 2 \frac{H \sin^2 x}{R} = H \sin (90 - 2x) = VI
\]

These formulae correspond to the modern formulae

\[
\sin \left(\frac{x - y}{2}\right) = \sqrt{\frac{(\sin x + \sin y)^2 + (\cos x - \cos y)^2}{2}}
\]

\[
\frac{(\cos x - \sin x)^2}{2} = \sin (45 - x)
\]

\[
1 - 2 \sin^2 x = \cos 2x
\]

respectively
These formulae imply a knowledge of the expansion of \( \sin (x \pm y) \) which is given in verses 21, 22 in the form

\[
H \sin (x \pm y) = \frac{H \sin x \cdot H \cos y \pm H \cos x \cdot H \sin y}{R}
\]

The formula \( H \cos (x \pm y) \) is got from VII by putting \( 90 - (x \pm y) \) for \( x \pm y \).

To construct the remaining sixteen \( H \) Sines Bhaskara directs us to use his formula IV wherein taking \( x = 27^\circ \), and \( y = 15^\circ \), we have \( H_2 \) which gives \( H_{23} \). From \( H_{23} \) we have \( H_{14}, H_7 \) and \( H_1 \) from \( H_2 \). Then \( H_{15}, H_{23} \) and \( H_{29} \) are got from \( H_{14}, H_7 \), and \( H_1 \) respectively. From \( H_{15} \) again we have \( H_8 \), and \( H_4 \) which in turn give give \( H_{23} \) and \( H_{26} \). \( H_{26} \) gives \( H_{13} \) which in turn gives \( H_{17} \). \( H_{22} \) similarly gives \( H_{11} \) which in turn gives \( H_{19} \). Thus the table is complete.

Verses 16 to 20. (Ibid.) give us the method of constructing the table of 90 \( H \) sines in a quadrant, through the formula \( H \sin (x + 1)^\circ = \)

This formula is got evidently by interpolating from his knowledge of \( H \sin 3^\circ \) and \( H \sin 3\frac{3}{4}^\circ \). He also gives that \( H \sin 5\frac{3}{4}^\circ \) is more correctly equal to 224', 51". In the table of 24 \( H \) Sines, he gives the formula

\[
H_{r+1} = Hr \left(1 - \frac{1}{497}\right) + H \cos x_r \times \frac{100}{1529}
\]

which could be similarly got by interpolation.

The determination of \( H \sin (A+B) \) from \( H \sin A \), \( H \sin B \), \( H \cos A \), \( H \cos B \) is called Samāsa-Bhaṅvanā and that of \( H \sin (A-B) \) is called Āntara-Bhaṅvanā, whereas computation of \( H \sin 2A \) from \( H \sin A \) and \( H \cos A \) is called Tulyabhāvanā. The word Vajrābhīyāsa is used for 'Cross-multiplication' in this context.

We shall now prove how the formula for \( H \sin (x+1)^\circ \) led Bhāskara to arrive at the differential formula.
\[ \delta (\sin x) = \cos x \delta x. \]
Since \( H \sin (x + 1)^\circ = H \sin x + \frac{H \cos x \times H \sin 1^\circ}{R} \) approximately, \( H \sin (x + 1)^\circ = \frac{H \cos x \times H \sin 1^\circ}{R} = H \cos x \times a \text{ constant.} \)

Hence Bhāskara could see that the variation in the function \( H \sin x \) is proportional to \( H \cos x \). Let it be now required to find the increment in \( H \sin x \) for an increment \( \delta x \) in \( x \) where \( \delta x < 60' \). Let \( H \sin (x + 1)^\circ - H \sin x = \frac{60'}{R} \times H \cos x = y \) where \( y \) is called the Bhogya-Khanda.

Then Bhāskara argues "If for an increment of \( 60' \), there is an increment of \( y \), what shall we have for \( \delta x \)?". The answer is \( \frac{y \delta x}{60} = \frac{60 \times H \cos x \times \delta x}{R} \times \frac{H \cos x \times \delta x}{60} = \frac{H \cos x \times \delta x}{R} \).

Hence \( H \sin (x + \delta x) - H \sin x = \delta (H \sin x) = H \cos x \times \frac{\delta x}{R} \) which corresponds to \( \delta (\sin x) = \cos x \delta x. \)

Bhāskara is thus the first mathematician to have perceived this differential formula 500 years before Newton and Leibnitz.

In the context of the preparation of the table of 24 \( H \) Sines which was there in Aryabhatiya as well as Sūrya-siddhānta, we have to offer the following remarks. The method of the construction of this table was the subject-matter of some study by S. N. Naraharayya and A. A. Krishnaswami Ayyangar. In this study it was supposed that the method was based on finite differences according to the verses of the Sūrya-siddhānta 15 and 16 of chapter II. The articles referred to reveal the difficulty in constructing the sine-table following this method. Some subsidiary corrections in the verse "पक्षविगाच्य विकाला".

from the Brahma Siddhānta by Ranganātha in his commentary of Sūryasiddhānta were alluded to in the articles cited but no satisfactory mathematical explanations were given by them. We shall give hereunder a satisfactory explanation of the matter discussed in the articles.

In the first place it may be noted that in the table of those 24 $H$ sines, the sixteenth as given by Bhāskara namely 2977 is more correct than that given in the Sūryasiddhānta namely 2978 (Lakshmi Venkateswara press edition 1955 Bombay).

In the course of the Commentary under the verses 15, 16 of the Sūryasiddhānta, Ranganātha gives the hint which must have been at the back of the mind of the author of the Sūryasidhhānta when he gave the rule to construct the table cited. Just as $\delta \left(\frac{H \sin \theta}{R}\right) = \frac{H \cos \theta \delta \theta}{R}$, Similarly the formula $\delta \left(\frac{H \cos \theta}{R}\right) = -\frac{H \sin \theta \delta \theta}{R}$ must have been known to the author. The negative sign means that the successive differences of the $H$ sines namely 225, 224, 222, 219 etc. are decreasing and also that the successive differences of these differences are increasing according to the $H$ sine. Just as Bhāskara could see that the $H$ sines were increasing and the successive differences of the $H$ sines were in Kotijyānupāta i.e. in direct ratio to the $H$ Cosine at their respective place, similarly, the author of the Sūryasiddhānta could see that the second differences cited above were in Kramajyānupāta as hinted by Ranganātha.

From the formula $\delta \left(\frac{H \cos \theta \delta \theta}{R}\right) = -\frac{H \sin \theta \delta \theta}{R^2}$ putting $\theta = 90^\circ$, we have the second difference numerically
\[ \frac{\times}{R} = 14' - 48'' - 30'' \]

Ranganâtha made a mistake in taking this to be
\[ = 15' - 16'' - 48'' \]

and a more correct value of the second difference would be
14' - 47'' approximately. Ranganâtha then argues that
taking this second difference to be 15 for the \( H \) sine 3438
'what will it be for the \( H \) sine 225' ?' The answer would
be \[ \frac{15 \times 225}{3438} = \frac{15 \times 25}{382} = \frac{375}{382} = 1' \]

So, the second difference in the beginning of the table
happens to be 1' ie \( \frac{225}{225} \). This led the author of the Surya
siddhânta to use the words "तद्विश्लेषलयोऽन". This being
an approximate formulation, naturally necessitated a second
formulation where the approximation led to an error of 1'
through the verse "एकविश्लेषच्य विश्लेषच्य etc." This second
formulation intended to make a correction, was done in
the wake of a correct calculation through the formulae I
& II which were known even prior to Bhaskara.

**Verses 10, 11.** To find the \( H \) sine of an intermediate
angle. Suppose it is required to find the \( H \) sine of an
angle \( \theta \) is \( \theta \times 60' \). Divide this by 225; the quotient
gives the previous \( H \) sine. Then \( \frac{R \times D}{225} \) where \( R \) is the
remainder, and \( D \) the difference between the previous and
next \( H \) sines, added to the previous \( H \) sine gives the \( H \) sine
required.

**Comm.** The formula is evidently based on an application
of rule of three.

**Verse 11.** To find the angle when the \( H \) sine is given
Suppose the \( H \) sine of an angle is given to be \( x' \). Subtract
the greatest \( H \) sine that could be subtracted from this.
Suppose the H sine of \( \theta^\circ \) could be subtracted. Let the remainder be \( r \). Then \( \frac{r \times 225}{D} \) where \( D \) is the difference between the previous and next H sines, added to \( \theta \) gives the angle corresponding to \( x' \).

*Comm.* Evidently this is the converse of the previous process and this also is based on Rule of three.'

*Verses 12—15.* The H sine of the obliquity of the ecliptic taken to be 24° is 1397. Now, the successive differences of the H sines will be given (on the basis of taking \( R = 120 \)) which are known as Laghu-Jyās intended for ease in Computations, namely 21, 20, 19, 17, 15, 12, 9, 5, 2. These are given for intervals of 10°, so that if it be required to find the H sine of \( x^\circ \), let \( q \) be the quotient and \( r \) the remainder when \( x \) is divided by 10. \( q \) gives the number of the previous H sine. Then \( \frac{r \times D}{10} \) where \( D \) is the next difference or jyākhandas as it is called, added to the previous H sine gives the required H sine. In this table the H sine of 24° is 48° — 45°. Also the H versines in this table are got by the reverse differences. To get the angle \( \theta^\circ \) for a given H sine say \( x' \) subtract the sum of as many differences (Jyā—Khandas) as could be from \( x \). Let the remainder be \( r \). Then \( \frac{r \times 10}{D} \) where \( D \) is the next jyā—Khanda added to the previous angle upto which the jyākhandas have been subtracted, gives the required angle. The H sine will be more accurate if the Bhōgya-Khanda or the next H sine—difference is rectified (as per the rule of interpolation next given).

*Comm.* H vers \( \theta = R - H \cos \theta = R - H \sin (90 - \theta) \) so that H verse 34° = 3433 — H sin (86°) = 3433 — 3431 = 7 as given in the previous table. Similarly in the above table of Laghu-Jyās, H vers 10° = R — H cos 10° = 120 — H sin 80° = 120 — (21 + 20 + 19 + ... + 5) = 2 so that the
above differences in the reverse order give the $H$ versines. The rest of the contents of the verses is simple, the processes being based on the 'Rule of three'.

**Verse 16.** Rectification of the next $H$ sine difference known as Bhogya—Khanda.

The difference of the previous and the following $H$ sine—differences being multiplied by the remaining degrees and divided by 20, the result is subtracted from the arithmetic mean of the previous and following $H$ sine—differences to give the rectified $H$ sine—difference, in question.

**Comm.** This is a formula for interpolation which agrees with the interpolation formula given by Ball in his spherical astronomy on page 18 in the form $y = y_0 + \frac{x}{h} \left( y_1 - y_0 \right) + \frac{x(x - h)}{2h^2} \left( y_2 - 2y_1 + y_0 \right)$. This formula is a re-statement of the formula enunciated by Brahmagupta in his work Brahma Sphuta Siddhanta as well as Ustara-Khandakhadya in the form "गदाकोयकुण्डकार्तन्त्रिकविभिन्नभागः प्रति \ शोभतेन शवते, तद्यथा दशे भोज्यद्वानिक भोज्यम्" wherein in the place of ten-degree-interval, a fifteen-degree-interval ie 900°-interval was taken. Rule of three is a linear formula of interpolation, whereas the above is a quadratic formula reflecting much credit on the mathematical genius of Brahmagupta.

We shall now see how the formula is applied and what mathematical significance it has. Suppose it is required to find the $H$ sine of $24^\circ$ from the previous table of $H$ sine—differences given for intervals of $10^\circ$ — from the table $H$ sin $10^\circ = 21$, $H$ sine $20 = 41$, $H$ sine $30 = 60$ where $R$ is taken to be 120. Now to find $H$ sin $24^\circ$, we are asked to rectify the next $H$ sine—difference namely $19'$, where the table is 21, 20, 19 etc. As a first approximation, applying rule of three $H$ sin $24^\circ = 41 + \frac{1}{10} \times 19 = 43'6 = 43' - 36''$. 


This is a crude approximation, the actual value being 48' — 48" — 14"'. Application of rule of three is justified if the H sine—differences are uniform, but they are not so, being in a decreasing order. So, the following H sine—difference namely 19' is to be rectified so as to be applicable at 24°. In other words we have to take such a H sine—difference which will hold good at 24°, not at 20° or 30° — from 10° to 20°, the H. S. d. (H Sine—difference) is 20', and from 20° to 30° it is 19'. If that be so what will be exactly at 24°? If should be less than 20' and greater than 19'. Now the argument advanced by Bhāskara is that the H Sine—difference at the mid-point of the 2nd and 3rd differences namely 20 and 19 should be \( \frac{20 + 19}{2} = 19'5 \).

The H sine—difference at the end of the third interval is 19. Then by the rule of three ‘If there is a decrease of 19'5 — 19 = 5, during the course of the 10° of the third interval, what should the difference be for 4°? (where we have to find the H sine at 24°) The result is \( \frac{4}{10} \times 5 = \frac{4 \times 1}{20} \) which is given by the words “यात्राध्व: खण्डकयोविशेषः शेषवाविश्वो नाखदन्”. This decrease makes the H sine—difference at 24°, 19'5 — \( \frac{4}{10} \times 19'3 = 19'3 \) at 24° —Now the argument to find the H sine at 24° is ‘If for an interval of 10°, the H sine—difference is 19'3, what should it be for 4°? ’ The answer is \( \frac{4}{10} \times 19'3 = 7'72 \). Hence the H sine of 24° is 21' + 20' + 7'72 = 48'72 = 48' — 43' — 12" which is nearer the truth 48' — 48" — 14" than what was obtained by the crude rule of three namely 48' — 36".

Here we have to explain Bhaskara's words more elaborately, because, Kamalakara happened to criticise Bhaskara's words in this context. Bhaskara Says "The H sine difference at the end of an interval is the arithmetic mean of the preceding and succeeding differences, whereas the succeeding one is that which holds good at end of the succeeding. In between, we have to apply the rule of three to
obtain the rectified difference." What Bhāskara means is this. At the end of an interval, to obtain the H sine, it is enough to add the H sine difference belonging to that interval to the preceding differences. But when it is required to find the H sine in the interior of an interval, we have to construe that the difference at the end of the previous interval is the arithmetical mean of the previous and succeeding differences. There is apparently a self-contradiction in Bhāskara’s words; for, at the end of the interval, according to his own words, the difference is that belonging to the previous interval and not the arithmetical mean as postulated. The contradiction will not be there when we read Bhāskara’s mind that he means “When we require to find the H sine in the interior of an interval only, the difference at the beginning of that interval is to be taken as the arithmetical mean of the previous and the current differences, and that at the end of the interval the current difference holds good.”

The truth of Bhāskara’s statement could be seen analytically as follows. The arithmetical mean of the preceding and succeeding H sine differences is (The context is to rectify the third H sine difference namely 19, for, we were finding the H sine of 22°) \( \frac{AB + BC}{2} \) (Ref fig. 8) where OA, AB, BC etc are the successive differences.

\[
AB + BC = H \sin 20 - H \sin 10 + H \sin 30 - H S 20
\]

\[
= \frac{H \sin 30 - H S 10}{2} = R \left( \frac{\sin 30 - S 10}{2} \right)
\]

\[
= R \times \cos 20 \sin 10 = \frac{H \cos 20 H \sin 10}{R}
\]

The numerical difference of the preceding and succeeding H sine differences is (i.e. यलैॊवकवनाॊ००). \( AB - BC = (H \sin 20 - H S 10) - (H \sin 30 - H \sin 20) = \)
\[
\frac{2H \cos 15 \sin 5}{R} - \frac{2H \cos 25 \sin 5}{R} = \frac{2H \sin 5}{R}
\]

\[(H \cos 15 - H \cos 25) = \frac{2H \sin 5}{R} \times \frac{2H \sin 20 \sin 5}{R}\]

Let now \(x\) be the point where we are to find the \(H\) sine (Here let us take it as \(x^o\) after the previous interval for generalisation). Then Bhaskara’s formula would give

\[
\frac{H \cos 20 \sin 10}{R} - \frac{2H \sin 5}{R^2} \times \frac{2H \sin 20 \sin 5}{20} \times x
\]

\[
= \frac{H \cos 20 \sin 10}{R} - \frac{x}{10 R^2} \times 2H \sin 20 \sin^2 5
\]

\[
= \frac{2H \cos 20 \sin 5 \cos 5}{R^2} - \frac{1}{R^2} \times \frac{x}{10} \times 2H \sin 20 \sin 5
\]

\[
= \frac{2H \sin 5}{R} \left\{ \frac{H \cos 20 \cos 5 - \frac{x}{10} \times H \sin 20 \sin 5}{R} \right\}
\]

Put now successively \(x = 0^o\) and \(10^o\) to get the rectified differences at \(B\) and \(C\) respectively; then those rectified differences would be respectively \(H \cos 20 \sin 10\) and \(\frac{2H \sin 5}{R} \times H \cos 25\). But \(\frac{AB + BC}{2} = \frac{H \cos 20 \sin 10}{R}\) (found above) and \(BC = H \sin 30 - H \sin 20 = 2 \frac{H \cos 25 \sin 5}{R}\) In other words the rectified differences at \(B\) and \(C\) are respectively what exactly has been stated by Bhaskara. Hence Kamalakara’s condemnation of Bhaskara is quite unjustified.

**Verse 17.** To rectify the arcual difference to obtain the arc for a given \(H\) sine.

Subtract as many \(H\) sine-differences as could be subtracted from the given \(H\)-sine. Half of the remainder multiplied by the difference of the preceding and succeeding \(H\) sine-differences and divided by the succeeding and
the result being subtracted from or added to as the case may be (added in the case of $H$ versines) the arithmetic mean of the preceding and succeeding $H$ sine-differences gives the rectified $H$ sine-difference while finding the arc for a given $H$ sine.

*Comm.* Let the given $H$ sine be that of $24^\circ$ found before i.e. $43.72$. We could subtract 21 and 20 from this and the remainder is $7.72$; half of this is $3.86$ which multiplied by $(20-19)$ is $3.86$. This divided by the succeeding difference namely 19 is $'2$ approximately. The arithmetic mean of the preceding and succeeding $\frac{20+19}{2} = 19.5$. If the above result is subtracted from this, we have $19.5 - '2 = 19.3$. If for $19.3$ we have $10^\circ$ increment what shall we have for $7.72$. The answer is $\frac{10 \times 7.72}{19.3} = 10 \times '4 = 4^\circ$. Hence the required arc is $20^\circ + 4^\circ = 24^\circ$.

The proof is analogous to the previous proof. Having subtracted 21 and 20, the remainder is $7.72$, (Ref. fig. 8)

![Diagram](image)

so that $BX = 7.72$. Now during the course of the succeeding interval of 19, there has been a decrease of $19.5 - 19 = '5$ or to put it in general terms, during the course of the succeeding interval $BC$ there has been a decrease
\[
\frac{x+y}{2} - y = \frac{x-y}{2}
\]
where \(x\) is the previous interval \(AB\).

\[\frac{x+y}{2}\]
is the \(H\) sine difference at \(B\). Hence the argument is "If for \(y\), there has been a decrease of \(\frac{x-y}{2}\), what will it be for \(D\) (Here \(D = 7.72\))?" The answer is

\[
D \left(\frac{x-y}{2}\right) \times \frac{1}{y} = \frac{D}{2} (x-y) \times \frac{1}{y} = \text{वेषतोष्णनाथम्} \times \text{गते}
\]

This result is to be subtracted from \(2\) i.e. \(2\) is to be subtracted from \(19.5\) to give the rectified \(H\) sine-difference.

**Verse 18.** Definition of \(K\)endra and assignment of sign thereto.

The excess of the longitude of the mean place over that of the apogee or aphelion as the case may be is called the mean anomaly. The excess of the longitude of the point called \(S\)ighroccha over that of the planet rectified by the first equation known as \(M\)anda-phala or equation of centre is known as the \(S\)ighra-anomaly. The equation of centre is positive or negative according as \(180 < m < 360\) or \(0 < m < 180\) where \(m\) is the mean anomaly. The case will be reverse in the case of the sign of \(S\)ighraphala, the second equation.

**Comm.** "चन्द्रसूया स्रष्टी स्यातं मान्द्रेनैकेन कर्मण।" i.e. The Moon and the Sun could be rectified by the equation of centre alone. This means that the Moon revolving round the Earth directly and that the Sun revolving relatively round the Earth are subject to only one correction namely the equation of centre for rectification. In the case of the Sun, though the fact is that he is revolving round the Earth relatively, assuming as Hindu astronomy does that he is revolving directly round the Earth and him to the correction of the equation of centre...
does not alter the mathematics that goes into his rectification. According to modern astronomy the Sun and the Moon, one relatively and the other directly go round the Earth, in ellipses, the Earth being in one focus whereas in Hindu astronomy both the Sun and the Moon are taken to be going round the Earth in eccentric circles i.e. circles whose centres do not coincide with the centre of the Earth. In fact, Bhāskara says in so many words “सुममेच्छ खलु मध्यस्याविषय मध्य यतं स्यात्, यथ्यत नुवले भावति लभरे नाहस्य मध्य कुमचे, भृस्थो द्रष्टा न हि मध्यस्य मध्यतुल्यां प्रपश्यत्, तस्मात् ततूः। कितत इहत्तू दोःथमम्खेरे i.e. The centre of the celestial sphere coincides with the centre of the Earth. The centre of the circle in which a planet goes does not coincide with the centre of the Earth. Hence an observer on the surface of the Earth finds the True planet’s position differing from that of the mean planet, so that what is called the correction of Bhujaphala is to be made in the mean position of the planet to get the True position.” Here it has to be noted that the Bhujaphala mentioned stands both for the equation of centre and the second equation known as Śīghrapahala as well. One may wonder how it could be so, but it may be noted that in the formulation of both the equations, the centre of the eccentric does not coincide with that of the Earth and also in both the cases the equation contains the term $H$ sine of the anomaly where the word anomaly whether it be of the first or second equation is known as ‘Bhuja’.

In the case of the other planets, the fact is that they go round the Sun in elliptic orbits, the Sun being in one focus. In Hindu Astronomy, we shall see that the centre of the eccentric circle in which these planets are taken to revolve coincides with the Sun. Hence, though the ancient Hindu Astronomy postulates geocentric motion, the mathematics that goes into the formulation of the second equation, makes the Sun’s centre the centre of planetary revolution. One may wonder again how the equation of centre
formulated by Hindu Astronomy agrees with its formulation in Modern Astronomy which enunciates elliptic motion; but it will be seen that the eccentric-circle theory also gives very approximately the same formula for the Equation of centre. There is just one point of difference, which does not matter. Whereas in Modern Astronomy the mean anomaly is reckoned from the perigee or perihelion as the case may be, it is reckoned in Hindu Astronomy from the apogee or aphelion. The difference is made up by prefixing the appropriate sign to the equation. The word Mandoccha stands for the apogee in the case of the Sun and the Moon and for the aphelion in the case of the other planets and the word Manda kendra stands for the mean anomaly in both the cases. In Hindu Astronomy the word 'graha' stands for not only the five planets Mercury to Saturn but also for the Sun and the Moon. Why that word is applied to the Sun and the Moon as well is, that both the Sun and the Moon also while moving among the stars along with the five other planets Mercury to Saturn, wield an influence on the residents of the Earth. The etymology of the word 'graha' is ग्रहणातीति वा ग्रहते अनेनेति वा प्रहः; ie. 'that which seizes upon the fates of the residents of the Earth', with this etymological significance only, even the lunar orbital nodes known as Rāhu and Ketu are also taken to be grahas in Hindu Astronomy. Hence translating the word graha as a planet and criticising Hindu Astronomy for taking the Sun, Moon and the lunar orbital nodes also as such is not right. In other words the translation should be pronounced wrong. Uranus, Neptunne and pluto were not mentioned in Hindu Astronomy.

We shall now elucidate the eccentric and epicyclic theories of Hindu Astronomy, which will be seen to give identical position to the planets. How they came to be postulated will be also elucidated. Incidentally we deal with the 'Bhagaṇopapatti' or the proof of the numbers of sideral revolutions of the various grahas enunciated in the
beginning of the Madhyādhikāra, Bhaganādhyaya in verses 1 to 6. Even Bhāskara gave such a proof as appealed to Āgama ie. ‘Authority’, which proof therefore will not be acceptable to a student of Modern Astronomy, who is likely to question how the Āgama came into existence.

How the Āgama came to formulate the number of sidereal revolutions of the grahas, we shall now see.

The forefathers of Hindu Astronomy (have been reported to be eighteen in number in the famous verses "सूत्रः पिनामहो व्यासो वसिष्ठोपनिः पराऱः कस्यो नारदो गमो मरीचिमनु- रक्षितः; कोमशः पौलिपाचैव व्यवसो भवनो भुः; शानकोव्यश्चादनेतस्युः व्यवित्तश्चास्त्रवैकः") Of these Brahmagupta mentions Brahma-Siddhānta, which he reports to have resuscitated. Varāha Mihiira gave a version of the old Sūryasiddhānta, mentioning that it accorded with observations. Aryabhata says that he revived his system from the then existing ocean of knowledge both good and spurious. The fact that none of these outstanding astronomers mentioned that they had derived their systems from a foreign source, and the reasonableness in presuming that all these three could not be impostors, make the author of this work feel strongly that there should have been some works in the name of Āgamas extant long before these Achāryās. The argument that the crude Vedāṅga-jyotisa alone should have existed before Āryabhata, simply because, no other work worth the name has been discovered, may not be correct. It is quite possible that crude works could exist side by side with advanced scientific works, just as even nowadays we have thinkers and works of a primitive type existing along with highly advanced thinkers as well as scientific works).

Bhaganāpapatti. In the first place, the forefathers of Hindu Astronomy must have noticed very easily that the Moon has a motion among stars, for, this could be detected
even by a lay man during the course of a single night. So, the period of a single sidereal revolution, could be roughly recognized by noticing the conjunction of the Moon with a luminous star. Having thus observed a good number of sidereal revolutions, which could be done even with the naked eye, the average period could be arrived at with sufficient accuracy within the course of a few years. Having thus obtained almost accurately the average of a sidereal revolution of the Moon, the sidereal revolution of the Sun could have been arrived at as follows. The moment of an eclipse solar or lunar could be observed with the naked eye. Observing a good number of eclipses within the course of a few years, the average of a lunation could be easily arrived at very accurately, for, in between the eclipses of the same nature an integral number of lunations elapse. That the Sun also has a motion among stars must have been noticed clearly during the course of a few months, for, observing at Sunset the star that was rising, it should have been noticed gradually even during the course of a month, that the Sun must have been approaching the star or vice versa and as the stars were found to keep the distances amongst them constant, it was the Sun that was approaching the star and not the star it was that was approaching the Sun. Having decided thus that the Sun was moving among stars from west to East, the approximate period that the Sun took to complete a sidereal revolution was arrived at. Then, as both the Sun and the Moon were having eastward motion, and as the Moon has a more rapid motion, the arc by which the Moon overtakes the Sun during a day was roughly noticed. Thus arriving at a rough estimate of a lunation within which a conjunction of the Moon with the Sun recurs, it was noticed that the excess of the sidereal revolutions of the Moon over those of the Sun gave the number of lunations. Since a correct estimate of both a sidereal revolution of the Moon as well as that of a lunation were previously arrived at, the number of sidereal revolutions of the Sun during
the course of a certain period were computed whereby a correct estimate of a sidereal solar year was arrived at. This period could also be checked simultaneously by observing the interval between the heliacal risings or settings of a particular star of the Zodiac as well. Thus far, we have seen how the sidereal periods of the Sun and the Moon were determined very accurately. It may be noted that these periods as determined by the Hindu astronomers were correct to a good number of decimal places.

When once the Sun and the Moon were found to be having eastward motion among stars, and when it was discovered that there were other luminous bodies like the Jupiter and Venus etc. moving among stars, it was attempted to determine their sidereal periods. It must have been done as follows. In the first place, it was noticed that these other luminous bodies which were five in number, namely Mars, Mercury, Jupiter, Venus and Saturn, were found to be having retrograde motion also unlike the Sun and the Moon. As these five bodies were looking like stars they were named Tārā-grahas i.e. grahas looking like stars. Also a distinction could be drawn very easily between Mercury and Venus on the one hand and the other three on the other, for, the former were always found oscillating about the Sun, never parting from him through long distances. Thus during the course of a sufficiently long interval, the geocentric sidereal periods of Mercury and Venus coincided with that of the Sun. In other words, it was taken that the geocentric sidereal periods of Mercury and Venus also were taken to be an year. This is clear from the statements made by all the Siddhāntas that in a Kalpa of 4320000000 years the Sun, the Mercury and Venus all the three make 4320000000 sidereal revolutions. Then with respect to the other three planets Mars, Jupiter and Saturn, it was noticed that they were having a pre-dominantly longer period of direct motion, though there was a retrograde motion for some time. This
gave the clue to arrive at an approximate estimate of their sidereal revolutions. But the correct estimates were arrived at not by observing their conjunctions with stars, for, that would take a very long period of observation in the case of Saturn, but, by observing a good number of their heliacal risings or settings. The interval between two consecutive heliacal risings or settings being a little greater than an year, ten or fifteen observations could be done very easily by a single person. Then the aforesaid argument given in the case of finding the sidereal revolution of the Sun, was also advanced in the case of these three planets Mars, Jupiter and Saturn. Let $x^\circ$ be the arc that the planet covers during the course of a day. Let $a^\circ$ be the arc covered by the Sun during the same period which was previously known and that correctly. So, during the course of a day the Sun overtakes the planet by $(a - x)^\circ$. Hence to overtake $360^\circ$, the period $S$ was computed. This period was observed as the interval between two consecutive heliacal risings or settings and known as the synodic period. So, from the equation $\frac{360}{a - x} = S$, the value of $x$ could be arrived at, wherefrom $p$ the sidereal period was determined. This sidereal period could be also determined in another way. Noting the distance covered among stars by a planet during the course of a synodic period, using rule of three, the sidereal period could also be arrived at with a good accuracy, for, the retrograde motion affects equally each synodic period. An average of such determinations made in two ways could give the sidereal periods of Mars, Jupiter and Saturn with a good amount of accuracy. It will be noted here, that the average of a good number of geocentric sidereal periods in the case of these planets (called Superior) is also the heliocentric sidereal period (for a proof of this statement reference may be made to page 80 of the author's 'A critical study of the ancient Hindu astronomy, published by the Karnataka University Dharwar). It is why the sidereal periods of these planets
as given by Hindu astronomy tally with the heliocentric sidereal periods given by modern astronomy. This is also one of the reasons why heliocentric motion of the planets could not be detected by Hindu astronomy, and also why a statement was made that "In the case of Mercury and Venus the Sun was the planet, and they are termed as Sīgrōchas, whereas in the case of Mars, Jupiter and Saturn, they are the planets while the Sun plays the part of Sīgrōcha "कुज्जीवशस्त्रोणं तु रवि: शीघ्रोचचनामक्र:, बसुक्कयो: ती शीघ्रनामको" (An elucidation of this statement will be given shortly)

In the case of the planets Mercury and Venus (Inferior planets) one may wonder how under the geocentric theory, their heliocentric periods could be arrived at, though they were not recognized as such but were pronounced as the "geocentric periods (not considered as heliocentric)" of two points known as their Sīgrōchas. Here we come across the peculiar concept of a Sīgrōcha which arose out of the fact of postulating a geocentric system in the place of the heliocentric. This concept is to be elaborated, in as much as confusion is there in the minds of many interpreters of Hindu astronomy in this behalf.

In the first place let us consider as to how the rectification of the Sun and the Moon, known as sphutikaraṇa was achieved. Having got their sidereal periods, their mean daily motions were calculated. Also a period was conceived, during which this Sun and the Moon would perform an integral number of revolutions. This period was termed as a Mahāyuga (or simply a yuga as we hereafter name it) whose duration was estimated as 4320000 solar years. That the yuga is an integral L,C,M, so to say of the sidereal periods of the Sun and the Moon (also of the other planets as we shall see shortly) could be seen from the statement of the Surya Siddhānta युगे सूर्यशुक्लाणि
"In a yuga, the Sun the Mercury and Venus perform 432000 sidereal revolutions as well as the Śīgrócchas of Mars, Jupiter and Saturn, whereas the Moon performs 57753336 revolutions" (It may be recalled here that the Mercury and Venus are oscillating about the mean position of the Sun; also it will be noticed that the Sun playing the part of the Śīgrócchas in the case of Mars, Jupiter and Saturn, their Śīgrócchas are also deemed as making the same number of revolutions as the Sun. In as much as the Śīgrócchas in the case of Mercury and Venus are looked upon as different from the planets, so in the case of Mars, Jupiter and Saturn also the Śīgrócchas are taken as different Divine entities though coinciding with the Sun in position). However smaller periods could be conceived as integral L.C.M.'s of the sidereal revolutions of the Sun and the Moon, but a presumption sponsored by a sense of orderliness in the Cosmos, that the planets should all have been started from the Zero point of the Zodiac, made the integral L.C.M. to be of such a dimension as 4320000 solar years in which period the other planets also would have made an integral number of sidereal revolutions. Here in this point the traditional Hindu astronomers place their faith in the Āgama, which said that the planets were all started at the Zero-point of the Zodiac in the beginning of the yuga and were ordained to return to the same point at the close of the yuga. Even a rational astronomer like Bhāskara, apparently placing faith in the Āgama, while adducing a proof in the name of Bhagaṇopapatti, states that after obtaining the mean daily motions of the planets, calculates them for the period of a Kalpa taking it on trust that the planets were started at the initial point of the Zodiac in the begin-
ning of the Kalpa. A modern astronomer, however, ques-
tions the assumption that the planets were all started at the
first point of the Zodiac, and even though they might all
have been in conjunction at that point in some remote
past, whether it was the initial point of the reported Kalpa.
Proceeding on the basis of the reported initial conjunction
of all the planets at the first point of the Zodiac, and calcul-
ating the number of days that have elapsed from the begin-
ning of the yuga, the mean positions of the Sun and the
Moon were computed. Noticing that these mean positions
did not exactly accord with the true observed positions, the
ancient astronomers tabulated the differences between those
mean and true positions. These differences were found to
be zero at two diametrically opposite points, and maximum
roughly at two points differing by a quadrant from them.
To account for these differences, the thought that occured
to their minds was that probably the Sun and Moon did
not move in a circle whose centre coincided with that of
the Earth but were moving in an eccentric circle i.e. a
circle whose centre is at some other point than the Earth’s
centre. This surmise could be made because unequal
motion was accountable only on varying distance from the
Earth’s centre and a celestial body appearing to move
fastest must be nearest whereas the same appearing to
move slowest must be farthest. Thus in the first place
seeing no reason for non-circular motions and also expec-
ting the celestial bodies to move only in circles, for, a
circular motion appealed to them as the most ideal motion,
the ancient astronomers later postulated an eccentric
circular motion with respect to the Sun and the Moon.
This postulation appeared to give good results as seen
below.
Eccentric circle theory.—Let $E_1$ be the earth's centre; let $M_1PA_1$ be the circular orbit in which the planet (here the Sun or the Moon) moves with a uniform motion. This planet is termed the Madhyagraha or the mean planet. Let $M_2A_2P_2$ be the actual orbit of the planet whose centre $E_2$ is removed a little away from $E_1$. Since the centre $E_2$ is moved in a vertical direction away from $E_1$, every point of the eccentric circle ($E_2$) will be vertically over the corresponding point of the mean circle. Thus $M_2$ will be the position of the actual planet where $M_2$ is vertically above $M_1$, the mean planet. Join $E_2M_2$ to cut the mean circle in $P$. Since $A_2$ is the position of the actual planet farthest from $E_1$ the Earth's centre, the planet should have
the slowest motion there. So this point $A_2$ is termed Mandoccha, Manda because it is point where the planet is slowest and Uccha because it is the highest or the farthest point from the Earth's centre. Corresponding to this Mandoccha in the eccentric circle $A_1$ is termed the Uccha in the mean circle. Also $p$ the point where $E_1M_1$, the line joining the Earth's centre to the actual planet and called the Mandakarna, cuts the mean orbit is taken to be the position where the apparent planet is situated. Thus 'p' is seen to be deflected from the mean planet $M_1$ towards the Mandoccha on which account the Mandoccha is considered to be attracting the planet 'रोक्त्तक्षयको मन्त्रित' as Bhāskara puts it. The angle $A_2E_2M_2$ is spoken of as the Manda-Kendra or the mean anomaly and it is equal to $Z_2E_2M_2 - Z_2E_2A_2 = $ longitude of the planet minus the longitude of the Mandoccha, where $E_1Z_1$ and the parallel $E_2Z_2$ are directions towards the Zero-Point of the Zodiac. This accounts for the statement 'पूर्वचेन हीनो ग्रहो मन्त्रकेंद्रम्' (of the verse under elucidation) i.e. the excess of the longitude of the planet over that of the Mandoccha is termed Mandakendra. While $M_1$ is termed the Madhya-graha in the mean orbit, $M_2$ is termed the prativratta-Madhyagrabha, and not spastagraha as might be deemed, while $p$ is spoken of as the spastagraha or the True planet or apparent position of the planet. The word Prativratta stands for the eccentric circle. Now $M_1P$ the difference between the mean and True positions is spoken of as the Mandaphala which corresponds to the modern 'Equation of Centre'. To find its value draw perpendiculars $PN_1$ and $M_2N_2$ on $Triangles E_2M_2Q_3$ and $M_1M_2N_2$ are evidently similar

Hence $\frac{M_2Q_2}{E_2M_2} = \frac{M_2N_2}{M_1M_2}$ so that $M_2N_2 = \frac{M_1M_2}{E_2M_2} \times M_2Q_2 = \frac{\rho}{\sin M_2E_2A_2}$ (1) (which is equal to $\rho \sin \hat{E}_2$ in modern terms) Now in the case of the Equation of centre which is
generally a small quantity \( M_2 N_2 \) is taken to be equal to \( PN_1 \). If, however, this approximation is not made, \( PN_1 = \frac{M_1 N_2 \times E_1 P}{E_1 M_2} \) (by the similarity of the triangles \( E_1 P N_1 \) and \( E_1 M_2 N_2 \)) so that the actual equation of centre is \( \frac{r}{R} \) \( H \sin E_2 \times \frac{R}{K} = \frac{r}{R} \) \( H \sin E_2 \) where \( K = \text{Mandakarna} E_1 M_2 \). As \( M_2 \) moves from \( A_2 \) to \( P_2 \), the equation of centre as given by (1) gradually increases from Zero to a maximum \( r \) when \( E_2 = 90^\circ \) and decreases from this maximum to Zero when \( E_2 = 180^\circ \). Thus from what was noticed from the tabulated differences between the computed mean positions and observed true positions, the fact that those differences vanish at \( A_2 \) as well as \( P_2 \) the diametrically opposite point of the Mandoocha (not called Sighroccha, for this word Sighroccha will be seen to have altogether a different connotation) was verified. The maximum value of \( M_2 N_2 \) \( 'r' \) is termed the Antyaphala-jya or the \( H \) sine of the maximum equation of centre from which the arc could be found. In the case of the Sun and the Moon from the maximum differences between the computed and observed positions, their \( H \) sines were found and taken to be equal to \( 'r' \) in the respective cases. From this value of \( r \), the circumferences of the circles whose radius equals \( r \), were found and termed as Mandaparidhis. Why the circumferences were found is that in all positions of \( M_1 M_2 \), the value of \( M_1 M_2 = r \) (in as much as the corresponding points of the two circles will be as much distant as the centres of the circles from each other so that \( E_1 E_2 = M_1 M_2 = \text{Constant} = r \)) so that \( M_2 \) will always lie on a circle whose centre is \( M_2 \) and radius \( r \). This circle is known as the Manda-Nichöcha Vritta or an epicycle, where the word Nicha stands for the \( P_2 \) which is nearest the earth, and the compound word Manda-Nichöcha-Vritta means that circle which makes the planet occupy the Nicha and the Uccha points; the word Manda pertains to the Manda-phala or the equation of centre in
contradistinction to the word Śighra which we shall shortly deal with.

In modern astronomy the equation of centre is given approximately to be equal to $2e \sin m$ where ‘$m$’ stands for the mandakendra so that $r = 2e$. It will be shortly seen from a subsequent table that this formulation of the equation of centre gives results which closely accord with their modern values. The true or apparent positions of the Sun and the Moon could be obtained fairly well from the above formulation, so that it is stated that चन्द्रस्य रूपते मान्द्रक्रियान्निमित्तः i.e., the Moon and the Sun could be rectified by the equation of centre alone." This is quite in order for, the Sun and the Moon may be taken to be going round the Earth in ellipses, with the earth in one focus, the former relatively and the latter directly.

After having formulated the method of rectification in the case of the Sun and the Moon, the next question was with respect to the Tārā grahas i.e. Mercury, Venus and Mars, Jupiter and Saturn. As these are going round the Sun and the Sun going round the earth relatively, the process of rectification got complicated. In the first instance, the ancient astronomers must have tabulated the differences between the mean computed positions and the observed true positions. In the case of Mercury and Venus, the case appeared different from what it was in the case of the other three planets, for the simple reason that the mean positions of the former were taken to coincide with the mean Sun. This meant that for rectification, the elongation had to be computed and added to or subtracted from the mean position of the Sun to get the apparent geocentric positions of Mercury and Venus. The analogy of the method of the formulation of the Mandaphala is taken here also by imagining (1) eccentric circular motion and (2) postulating an Uccha. In the case of the Mandaphala, the equation was zero when the mean planet coincided with the Mandoccha. Here the equation is zero when
elongation is zero, i.e. when the apparent geocentric position of the planet coincides with the Sun, who is taken to be the mean planet. Naturally therefore the Uccha is taken to coincide with the Sun the mean planet, when the planet is in conjunction with the Sun. The maximum equation was had in the case of the Mandrahala when the arc between the Uccha and the mean planet was a right angle. So, here also, the maximum equation i.e. the maximum elongation should be had when the Uccha is at right angle from the Sun. Thus an Uccha was postulated with the following criteria namely (1) It should be a point moving in a geocentric circle (2) It should coincide in direction with the Sun when the planet is in conjunction with the Sun (3) It should be removed by a right angle from the Sun when the elongation is maximum (3) It should be removed from the Sun by 180°, again when the planet coincides in direction with the Sun (4) It should have a longitude exceeding that of the Sun by 270 when again the elongation is a maximum on the other side and finally (5) It should complete a circle with respect to the Sun when again the planet coincides in direction with the Sun.

When such a point was conceived it is clear that this Uccha is not the same as the planet, as some have misconstrued, because while the planet oscillates about the Sun by a particular angle (29° in the case of Mercury and 45° in that of Venus) in Uccha completes a circle with respect to the Sun and further as Hindu Astronomy postulated geocentric motion, the Uccha is a point construed as going in a geocentric circle. By the above postulation the synodic period of the Uccha is equal to the period of oscillation of the planet about the Sun. But the latter period is no other than the synodic period of the planet so that the synodic periods of the Uccha and the planet coinciding their sidereal periods should be equal. In other words the Uccha so conceived is a point other than the planet going round in a geocentric circle and having a geocentric side-
real period equal to the heliocentric sidereal period which again means that the geocentric longitude of the Uccha is the heliocentric longitude of the Planet. Thus the radius vector to the planet from the Sun is parallel to the geocentric radius vector of the Uccha. This accounts as to how the heliocentric sidereal periods of Mercury and Venus could be found under a geocentric concept and also as to how the heliocentric planets are themselves spoken of as their respective Uchhas, while their mean planet is the same as the Sun. On this count it was mentioned by the Hindu Astronomers बुधः प्रदः कुञ्जयाः स्रोऽ भगवः तैः शीतपानाकौ ie. The mean Planet of Mercury and Venus is the Sun himself where as they are themselves spoken of as their Uchhas. The phrase ‘they are themselves’ in the above statement is significant as it connotes that the word ‘they’ stands for the heliocentric planets, though it was not stated in so many words. Shortly we shall see also that the centre of the eccentric circle coincides with the centre of the Sun also and applying Bhāskara’s statement ‘यस्मिन्निश्चये अमति ख्याने नाः स्यं मध्ये कुम्भे’ i.e. the centre of the circle in which the planet moves does not coincide with that of the Earth’, the Sun was, though unwittingly taken as the centre of the Planetary motion. Thus we see how even the geocentric postulation also could help computation of the Planetary positions, the mathematics behind revealing heliocentric motion. What Copernicus achieved was that he identified that the point about which the planets revolved which was construed by the Hindu astronomers as an imaginary point not coinciding with the earth’s centre, was no other than the Sun himself.

In the case of Mercury and Venus the so-called Sīghra-phala came to be discovered first and we shall presently see why their elongation was called Sīghra-phala and how the Uccha postulated as above came to be termed Sīghroccha. Since initially the equation was to be zero, when the Planet and the Uccha coincided with the Sun
and then the elongation has to increase as the Uccha gained over the Sun, the initial conjunction was the modern Superior conjunction. The other position of the Uccha when again the elongation i.e. the equation is Zero must be therefore the Inferior conjunction. Also at the motion of Superior conjunction, the planet must be having the maximum daily motion, as it is clear from a heliocentric figure that at that point the relative motion of the planet with respect to a geocentric observer is the sum of the velocities of the planet and the earth. Hence this Uccha is spoken of as the Sighroccha also because the Uccha being a geocentrically moving point having heliocentric angular motion, its velocity is always greater than that of its planet namely the Sun. The word Uccha is applied because at the Superior conjunction the planet is farthest or highest from the earth. The excess of the longitude of this Uccha over the longitude of the planet i.e. the Sun is known as the Sighrkendra or anomaly as it is said in the verse under commentary ‘चलोचच प्रशोधं भवेत् शीघ्रकेन्द्रम्’. Thus in the case of Mercury and Venus, the Sighraphala came to be discovered first. This being discovered, formulated as will be shortly shown, and applied to the mean Sun as the planet, still it was found that there was a difference between the computed position and the observed position. Such differences were tabulated. By analogy from the case of the Sun and the Moon, it was thought that there should be also a Mandoccha here also, so that the point indicated by the position of the mean planet after being corrected by the equation where the above tabulated difference was zero, was identified as the Mandoccha. From the H Sine of the maximum difference taken as the radius of the Manda epicycle, its circumference was then computed.

In the case of the superior planets, we have already said that the geocentric sidereal periods accord with their heliocentric ones. Calculating the mean position of the and finding its difference from the observed true
position, such differences were tabulated. It was discovered that these difference almost vanished when the planet was in conjunction with the Sun and attained a maximum when the elongation was nearly a right angle from an analogy from the Manda-Karna i.e. process of obtaining the Manda-phala. Since the differences attained their maximum value when the elongations were nearly a right angle it could be seen that the Sun played the part of the Uccha in this case. As the Sun has a quicker motion than the planet and also as at conjunction the planet has the quickest motion relative to the Earth while it is farthest from the Earth the Uccha is the Sun here, is termed S'ighroccha. The excess of the longitude of the Sun over that of the planet is termed accordingly the S'ighra-kendra and the S'ighra-phala the equation was formulated as will be shown. Applying this S'ighra-phala to the mean position, the differences still found between the position so obtained and the observed true position were tabulated. The point indicated by the above position where the difference was found to be zero, was identified as the Mandoccha, and through the maximum difference, the Mandaparidhi was formulated.

In the above discourse, we have tried to give an account of how the originators of Hindu Astronomy could give us a workable system. We never assumed that an Āgama gave us the numbers of sidereal revolutions of the planets or the measures of the epicycles Manda or S'ighra. But in the explanation given by Bhāskara under Bhagañopapatti, one will notice that when Bhāskara gave the proof of the Moon’s sidereal revolutions, he said that having got the true positions of the Moon on two consecutive days, the mean positions were computed from the true by an inverse process of applying the equation of centre, and getting the mean daily motion of the Moon from those mean positions, the number of sidereal revolutions in a Kalpa were obtained. Here the Upapatti or the proof adduced by Bhāskara was not a proof but only a verification in as much as (1) he assumed the formulation
of the Mandaphala from the Āgama without pointing out how it was formulated and (2) he assumed the period of a Kalpa and that at the beginning of the Kalpa the planets were all in conjunction at the first point of Aries. Similarly in the Upāptti adduced by Bhāskara with respect to the Mandocchas of the planets, he assumed the formulation of Sīghra-phala on the basis of Āgama without proving how the concept of Sīghra-phala was arrived at by the founders of Astronomy and how the difference between the observed apparent positions and the computed mean positions, was resolved into two equations the Mandapha'ā and the Sīghrā phala. In the proof adduced with respect to the Sīghroccha of Mercury and Venus also Bhāskara did not mention anything as to how their heliocentric sidereal periods could be obtained but simply assumed the formulation of the Mandapha'ā and Sīghra-phala as already being there on the basis of Āgama.

We shall now proceed to describe the method of formulation of the Sīghra phala with respect to the five Tara-grahas, star planets namely Mercury, Venus and Mars, Jupiter, Saturn and show how so different a set of geometries of the ancients and the moderns the one geocentric and the other heliocentric could give identical formulation with respect to Sīghra-phala. Let us consider the case of Mercury and Venus in the first instance.

Having taken the mean Sun to play the part of the 'Graha' in the case of Mercury and Venus and having formed a concept of Sīghroccha as mentioned before, whose geocentric period of revolution was determined, without suspecting it to be the heliocentric sidereal period of the planet the ancient Hindu astronomers assumed by analogy from the case of Mandaphala with respect to the Sun and Moon, that the centre of the circle in which these planets revolve does not coincide with the centre of the Earth. In other words, they continued eccentric circle theory here so that without suspecting heliocentric
their mathematics led to them to make the centre of the eccentric circle coincide with the Sun himself. On this basis alone it was given to Copernicus to formulate heliocentric theory, sponsored by a thought that the Heavenly Sun could not be deemed as a satellite of the ‘Mundane’ Earth.

(Re" fig. 9). The same figure 9 will also serve the purpose to obtain the Sighrapala, only $M_1 M_2$ will be now on the right hand side of the Sighrocchas $A_1 A_2$, for, the latter will be taken to be in advance of the mean planets Kaksha-Vrittiya Madhyagraha $M_1$ (i.e. mean planet of the deferent) and prati-Vrittiya Madhyagraha $M_2$ (i.e. mean planet of the eccentric). The points $A_1$ and $A_2$ are themselves called the Kaksha-Vrittiya Sighroccha and prati-Vrittiya Sighroccha respectively. As was shown in the case of the Mandaphala from the eccentric figure 9, $M_2 N_2 = \frac{r}{R} H \sin ( \text{Kendra})$ so that $PN_1 = \frac{r}{R} H \sin ( \text{Kendra}) \times \frac{R}{K}$

$= \frac{r}{K} H \sin ( \text{Kendra}) = \frac{\text{Antyaphalajya} \times \text{Sighrakendrajya}}{\text{Sighrakarna}}$.

We shall take this for elucidation in the appropriate context.

Verse 19. Three Rasis each of 30° constitute a quadrant, and there are four quadrants in a circle which are respectively odd, even, odd and even. In the odd quadrants the Kendra covered is itself called Bhuja whereas in the even ones, the complement thereof is called Bhuja. Also, the complement of the Bhuja is called the Koti.

Verse 20. $R - H \sin e = \text{Co.} \ H \text{versine}$ and $R - H \cos e = H \text{versine}$ and $R - \text{Co.} \ H \text{versine} = H \sin e$, $R - H \text{versine} = H \cos e$.

Verse 21. Also $\sqrt{R^2 - H \sin e^2} = H \cos e$,

$= H \sin e$. Similarly

$Dyujya, \sqrt{R^2 - Dyujya^2} = \text{Kräntijya}; \sqrt{R^2 - \text{Drig-jiya}^2}$
= S'anku, $\sqrt{R^2 - S'anku^2} = Drig-ju$. In all the cases cited above, the radius R happens to be the hypotenuse.

ماذا. The convention in verse 19 corresponds to saying in modern trigonometry that \( \sin 90^\circ + \theta = \cos \theta \)
\( \sin (180^\circ - \theta) \sin \theta, \sin 180^\circ + \theta = -\sin \theta, \sin 270^\circ - \theta = -\cos \theta, \sin 330^\circ - \theta = -\sin \theta. \)
In Hindu trigonometry, the sign is understood and not explicitly mentioned.

Kräutijä, Dyujjä, Drig-ju and S'anku, are respectively H sin $\delta$, H cos $\delta$, H sin Z, H cos Z where $\delta$ is declination and Z the Zenith-distance of a celestial body. Taking the radius of the celestial equator to be R, the radius of the diurnal circle of a celestial would be equal to $R \times \cos \delta = H \cos \delta$ which is called Dyujjä because it is the radius of the diurnal circle.

Verse 22. The lengths of the circumferences of the Manda—epicycles are respectively 13°—40′, 31°—36′, 70°, 38°, 33°, 50,* for the Sun, Moon, Mars, Mercury, Jupiter, Venus and Saturn.

ماذا. Bhāskara has given these measures reportedly on the basis of Āgama or ancient authority. The peculiarity of measuring the circumferences in degrees less than 360°, is due to the idea that these circumferences are measured in relation to that of the deferent or Ākṣā śrāstrī. In other words, circumference of the epicycle of a planet as given above: circumference of the mean orbit :: x : 360 = radius of the epicycle : radius of the mean orbit where x is the measure of the circumference of the planetary epicycle. It may be mentioned once again that the radius of a planetary epicycle is the measure of the greatest

* The printed book of Brahma Sphuta Siddhānta gives in the case of Saturn 30° only which might have been the mistake of the scribe (Vide verse 36, Spashtādhihikāra B. S.). In the place of भूयरामा: it ought to have been
equation of centre pertaining to the planet which may be taken to be equal to $2e$ as a first approximation where $e$ is the eccentricity of the elliptic orbit of the planet.

It may be further mentioned here that in Sūryasiddhānta, as well as elsewhere in this work, the circumferences are given to vary continuously. This variability curiously achieves ellipticity in the orbit as may be seen as follows.

In the case of the Sun, the epicycle has a periphery of $14^\circ$ when $m = 0$ or $180^\circ$ and of $13\frac{3}{5}^\circ$ when $m = 90^\circ$ or $270$ according to Sūryasiddhānta, where $m$ is the Manda-kendra or mean anomaly. At any arbitrary point, where

![Diagram](image-url)
the mean anomaly is \( m \) the periphery is given to be 
\[
14^\circ - \frac{20' \ H \ sin \ m}{R}.
\]
The corresponding radius will therefore be 
\[
r = r - \frac{20'}{2\pi} \ \frac{H \ sin \ m}{R} = r - H \ sin \ m \ (\text{say}) \ 	ext{where} \ r = \frac{14^\circ}{2\pi}.
\]
(Ref. fig. 10) Let \( \mathcal{C} \) be the earth's centre, \( A \) the position of the apogee \( EE_1 \), \( = \) the radius of the epicycle measured along \( \mathcal{C}A \), \( B \) any arbitrary position of the mean planet and \( b \) the position of the planet in the epicycle. Here the radius \( BB_1 \) is not equal to the max. radius equal to \( EE_1 \), i.e. \( r \) but equal to \( r - H \ sin \ m \). Take \( E_1 \) as the origin and \( E_1A \) as the \( y \)-axis and a perpendicular to \( E_1A \) through \( E_1 \) namely \( E_1x \) as the positive direction of the \( X \)-axis. If the mean anomaly \( BH \mathcal{A} \) be \( m \), then the coordinates of the true planet are given by 
\[
x = BL = H \ sin \ m \ (1) \ y = E_1L + BB_1 = EL - EE_1 + r - \lambda H \ sin \ m = H \ cos \ m - r + (r - \lambda H \ sin \ m = H \ cos \ m - \lambda x).
\]

\[
\therefore \ y + \lambda x = H \ cos \ m \ (2) \text{ Squaring and adding } I \\
\text{and } II \ x^2 + (y + \lambda x)^2 = H \ sin^2 m + H \ cos^2 m = R^2
\]

\[
\therefore \ x^2 (1 + \lambda^2) + 2\lambda xy + y^2 = R^2 \text{ which is an ellipse with centre } E_1.
\]

Verses 23, 24, 25. The peripheries of \( \text{S} \)ighra epicycles. The peripheries of the epicycles of the star-planets Mars, Mercury, Jupiter, Venus and Saturn are respectively \( 243^\circ - 40', 132^\circ, 63^\circ, 258^\circ \) and \( 40^\circ \). The \( H \) sine of the \( M \)anda mean anomaly of Venus being multiplied by \( 2 \) and divided by \( 348 \), and the result being subtracted from the periphery gives the rectified \( M \)anda periphery. The \( H \) sine of its \( S \)ighra mean anomaly being multiplied by \( 5 \) and divided by \( R \), and the result being added to the \( S \)ighra periphery gives the rectified periphery. The smaller of the \( H \) sine or \( H \) cosine of the \( S \)ighra anomaly of Mars being multiplied by \( 6\frac{3}{8} \) and divided by \( H \ sin 45^\circ \) and the result in degrees being subtracted from or added to as the case may be,
the aphelion gives the rectified aphelion. The $S$ighra periphery being reduced by the above degrees gives the rectified $S$ighra periphery in case the $S$ighra anomaly is $90^\circ < m < 180$ or $270^\circ < m < 360^\circ$.

**Comm.** In the commentary Bhāskara adds that in the case of Venus the Mandaperiphery of $11^\circ$ as given is at the end of even quadrants whereas at the end of odd quadrants it is $9^\circ$, wherefore the enunciated rectification. Similarly in the case of his $S$ighraphala, the periphery of $245^\circ$ mentioned is at the end of even quadrants whereas at the end of odd quadrants it is $263^\circ$, and so the suggested rectification. Again in the case of Mars, the aphelion as computed is the same at the end of all quadrants whereas in the middle of the quadrants it is to be increased or decreased by $6\frac{3}{5}^\circ$ when the anomaly as stated. Also in the case of this Mars, the $S$ighra periphery mentioned is at the ends of quadrants. In the middle of the quadrants the periphery is to be reduced as suggested. In all these interpolations Bhāskara accepts the Āgama as enunciated by Brahmagupta.

We shall deal with the geometrical nature of these $S$ighra peripheries shortly in the appropriate place.

**Verse 26.** To obtain what are called Bhujaphala and Kotiphala both in the case of Mandaphala as well as $S$ighraphala. The $H$ sine and $H$ cosine of the Manda or $S$ighra anomalies multiplied by the respective peripheries and divided by $360^\circ$, or multiplied by $r$ and divided by $R$ gives the Bhujaphala or Kotiphala where $r$ and $R$ are respectively the radius of the Manda or $S$ighra peripheries and $R$ the radius of the deferent taken to be $3433^\circ$. If the radius $3438^\circ$ be respectively multiplied by the Manda or $S$ighra peripheries and divided by $360^\circ$, the result will be the $H$ sine of the maximum Mandaphala or $S$ighraphala, known as in either case.
Comm. As per the formulation.

Bhujaphala = \( \frac{H \sin m \times c}{360} = \frac{H \sin m \times r}{R} \)

Kotiphala = \( \frac{H \cos m \times c}{360} = \frac{H \cos m \times r}{R} \)

in the case of Mandaphala or Sīghraphala where \( c = \) periphery of the Manḍa or Sīghra periphery, \( r = \) Antyaphalajāyā defined above \( R = 3438' \) and \( m \) stands for the Manḍa or Sīghra anomaly. These Bhujaphala and Kotiphala will be used in their respective contexts.

Verses 27, 28, 29. Calculation of what is known as Sīghrakarpā.

\[(H \cos m \pm r)^2 + H \sin^2 m = K^2 \quad (1)\]
\[(R \pm Kotiphala)^2 + Bhujaphala^2 = K^2 \quad (2)\]
\[R^2 + r^2 \pm 2 R \times Kotiphala = K^2 \quad (3)\]
\[R^2 + r^2 \pm 2 r \times H \cos m = K^2 \quad (4)\]

The arc of the \( H \) Sine of the equation of centre is called the Mandaphala.

Comm. Ref. fig. 9. From triangle \( E_1M_1M_2 (E_1M_1 + M_1N_2)^2 + M_1N_1^2 = E_1M_2^2 = K^2 \). But \( M_1N_2 = Kotiphala \) and \( M_2N_2 = Bhujaphala \) defined previously so that we have the second formula for \( K \) enunciated above. From the similarity of the triangles \( E_1M_1Q_1 \) and \( M_1M_2N_2 \) we have \( \frac{M_2N_2}{M_1Q_1} = \frac{M_1M_2}{E_1M_1} \) so that \( M_2N_2 = \frac{r}{R} \times M_1Q_1 \); Since \( M_1Q_1 \) is called the Bhuja, the corresponding \( M_2N_2 \) in the Antyaphalajāyā triangle is called the Bhujaphala. Similarly \( M_1N_2 \) is called the Kotiphala.

Again \( (E_1Q_1 + Q_1Q_2)^3 + M_2Q_2^3 = E_1M_2^3 \) where \( E_1Q_1 = H \cos m \), \( Q_1Q_2 = M_1N_2 = r \) and \( M_2Q_2 = M_1Q_1 = H \sin m \). From this we have the first formula enunciated.
Again expanding \((E_M + M_N)^2\), \((E_M + M_N)^2 + M_N^2 = E_M^2\) we have the third formula; similarly expanding \((E_Q + Q)^2\), \((E_Q + Q)^2 + M_Q^2 = E_Q^2\) we have the fourth formula.

Since the \textit{Sighraphala} has been defined to be equal to \(\frac{r}{K} \sin m\), we have had the necessity of knowing the value of \(K\). The convention of signs mentioned in the formulation in the words ‘\(योगे युगादायय कक्षाधारी केन्द्रेष्टरम्\)’ is due to the fact that cosine is positive in the fourth and first Quadrants and that the \textit{Kotiphala} becomes negative in the 2nd and 3rd Quadrants as could be seen by drawing the figure in those Quadrants.

Now we shall prove what is most important, namely that postulating an entirely different geocentric motion how the Hindu Astronomers could formulate the \textit{Sighraphala} which accords exactly with the heliocentric theory, assuming of course coplanar circular orbits. Let figures 11 and 12 pertain to the modern heliocentric geometry,

![Fig. 11](image)

![Fig. 12](image)

the former with respect to the Inferior planets Mercury and Venus signified by \(V\), and the latter to the superior planets Mars, Jupiter and Saturn signified by \(J\). Let fig. 13 pertain to the Hindu geocentric geometry dealing with both the Inferior and Superior planets as well. In the heliocentric figures let \(S = \text{Sun}, E = \text{Earth}, SA = \text{direction to } \mu\), the Zero-point of the Zodiac from the Sun, \(\mu\).
geocentric direction towards Aswini. Let $SV, EV^1$, be the heliocentric and geocentric directions of the $S$ighroccha where $V$ is the actual planet and $V^1$ an imaginary point. Draw $EJ' 11$ to $SJ$. Let the radius of the inner and outer heliocentric circles be respectively $r$ and $R$. Let $K$ be the radius vector to the planet in both the figures.

Let in fig. 13, $E_1 =$ Earth's centre, $E_2 =$ the centre of the eccentric circle which we shall presently show to be coinciding with the Sun's position. Let $M_1, M_2$ represent the mean planets in the deferent and the eccentric known as Kaksha-Vrittiya Madhyaagraha and prati-Vrittiya Madhyaagraha. Let $A_1, A_2$ be the $S$ighrocchas in the deferent and the eccentric. Let $E_1E_2 = r$ known as Antyaphalajya and $R$ the radius of both the deferent and the eccentric. Let $K$ be the radius vector to the planet known as Sighra-Karna.
We shall prove that $m$ the Sighra anomaly of the Hindu figure will be the same as 'm' as marked in the heliocentric figures. Sighra anomaly is defined as longitude of $\text{Sighroccha} -$ longitude of the $\lambda_1 - aE_1M_2 = aE_2A_2$.

$aE_2M_2 = m$ fig. 13. In the heliocentric fig. 11, since $A^1E^1$ is the longitude of the $\text{Sighroccha}$, and $A^1E \hat{S}$ the longitude of the Sun treated as the Madhyagraha of the Inferior planet $A^2E^1 - A^2E \hat{S} = V^1 \hat{E} \hat{S} = VSS^1 = m =$ the Sighra anomaly. In fig 12, $m = S^1 \hat{S} J = SEJ = A^1 \hat{E} \hat{S} - A^1EJ^1 = A^1E \hat{S} - A \hat{S} J =$ Longitude of the Sun treated as the $\text{Sighroccha}$ of the Superior planet minus longitude of the heliocentric planet known as Mandaphutagraha or the planet rectified for the Mandaphala or equation of centre =$\text{Sighra anomaly}$.

In the case of the Inferior planet the heliocentric direction of the planet is equal to the $\text{Sighroccha}$. Now consider the triangles ESV, JSE, $E_1 \hat{M}_1 \hat{M}_2$ of the three figures. Evidently $E \hat{S} V = J \hat{S} E = E_1 \hat{M}_1 \hat{M}_2 = 180 - m =$ Supplement of Sighra anomaly. If, further it is shown and (it will be shown subsequently) $\frac{SV}{SE} = \frac{SE}{SJ} = \frac{M_1M_2}{\text{the similarity}}$ of the triangle $E_1 \hat{M}_1 \hat{M}$ geocentric figure separately with ESV and JSV will have been established. Taking this similarity to have been established, $M_1 \hat{E} \hat{M}_2$ known as Sighraphala will be equal to SEV in fig. 11 and SJ$E$ in fig. 12. In fig. 13, $M_2$ the prativritta Madhagraha is also known as the pāramārthikagraha or the actual planet where as its geocentric position on the Kakshāmandala is taken to be the true planet or apparent position of the planet. In figures 11 and 12, EV and EJ are the directions to the true planets V and J so that the angles between the Sphutagraha and the Madhyagraha (ie the Manda Sphutagraha =
(in fig 11) and \(= A^1 E J \).

Sigraphala. Once the similarity of the triangles fig. 12 is established, the equality of the Sigraphala will be established. Also due to the similarity mentioned above the formulae for \(K\) as given in Hindu Astronomy should also accord with that in the heliocentric figures. In fact in the heliocentric figures \(K^2 = R^2 + r^2 + 2Rh \cos m = R^2 + r^2 + 2RH \cos m\) which is identical with the four formulae given before as per verses 27, 28, 29. It will be seen that the epicycle \((M_1)\) with radius \(M_1 M_2\) will be identical with the inner circles in the heliocentric circles, whereas the Kakshamandal \((E_1)\) with radius \(R\) will be identical with the outer circles of the heliocentric figures.

Before we proceed further, we shall annex the table wherein the ratio \(r/R\) as given in Hindu Astronomy will be seen to accord with that in modern astronomy.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Periphery of the Sigrapha-epicycle</th>
<th>Periphery of the deferent</th>
<th>Ratio</th>
<th>Value in modern astronomy taking Earth's radius to be unity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>132(^\circ)</td>
<td>360(^\circ)</td>
<td>(\frac{132}{360} = .37)</td>
<td>.387</td>
</tr>
<tr>
<td>Venus</td>
<td>258(^\circ)</td>
<td>360(^\circ)</td>
<td>.716</td>
<td>.723</td>
</tr>
<tr>
<td>Mars</td>
<td>249(^\frac{1}{2})(^\circ)</td>
<td>363(^\circ)</td>
<td>1.5</td>
<td>1.52</td>
</tr>
<tr>
<td>Jupiter</td>
<td>68(^\circ)</td>
<td>360(^\circ)</td>
<td>5.3</td>
<td>5.2</td>
</tr>
<tr>
<td>Saturn</td>
<td>40(^\circ)</td>
<td>360(^\circ)</td>
<td>9</td>
<td>9.5</td>
</tr>
</tbody>
</table>

In the light of this table the similarity of the triangles ESV, and JSE with \(E_1 M_1 M_2\) is now established.
Formula for Sighraphala from the heliocentric figures

In fig. 11, \( \frac{r}{K} = \frac{\sin SEV}{\sin ESV} \) so

\[ \frac{r}{K} \times \sin m \]

In fig. 12 \( \frac{r}{K} = \frac{\sin SJE}{\sin ESJ} \) so that \( \frac{\hat{S}JE}{K} \sin m \)

Both these accord with the Hindu formula.

It will be interesting to point out here that in fig 11, keeping the earth constant and supposing the Sun S to go in a circle with centre E and radius ES, the orbit of the Inferior planet V will play the part of the epicycle of Hindu Astronomy. Thus in the case of the Inferior planets, the epicyclic theory is only a different version of the heliocentric theory. In the case of the Superior planets, however, (fig. 12) cut off EJ \( J \) = SJ along EJ"parallel to SJ; then \( JJ \) will be parallel to ES just as \( MM_1 \) is parallel to \( E_1E_2 \) in fig. 13. Then the circle with E as centre and EJ \( J \) as radius corresponds to the deferent of fig 13, whereas the circle \( J^1 \) with centre \( J^1 \) and radius \( J^1J \) corresponds to the epicycle. The circle with S as centre and radius SJ corresponds to the eccentric.

Verse 30. The equation of centre pertaining to the Sun and the Moon using a simpler table of H sines where the radius = 120 units. The H sines of the mean anomaly as found from the simpler H sine table where radius = 120, multiplied by 20, and divided by 1103 and 477 respectively gives the equation of centre of the Sun and the Moon in degrees.

Comm. The maximum equation of centre with respect to the Sun is \( 2^\circ-10'-31" \). Then the argument is “If by the H sine of the anomaly equal to the radius 120, we have the above max. equation what shall we have for H sin m?”.
The answer is \[ \frac{H \sin m \times 2^{\circ}-10'-31''}{120} = \frac{2^21.0^\circ}{120} \times H \sin m \]
very approximately \[ \frac{251}{14400} H \sin m = \frac{20}{1103} H \sin m \].

Similarly in the case of the Moon, the maximum equation of centre is \(5^\circ-2'-8''\). By the same argument as above we have:

\[ \frac{H \sin m \times 1133}{225 \times 120} = \frac{H \sin m \times 20}{54000} - \frac{H \sin m \times 20}{1133} \]

**Verse 31.** Rectification of the mean daily motion of the Sun and the Moon.

The \(H\) cosine of the mean anomaly divided by \(54\) in the case of the Sun and in the case of the Moon multiplied by \(4\) and divided by \(7\) gives the increment or decrement in the respective mean motions according as \(90^\circ < m < 270\) or \(270 < m < 360 + 90\).

**Comm.** We have Equation of centre \(= \frac{r}{R} H \sin m = E\) (say) so that differentiating \(\delta E = \frac{r}{R} H \cos m \frac{\delta m}{R}\). But \(\frac{r}{R} H \cos m\) is called kotiphala and \(\delta m\) is called Kendragati so that \(\delta E = \frac{\text{Kotiphala} \times \text{Kendragati}}{R}\). Since Kotiphalia is negative when \(90 < m < 270\) \(\delta E\) is negative but in Hindu Astronomy we measure the Kendra not from perigee as in modern astronomy but from aphelion so that the equation of centre is strictly \(-\frac{r}{R} H \sin m\) if sign is also taken into consideration. Hence \(\delta E\) must be + ve. Since \(M + E = S\) where \(M\) is the mean planet, \(E\) the equation of centre and \(S\) the true planet \(\delta S = \delta m + \delta E\) so that the true motion is equal to the mean motion plus \(\delta E\). As \(\delta E\) is + ve when \(90 < m < 270\) as mentioned above we have to add this to the mean motion to get the true motion. This
is called gatiphala or what is to be added to the mean motion to give the true motion. Incidentally we have commented on the contents of verse 37.

**Verse 32.** The Sīghra phala with respect to the Star-planets.

\[
\frac{H \sin m \times r}{R}
\]

being multiplied by the radius R or the product of \( H \sin m \) and \( r \) being divided by \( K \) the arc of the result gives the Sīghraphala.

**Comm.** In the verse it is mentioned that the product of the Bhujaphala and the radius is divided by \( K \) so that the formula for the Bhujaphala being \( \frac{H \sin m \times r}{R} \) the Sīghraphalajya will be \( \frac{H \sin m \times r}{K} \) which is stated in the alternative. We have already derived this formula before where we got \( H \sin E_a = \frac{H \sin m \times r}{K} \). The arc of this will be \( E_a \) i.e. the Sīghra-phala, (\( E_a \) because we take \( E_1 \) as the Mandaphala).

**Verses 33 and 34.** An alternative formulation of Sīghra-phala.

\( H \sin m \) being multiplied by \( R \) and divided by \( K \) and the difference between the arc of the result and \( H \sin m \) will be the Sīghra-phala. Here \( H \sin m \) belongs to the eccentric. The arc of the maximum Sīghra-phalajya added to or subtracted from 90° will give respectively the Quadrants and the \( H \) sine will have to be taken of the elapsed Kendra or its Koti according as the Quadrant is odd or even.

**Comm.** Ref. fig. 13. From the similarity of the triangles \( E_1 M_2 N \) and \( E_1 PM \),

\[
= \frac{R}{K} \times M_2 N
\]
But PM is the $H$ sine of $m^1$ where $m^1$ is called Sphuta kendra ($m$ is called the madhya-Kendra). Hence

$$H \sin m^1 = \frac{R}{K} \times M_2N = \frac{R}{K} \times Bhujajya\text{ (in the eccentric)}$$

$$\therefore m^1 = H \sin^{-1} \left( \frac{R}{K} \times Bhujajya\right) = PA_1$$

$$\therefore M_1P = S'ighraphala = M_1A_1 - PA_1 = Kakshya-mandala Babu minus the obāpa $m^1$ - Here a clear understanding of the word Bāhu or what is the same Bhuja should be had. The arc pertaining to the angle $m$ in the eccentric is known as the Bāhu in the eccentric and that to the angle $m$ in the deferent as the Bāhu in the deferent. When $0 < m < 90$, $m$ is itself spoken of as Bāhu; when $90 < m < 180$, $180 - m$ is spoken of as the Bāhu; when $180 < m < 270$, $m - 180$ is the Bāhu and when $270 < m < 360$, $360 - m$ is the Bāhu. Thus the Bāhu is that angle whose $H$ sine will be $H \sin m$ numerically. When it is said in the verse ‘विज्याहतं कथितं भुजज्यं’ the word Bhuja is the arc $M_2A_2$ as is mentioned in the same verse ‘वेयो जुः बाहुः प्रतिमण्डलस्य’. The second part of the verse divides the eccentric circle into such quadrants that in them S'ighraphala increases from Zero to a maximum, decreases again from a maximum to zero, again in creases from zero to a maximum and again decreases from a maximum to zero. Thus at $A_2$ of the eccentric the S'ighraphala is zero; at $a_1$, it is a maximum namely the arc $b_4c_1$ where $H \sin b_4c_1 = a_1c_1 = r$; Thus in the course of $A_2a_1$ the arc of the eccentric the S'ighraphala gradually increases from zero to a maximum and in the course of $a_1a_2$ the S'ighraphala decreases from a max to zero. Again from $a_2$ to $a_3$ it increases from zero to a max and from $a_3$ to $A_2$ it decreases from the maximum to zero. Thus the quadrants in the case of S'ighraphala arc not of 90° but arcs $A_2a_1, a_1a_2, a_2a_3$ and $a_3A_2$, which are respectively of magnitude $90^\circ + H \sin^{-1}r$, $90 - H \sin^{-1}r$, $90 - H \sin^{-1}r$ and $90 + H \sin^{-1}r$. In the case of Mandaphala also, the quadrants should have been of the same magnitude if the so-called Karnānupāta has been postulated i.e. reducing the Manda-
phala from the extremity of the Karna to the extremity of the radius in the deferent; but as this Karnānapāta is not adopted, the difference being negligible the quadrants are all of equal magnitude i.e. each of 90°.

In the course of the commentary of this verse Bhāskara mentions that for Mercury, as the maximum Sīghraphala is 21°-31’-43”, the quadrants arc of magnitude 3-21-31-43, 2-8-28-17, 2-8-28-17 and 3-21-31-43 respectively.

Also in the commentary Bhāskara adds that a₁ which is the point of intersection of the eccentric with the horizontal diameter E₁b₁ the Sīghraphala is maximum and that at that point the mean motion is itself the true motion.

That the Sīghraphala at a₁ and a₂ is maximum is clear from the figure 13, where it is equal to the arcs b₁c₁ and b₂c₂ whose H sine is equal to r. To prove that the mean motion is itself the true motion, we have the equation M₂ + E₂ = S where M₂ is the mean planet here or the Mandasphutagraha or planet rectified for the equation of centre, (by the equation M₁ + E₁ = M₄, M₁ being the original mean planet and E₁ the equation of centre) so that δM₂ + δE₂ = δS where δM₂ is the mean motion here, δS the true motion and δM₂ is the variation in the Sīghraphala; but at a₁ and a₂ E₂ the Sīghraphala being maximum δE₂ is zero. Hence δM₂ = δS which means that the mean motion is itself the true motion.

Verse 34. Latter half, 35 and 36 former half.

The mean planet rectified for the equation of centre or Manda-phala is called Mandasphuta. Then subtracting the longitude of the Mandasphuta from that of the respective Sīghroccha, the result will be the Sīghra anomaly from which the Sīghra-phala is to be obtained. Rectifying the Mandasphuta for this second equation namely Sīghraphala, again obtaining therefrom the equation of centre effecting-
this in the original mean planet and again correcting for Sīghraphala and repeating the process till a constant value is obtained, the true planet is had with respect to the star-planets other than Mars. But with respect to Mars, let first the mean planet be corrected for half of the equation of centre. Then make half of the correction of Sīghraphala. Take the resulting planet to be the mean planet and again finding the equation of centre, make this whole correction in the original mean planet. Again taking the resulting planet to be the Mandasphuta effect the entire Sīghraphala. Then we have the true planet.

Comm. The Suryasiddhānta stipulates the same kind of correction in the case of all the star-planets.

"देवने फलार्थ प्रथम ततो ग्रहे ||" i.e. In the first place half of the Sīghraphala is to be effected in the mean planet; taking that to be the mean planet and computing the equation of centre half of it is administered; taking the resulting to be the mean planet and computing the equation of centre, the entire equation of centre is now to be administered in the original mean planet; taking the result to be the Mandasphutagraha, and computing the Sīghraphala, it is to be administered in full in the Mandasphuta. Then we have the true planet.

In modern astronomy, the equation of centre is first done and the result will be the planet in its heliocentric elliptic orbit. To reduce it to the geocentric position, the second correction is made which corresponds to the Hindu Sīghraphala. Thus the two corrections administered successively gives the apparent or true geocentric planet.

Though the mutual relationship as conceived between the equation of centre and the Sīghraphala in Hindu Astronomy is deemed irrational by modern interpreters of Hindu Astronomy, there is some rationale in the process as ex-
plained by this author in his work 'A critical study of Ancient Hindu Astronomy' (published by the Karnatak University) page 98.

*Verse.* Cited from Golādhyāya.

The equation of centre is to be applied to the mean planet to obtain the centre of the Sīghraepicycle; then to obtain the true position the Sīghraphala is to applied to the Mandasphutagraha; hence the two equations are mutually related so that the true position is obtained after repeated application of the two equations.

*Comm.* Explained above.

*Verse* 36 latter half and 37. The true daily motion of the planet is the excess of the longitude of the true planet of the next day over that of the true planet of the previous day.

The Kotiphala being multiplied by the daily motion of the Manda mean anomaly and divided by the radius, and the result being added to or subtracted from the mean motion, gives what is called Mandasphutagati.

*Comm.* Already explained before. If $M_1$, $M_2$ and $S$ be the mean planet, Mandasphutagraha, and the true planet respectively, and if $E_1$ and $E_2$ be respectively the two equations, then

$$M_1 + E_1 = M_2, \quad M_2 + E_2 = S$$

so that

Here $\Delta M_1$ = mean daily motion, $\Delta E_1$ = daily variation in the Mandaphala, $\Delta M_2$ = daily motion of the Mandasphuta graha or what is the same Mandasphutagati, $\Delta E_2$ = daily variation in the second equation and $\Delta S$ = True daily motion.
Lallāchārya formulates the Mandasphutagati in a different manner which is equally correct (Ref. verse 45 Spāṣṭādhikāra Siṣyadhī Vṛiddhida “विज्ञाहव ग्रहगति: मुदुकर्षणका मन्द्रसुत्ता भवति” i.e. $\frac{\delta m \times R}{K} =$ Mandasphutagati where $\delta m =$ Madhyagati or mean motion and $K$ is the Manda Karna equal to $\sqrt{R^2 + r^2} = 2 R \cos m$

\[
\therefore \text{Mandasphutagati} = \sqrt{\frac{\delta m \times R}{1 \pm 2 \frac{r}{R} \cos m + \frac{r^2}{R^2}}} = \delta m \left(1 \pm \frac{2r}{R} \cos m\right) - \frac{r^2}{2R^2} \text{ within brackets}
\]

\[
= \delta m + \frac{r H \cos m}{R^2} \delta m = \delta m + \frac{r H \cos m}{R} \times \frac{\delta m}{R} = \delta m + \text{Kotiphala} \times \text{Mandakendragati} \text{ as given by Bhāskara.}
\]

Verse 38. In the case of the Moon, obtaining the true Moon for a particular moment and his daily motion for the day, the ending moment of the tithi near at hand is to be computed with that daily motion, and the method of successive approximations is to be used to rectify the ending moment. In the case of the ending moment of the tithi being sufficiently far away, then it does not matter even if the above daily motion is applied to get the approximate ending moment. In as much as the Moon's daily motion is great and varies from moment to moment the motion at the moment is to be used.

Comm. Strictly speaking the ending moment of every tithi is to be computed by the method of successive approximation. That is why in the computation of eclipses,
Moon's hourly motion is given in modern almanacs. Since it is very cumbersome to use the method of successive approximation to determine the ending moment of every tithi, the Hindu almanac-makers generally compute the ending moment of a tithi using the daily motion of the moon computed for the moment of Sun-rise of the day. Only for ritual purposes, the method of successive approximation is used and also in the computation of eclipses.

**Verse 39. Computation of the Sīghragatiphala.**

\[
\frac{H \sin (90 - E_a)}{K} \times \delta m = \Delta S \quad \text{where} \quad \Delta S \quad \text{is the daily motion of the Sīghrācchā,} \quad E_a = Sīghraphala, \quad \delta m = \text{daily motion in the Sīghra mean anomaly,} \quad K = Sīghrakarna, \quad \text{and} \quad \Delta S \quad \text{the true motion of the planet. If} \quad \Delta S \quad \text{is negative the planet is retrograde.}
\]

**Comm.** This is mathematically an important verse, and the proof given by Bhāskara really reflects his genius. Before we attend to his proof, we shall give this a modern treatment. (Ref. figs. 14, 15). SA and EA¹ are the heliocentric and geocentric directions to Aswini the Hindu Zero-point of the Zodiac; S = Sun, E = Earth; V = Inferior planet Venus or Mercury; J = Superior planet; E = Sīghra-phala, K = Sīghra Karna; M = Sīghra anomaly.
True motion of the planets = $\delta (A'EV)$ or $\delta (A'EJ)$

But $\delta (A'EV) = \delta (A'EV' - n)$ and $\delta (A' EJ) = \delta (A' ES' - n)$

In the case of the Inferior planet $\delta (A'EV') = \delta (ASV) = S'ighrōcchagati$ and $\delta n$ is Sphutakendragati where $n$ is called Sphutakendra, $m$ being called Madhyakendra. In the case of the Superior planet $\delta (A'ES) = S'ighrōcchagati$ because the Sun plays the part of Sīghrōcha in the case of a Superior planet and $\delta n = Sphutakendragati$ as before. Hence in both the cases, Sphutagati = Sīghragati — Sphutakendragati. We have now to find Sphutakendragati to obtain Sphutagati, as Bhāskara remarks rightly "स्थायितिमेव सप्तीक्रता’’. In other words we have to find $\delta n$. From the figures $K \cos n - R \cos m = r$ (i) Differentiating this we have $- K \sin n \, \delta n + \cos n \, \delta K + R \sin m \, \delta m = 0$ (ii). But $K^2 = R^2 + r^2 + 2Rr \cos m$ so that $2 \delta K \times K = -2Rr \sin m \delta m$ (3) Eliminating $\delta K$ between (2) and

$$3 - K \sin n \, \delta n - R \sin m \delta m \left(1 - \frac{r}{K} \cos n \right)$$

$$- R \sin \frac{m \delta m}{K} (K - r \cos$$
But $K - r \cos n = R \cos E$.

\[ \therefore \delta n = \frac{R \sin m \delta m \times R \cos E}{K^2} \]

But $R \sin m = K \sin n$

Cancelling $\delta n = \frac{R \cos E \delta m}{K} = \frac{H \cos E \delta m}{K}$ as given by Bhaskara. In the formula $H \sin E = \frac{r \cdot H \sin m}{K}$, Bhaskara perceived the variability of both $H \sin m$ and $K$ on the right hand side and he exclaims "न हि केन्द्रगतिमये फलयोरत्तरं स्याते, किर्तिन्यथावपिष्ट अथतन्युज्ज्वलैश्वर्त्त-अस्तुर्यत्तररे तिज्ञयुगुणे अथतनकर्यत्ततै तात्रो फलं न तात्रो श्वस्तनकर्यतातै, स्वल्प-नतरं विषयम्यं कण्ठो मातस्त्य बहुत्तात्त बहन्तरं श्वादिशेवद्यं नित्यवा अन्यत्त महामातिमक्त: कवित्तम्, तथ्या केन्द्रगतिरेत्र स्पर्शक्ता" i.e. "The variation in the Sighrapahala is not entirely constituted by the variation in $m$ but also by that in $K$.....So leaving the method of seeking $\delta E$ through the formula $H \sin E = \frac{r \cdot H \sin m}{K}$ the great intelligent astronomers used the formula.

Sphutabhukti = Sighra Bhukti - Sphuta Kondrabhukti (Bhukti means gati) wherein it was sought to obtain the variation in Sphutakendra i.e. $n$ in the figures.

We have given a proof of Bhaskara's formula, which circumvented finding Sighragatipahala but which sought directly Sphutabhukti, by using the modern heliocentric figures. We shall now see how Bhaskara could deal with such a tough problem. Refer fig 16. Let $P_1$, $P_2$ be two positions of the planet on two consecutive days relative to the Sighra $A_2$, so that $P_1, E, P_2$ is the Sphuta Kendragati spoken of. It will be noted that it is not Sphutagati because $P_1$, $P_2$ are positions of the planet relative to $A_2$ which is itself moving (as rightly remarked by Bhaskara). It is Sphuta Kendragati because $A_2, E, P_1$, and $A_2, E, P_2$ are the
Kendras on two consecutive days whereas Madhyakendragati is $P_1 \hat{E}_2 P_2$. Also, we have the equation.

$$S'\text{ighra} - \text{Sphutagraha} = \text{Sphutakendra so that } S'\text{ighragati} - \text{Sphutagati} = \text{Sphutakendra}. \text{ Hence } \text{Sphutagati} = S'\text{ighragati} - \text{Sphutakendra}. \text{ So we have now to seek the value of } P_1 \hat{E}_1 \hat{P}_2. \text{ Let } P_1 a \text{ stand for the } S'\text{ighraphala of the first day which is equal to } e f, f \text{ being the true planet of the first day. } E_2 d \text{ will be parallel to } E_1 f \text{ because } P_1 e \text{ being parallel to } E_1 E_2 \text{ and } e f \text{ being equal to } P_1 d, d f 11 P_1 e \text{ (parallels to } E_1 E_2 \text{ cut off equal arcs on the two circles). This may be seen also as follows. Since } P_1 d \text{ is taken to be equal to } e f, e \text{ being the mean planet and } f \text{ the true on the first day } e \hat{E}_1 f = P_1 \hat{E}_2 d. \text{ But } E_1 e 11 E_2 P_1 \therefore e E_1 f = E_1 \hat{P}_1 E_2 \therefore P_1 \hat{E}_2 d = E_1 \hat{P}_1 E_2 \text{ and alternate angles being equal } E_1 P_1 11 E_2 d. \text{ } P_1 a \text{ is the } H \text{ sine of } P_1 d \text{ where } P_2 b \text{ is the } H \text{ sine } P_2 d. \text{ Looking upon } P_1 P_2 \text{ as an increment in } P_1 d \text{ i.e. looking upon the Kendragati } P_1 \hat{E}_2 P_2 \text{ as an increment in } S'\text{ighraphala, Bhāskara uses the method of Bhogyakhandaka sphuti Karana to obtain the Sphutakendragati. From the figure.}

$$P_2 c = P_2 b - P_1 a = H \sin (G + \delta m) - H \sin E
$$
$$= H \sin E H \cos \delta m + H \cos E H \sin \delta m \frac{R}{R} - H \sin E$$
Taking $H \cos \delta M = R$ and $H \sin \delta m =$

$$P_xc = \frac{H \cos E \delta m}{R}$$

This is at the end of $E$, $P_x$ i.e. at the end of $K$; so to get the corresponding chord in the deferent we do Karnānupāta so that the result is

$$\frac{H \cos E \delta m}{R} \times \frac{R}{K} = \frac{H \cos E \delta m}{K}$$

as given by Bhāskara.

We have cut short Bhāskara's method of Bhōgyakhanda Sphutikarāṇa to make it clear to a modern student. Since $P_xc$ is small, passing on from the $H$ sine to the arc is not necessary, for, the $H$ sine of a small arc is equal to the arc itself.

Bhāskara’s argument, however, is as follows:—“If for 225, we have Bhōgya Khandā, what for $\delta m$?” The result is $\frac{B \times \delta m}{225}$; Then $B$ is rectified as follows:—“When the $H$ cosine $E$ is equal to the radius, i.e. initially in the $H$ sine table, the Bhōgyakhanda is 225, then what is it for $H \cos E$?”

The result is $\frac{H \cos E \times 225}{R}$ which we have to substitute for $B$. Then if this be at the end of $K$ what is it at the end of $R$?” The result is

$$\frac{H \cos E \times 225}{R} \times \frac{\delta m}{225} \times \frac{R}{H \cos E \times \delta m}$$

Before we proceed to explain "we shall explain what Bhāskara pointed out as a mistake in Lallāchārya.

Verse 40. Let Mathematicians understand that what formula was given by Lallāchārya for Sīghragatiphala is not correct. When the anomaly is 90° or 270°, the gatiphala vanishes and there will be gatiphala at the points where it ought to be Zero according to his formula.
Comm. Ref. verse 45, Spāṣṭādhikāra, Siṣya Dhī-vyāḍhīda

\[ \text{i.e. } \quad \left( \frac{\text{Sīghraphala Bhög yakanda}}{225} \right) \times \frac{R}{K} = \text{Sīghragatiphaha}. \]

Here the quantity within the brackets is \( \delta m \). What Lallācharya had in his mind is as follows. ‘If for the Ādyā Khandha 225 we have \( H \cos E = R \), what shall we have for the Bhögya Khandha of the Sīghraphala?’ The result would be

\[
\left( \frac{\sin E \cos 225 + H \sin E}{H \cos E} \right) \times \frac{R}{225} = \frac{H \cos E \times 225}{225} \times \frac{R}{225} = H \cos E. \]

So Lallācharya’s formula would become \( \frac{\delta m R}{K} \times H \cos E \) as given by Bhāskara. The charge levelled at Lallācharya is due to the fact that by the word ‘Āsu-chāpa’ by which Lallācharya meant Bhāskara meant ‘आषुकेत्र साप’. When the Kendra = 90 or 270, the Bhög yakhanda being Zero, the Sīghragatiphaha would be Zero. Also where it ought to be zero namely \((90 + H \sin' r) \& (270 - H \sin' r)\) it would not be zero. If Lallācharya had really meant what Bhāskara attributed to him, the formula would be \( \frac{H \cos m \delta m}{K} \)

it is very unlikely that Lallācharya would have meant this wrong formula, for, even if \( K \) were taken by him to be steady, \( \delta \left( \frac{R \sin m}{K} \right) = \frac{R H \cos m \delta m}{K} \) and Lallācharya’s formula does not contain ‘r’. The only non-rigorous part in Lallācharya’s formula is at the point where he took
\( H \cos 225 = R \) and \( H \sin 225 = 225 \) which is rather crude. Bhāskara of course improved on this crudeness by taking to be an increment in \( E \).

In the commentary under this verse, Bhāskara, having misinterpreted Lallācharya’s phrase आगुत्तिकार as meaning आगुत्तिकार and not as आगुत्तिकार which was in the mind of Lallācharya, goes on recounting examples where the wrong formula attributed to him would give wrong results. So, we need not enter into those details.

Now we shall correlate Bhāskara’s formula with its modern counterpart. Assuming coplanar heliocentric circular orbits for planets, let us see at what points two planets appear mutually stationary, that is, have a Zero relative angular velocity before they appear mutually retrograde.

Let \( S = \) Sun, \( E = \) Earth, \( J = \) Jupiter, \( u = \) Earth’s linear velocity, \( v = \) Jupiter’s linear velocity \( r \) and \( R \) the orbital radii of the Earth and Jupiter respectively. Let \( EE' \) and \( JJ' \) be perpendiculars to \( EJ \) so that when the relative velocity of Jupiter with respect to the Earth perpendicular to \( EJ \) is Zero, Jupiter will appear stationary as seen from the Earth. This

![Diagram](image)

Fig. 17

means that \( u \cos \theta + v \cos \xi = 0 \) 

\[ \therefore \quad \frac{u}{v} = \frac{-\cos \xi}{\cos \theta} \]  

But from triangle \( ESJ \),

\( R \cos \phi + K \cos \theta = r \)  
\( r \cos \phi + K \cos \xi = R \)

I

II

III and

IV
From III & IV \[ \frac{\cos \xi}{\cos \theta} = \frac{r \cos \phi - R}{R \cos \phi - r} \]

Equating \[ \frac{\cos \xi}{\cos \theta} \] from II and V

\[ - \frac{u}{v} = \frac{r \cos \phi - R}{R \cos \phi - r} \]
so that \[ \frac{ru + Rv}{rv + Ru} = \cos \phi \]

If \( m \) be the S'ighra anomaly \( m = 180 - \phi \)

so that \[ \cos m = \frac{(ru + Rv)}{(rv + Ru)} \] VI

Now we shall show that Bhāskara's formula accords with this \( \text{Spaṭagati} = \text{S'ighragati} - \frac{H \cos E \delta m}{K} \). As per Bhāskara

\[ \text{Spaṭagati} = 0 \text{ if } \text{S'ighragati} = \frac{H \cos E \delta m}{K} \] VII

ie. Jupiter appears stationary as seen from Earth, if \( \text{S'ighragati} = \frac{H \cos E \delta m}{K} \)

The angular velocities of Earth and Jupiter are respectively \( \frac{u}{r} \) and \( \frac{v}{R} \) so that the Sun's apparent velocity is also

\[ \frac{u}{r} \] and \( \delta m = \text{Kēndra gati} = \text{Sun's apparent velocity} \]

minus Jupiter's heliocentric velocity = \[ \frac{u}{r} - \frac{v}{R} \]

\[ \therefore \] Substituting in VII

\[ \text{S'ighragati} = \frac{u}{r} = \frac{H \cos E}{K} \left( \frac{u}{r} - \frac{v}{R} \right) \]

\[ \therefore \frac{u}{r} \left( \frac{H \cos E}{K} - 1 \right) = \frac{H \cos E}{K} \times \frac{v}{R} \]

ie. \[ \frac{u}{r} \left( \frac{H \cos E - K}{K} \right) = \frac{H \cos E}{K} \times \frac{v}{R} \]
But \( H \cos E = R \cos E = R \cos \xi \) (here) \( \xi \) standing for \( E \) and \( R \cos \xi - K = -r \cos \theta \)
so that \( \frac{u}{r} \otimes -r \cos \theta = u \cos \xi \)
\[ \therefore \quad u \cos \theta + u \cos \xi = 0 \]
This is equation I derived from modern methods, so that Bhāskara's formula accords with I and the rest follows as before i.e. \( \cos m = -\left(\frac{ru + Rv}{rv + Ru}\right) \)

Substituting the values of \( r \) and \( R \) for each of the planets and noting that \( r = \) epicyclic radius or Antyaphalajyā \( R = 3439' \), \( u = \) mean velocity of the Sun and \( v = \) mean velocity of the planet, we have the respective values of \( m \) when planets appear stationary or, what is the same we could more easily use equation I noting that \( \theta = m - \xi \) and thereby get the values of \( m \) for stationary values.

We shall give here some points of observation pertaining to Sīghraphala and Sīghragatiphala.

We have \( M_2 + E_2 = S \) I where \( M_2 = \) Mandasphutagraha and \( E_2 = \) the Sīghraphala where from we have \( \delta M_2 + \delta E_2 = \delta S \) II i.e. Mandasphutagati + Sīghragatiphala = Spastagati (a) Let \( E_2 \) be maximum so that \( \delta E_2 = 0 \), then \( \delta M_2 = \delta S \). This means that in the heliocentric figures 11 and 12, at the points \( a \) and \( b \), the Sīghraphala being maximum, the Mandasphutagati will be itself the Spashtagati. This line \( ab \), it will be seen corresponds with the so called "कष्णायथयमलियमेकविद्युतकर्तरेकस " i.e. the line cutting the eccentric, drawn through the centre of the deferent.

(b) The planet begins to retrograde only after the Spashtagati vanishes i.e. after \( \delta M_2 + \delta E_2 \) becomes Zero. Taking \( \delta M_2 \) to be almost a constant since the Mandagatiphala is small, the negative value of \( \delta E_2 \) must cancel \( \delta M_2 \) in order that the Spashtagati may be zero. \( \delta E_2 \) becomes
negative when the planet courses the arc of the smaller segment ab, because $E_2$ decreases from a maximum value to zero. As a matter of fact $\delta E_2$ negatively increases along ac and again increases from a negative minimum at c to zero at b as we course along cb (vide fig. 11 & 12). Thus the planet will assume zero velocity at two points symmetrical about c and in between ab. In other words the planet will be retrograde along the arc dc, cd, not entirely along ab as some have misconstrued. Regarding $E_2$, it is clear that it is zero at $S'$, then gradually increases to a maximum as the planet traces $S'a$, the maximum being assumed at a, then it decreases from the maximum to zero as the planet courses the arc ac, and then increases from zero to a maximum at b and finally decreases from that maximum to zero at $S'$ again. Keeping the Earth constant it will be noted that an Inferior planet always goes anticlockwise whereas a Superior planet always goes clockwise. Also it will be seen that the Sighraphala is positive as the planet courses the arc $S'ac$, whereas it is negative as it courses cb$S'$.

(c) The values of the Spashtagati at $S'$ and C will be respectively putting $H \cos E = R$ in the formula.

$$\text{Spashtagati} = S' \text{ighragati} - \frac{H \cos E \delta m}{K}.$$

Here for $S' \text{ighragati}$ we may put $U$ and $U - V$ for $\delta m$, and putting $K = R + r$ at $S'$ and $R - r$ at C, the values of the Spashtagati would be

$$\frac{U - R(U - V)}{R + r} \quad \text{and} \quad \frac{U - R(U - V)}{R - r} \quad \text{ie.} \quad \frac{RV + ru}{R + r} \quad \frac{RV - ru}{R - r}$$

respectively.

$$\frac{ru}{r} > \frac{RV - ru}{R - r} \quad \text{if} \quad R^2V - rRV + Rru - r^2U > R^2V - Rru + rRV - r^2U$$
ie. if \( rR(U - V) > Rr(V - U) \), \( U - V \) is \(+\) ve and equal to \( V - U \).

So the positive velocity at \( S' \) of the planet will be equal to its negative or retrograde velocity at \( C \). A quantity assuming values \( K, O, -K, O, K \) must have a value numerically less than \( K \) in between. Thus the velocity direct or retrograde at any point of the orbit is less than the numerical value of the velocity at \( S' \) and \( C \).

**Verse 41.** Retrograde motion.

The planets Mars, Mercury, Jupiter, Venus and Saturn will be retrograde when the \( S' \)ighra anomaly assumes values 163, 145, 125, 165 and 113 respectively and the direct motion again ensues at \((360 - 163), (360 - 145), (360 - 125), 360 - (165) \) and \((360 - 113)\) respectively.

**Comm.** Since we have had the formula VI

\[
\cos m = \left( \frac{rU + Rv}{rV + RU} \right)
\]

noting that \( \cos (180 - \theta) = -\cos \theta = \cos (180 + \theta) \) the stationary points are symmetrically situated with respect to \( S' \) of figures 11 and 12. As mentioned before substituting the values of \( U, V, r, R \) for all the planets we can prove the veracity of Bhāskara's statement.

**Verse 42.** Heliacal rising and setting of planets.

Mars rises heliacally in the East by \( 28^\circ \), Jupiter by \( 14^\circ \), Saturn by \( 17^\circ \) of \( S' \)ighra anomaly and set heliacally in the west by degrees which are the differences of the above and \( 360^\circ \) respectively.

**Comm.** The Sun's velocity being greater than that of the Superior planets, the Sun overtakes them so that they set in west and rise in the East. When these planets are
situated within particular limits from the Sun, they will be invisible in the rays of the Sun. As these superior planets will be near the Sun, near the moment of conjunction, they will not be seen at conjunction and within particular limits from the position of the Sun. The limits cited above mostly depend upon their respective brilliance and to some extent upon their distances from the Sun too. Their brilliance again depends upon the extent of their gibbosity. The formula given for the phase in modern astronomy is phase = \( \frac{1+\cos E_p S}{2} \) and noting that \( \widehat{E_p S} = E_a \) = the Sighraphala, phase = \( \frac{1+\cos E_a}{2} \). Hence the Superior planets are always gibbous i.e. the disc illuminated will be always greater than \( \frac{1}{2} \). Though at conjunction \( E_a = 0 \) and the entire discs of the major planets will be illuminated, we cannot see them as they are immersed in the rays of the Sun. As they emerge out of conjunction gradually \( E_a \) will be increasing so that the discs will be illuminated lesser and lesser gradually. But \( K \) decreasing, the brilliance will not be so much effected. Along the arc acb (figs. 11, 12) the planets gradually gain in illumination and will be brightest when they are in opposition both as \( \cos E \) increases and \( K \) decreases. In other words the Superior planets appear more and more brilliant when they are retrograding, being most brilliant at ‘C’. The spherical radii of Jupiter Saturn and Mars being in decreasing order, their brilliance will be in decreasing order so that they will be rising at distances from the Sun which are in increasing order. The inverse square law of courses works here but combining the two factors (1) the spherical radius of the planet and (2) the inverse square law, we may take it that Bhāskara’s numbers cited above namely 28°, 17° and 14° for Mars, Saturn and Jupiter respectively accord with truth, for, these numbers are given by Bhāskara or his authority Brahmagupta only after observing the planets rising heliacally.
That the positions of the planets while setting or rising heliacally will be situated symmetrically with respect to the Sun, goes without saying.

But one thing. In the chapter called Udayāstāmayādhyāya, the degrees known as Kālamsas which are given as the arcs between the planets and the Sun for heliacal rising or setting, are different from the numbers given above, for, the latter are the values of the Sīghra anomaly. In the case of Mars when the Sīghra anomaly is 28°, the Sīghraphala will be 11°, so that the apparent distance of the planet will be 28°−11°=17° which are the Kālamsas for Mars. Similarly in the case of Jupiter, when the Sīghra anomaly is 14°, the Sīghraphala will be 3°, so that the distance between the planet and the Sun as seen from the earth will be 11°, which are given as Jupiter’s Kālamasas. Also in the case of Saturn, the Sīghraphala for 17° of anomaly will be 2°, so that the Kālamsas would be 15°.

Verse 43. Mercury and Venus rise in the West by 50° and 24° of Sīghra anomaly respectively, and set in the West by 155°, and 177° respectively. They rise in the East by 205° and 183° of Sīghra anomaly and set there by 310° and 336° respectively.

Comm. Regarding the Inferior planets, they rise heliacally in the East after Inferior conjunction and then they are retrograde. They attain gradually the maximum elongation in the East and after they revert to direct motion, their elongation gradually decreases. They then set in the East and heliacally rise thereafter in the West. There again their elongation attains a maximum value; then they begin to retrograde and gradually set in the West only to rise in the East. This is all clear from the heliocentric figure 11. When the Sīghra anomalies of Mercury and Venus happen to be respectively 50° and 24°, their Sīghraphalas would be 13° and 11° respectively, so that they are themselves the Kālamsas, in as much as in
the case of the Inferior planets, the Sighraphala will be itself their elongation eastern or western. Then they rise in the West, being near Superior conjunction. When again their Sighra anomalies equal respectively 155° and 177°, the same Sighraphalas will arise so that they set heliacally in the West. Then as the Sighra anomalies attain the symmetrical values on the other side ie. (360-155) and (360-177) ie. 205° and 183° the Sighraphalas being the same, they rise in the East. Again when they attain the values (360-50) and (360-24) ie. 310° and 336°, they set in the East on account of the same Sighraphalas or Kalamsas.

Verse 44. When the Sighra anomalies have particular values, to decide when the planets rise or set heliacally, we have to take the difference of those particular values and the numbers given above for the respective Sighra anomalies, convert them into minutes of arc and divide the results by the daily motion in the Sighra anomalies in minutes of arc. Then we have the number of days in which the rising or setting takes place thereafter.

Comm. Suppose it is required when Mercury rises in the West. Suppose we want to compute this on a particular day when Mercury's Sighra anomaly is $x^\circ$. Then because we know that Mercury rises in the West when his Sighra anomaly is 50, we have to calculate by how many days the difference $|x-50|$ of the Sighra anomaly would be covered. Let the daily motion of the Sighra anomaly be $y^\prime$ per day. Then by rule of three $\frac{|x-50| \times 60}{y}$ will be the number of days before or after as the case may be for Mercury to rise in the West.

Verse 45. To obtain the Mean planet knowing the True.

Assume the True planet to be the Mean; compute the Manda and Sighraphalas and applying them inversely,
we have an approximation of the Mean planets. Treating these as the Mean planets, again obtaining the Manda and Stighraphalas and again applying them inversely and repeating the process till constant values are obtained, we have by this method of successive approximation the Mean planets required.

Comm. The method of successive approximation is clear.

Verse 46. To obtain the equinoctial shadow.

Convert the Ayanāmsas into minutes of arc, and divide by the mean daily motion of the Sun; then we have the number of days before the Māṣa or Tulā Samkrānti day, or before Makara and Karkataka Samkrānti days, when the Sun will be in equinoxes or Solstices respectively. The mid-day shadow of the Sun cast by the gnomon on such an equinoctial day, will give us the equinoctial shadow required.

Comm. The palabha or equinoctial shadow as it is called is the length of the shadow cast by a gnomon taken to be of 12 units in length, (measuring the shadow also in the same units), at noon of an equinoctial day. In other words, if this shadow be of $s$ units, clearly $\frac{s}{12} = \tan \phi$ (Vide fig. 18).

The Ayanāmsas are the degrees of the arc of the ecliptic in between the Hindu Zero point of the Zodiac and the first point of Aries which is now behind the former due to the phenomenon known as the precession of the equinoxes. They are called Ayanāmsas because the solstices are also behind the Makara and Karkataka Samkrānti points of the Hindu Zodiac or points which have Hindu longitudes 270° and 90° reply, by the same arc. The Hindu astro-
nomers came to know that the solstices are preceding by observations made with the gnomonic shadow at mid-day around the solstitial days. The day on which the maximum mid-day shadow is cast by the gnomon is the Winter solstitial day whereas the day on which the mid-day gnomonic shadow is maximum in the southern direction (assuming the place to be of northern latitude and $\phi < \omega$) or minimum in the northern direction ($\phi > \omega$) is the summer solstitial day. Calculating the Sun's longitude on that day at noon, we know how far the solstitial points have preceded behind the points of the Hindu Zodiac which have Hindu longitudes 90° and 270°. The word अयनविलोकनायः has therefore the meaning अयनविलोकनायः: where the Samāsa may be viewed as a

Verses 47, 48. Calculating the five fundamental $H$ sines of a point of the Zodiac pertaining to a point of the ecliptic.

The declination has to be computed from the sum of the Hindu longitude of that point and the Ayanamsas. Similarly if it be required to find the time before or after the rise of a point of the ecliptic, we have to compute them from the sum of the longitude of that point and the Ayanamsas.

$$\frac{H \sin 24^\circ \times H \sin \lambda}{R} = I$$

$$H \sin^2 \delta = H \cos \delta = Dyujyā \quad II$$

$$\delta = H \sin^{-1} (H \sin \delta) \quad III$$

$$\frac{s}{12} \times H \sin \delta = Kuujyā \quad IV$$

$$\times R$$

$$H \cos \delta$$

$$H \sin^{-1} (Charajyā) = Charam \quad VI$$

Comm. (1) Let $rA\tau$ be the ecliptic where $r =$ Vernal Equinox, A = first point of the Hindu Zodiac,
in between \( r \) and \( \odot \), not shown in the figure \( \odot = \) the Sun. Let \( rA = a^\circ \) = Ayanamsas defined before so that \( r \odot = (a + \lambda) \). From the spherical triangle \( r \odot M \), by Napier’s rule, 
\[
\sin \delta = (\sin \lambda + a) \sin \omega \]
which in Hindu trigonometry becomes \( H \sin \delta = \frac{H \sin (\lambda + a) \times \sin \omega}{R} \).

In Hindu Astronomy the obliquity of the ecliptic was taken to be \( 24^\circ \). The value of the obliquity is now \( 23^\circ - 27' \) approximately and it has been know that this has been decreasing. At the time when the Hindu Astronomers observed this, it should have been greater than \( 23^\circ - 27' \) so that if it was taken to be \( 24^\circ \), their observations were not far from truth. This means that the antiquity of Hindu Astronomy might be far more than what the Moderns estimate it to be.

(2) How the formula (1) was derived in Hindu trigonometry was as follows. Let \( \odot \) be the summer solstice \( \odot C \), \( \odot B \), the perpendiculars dropped from \( \odot \) and \( \odot \) on the line of intersection of the Equatorial and Ecliptic planes namely \( rBC \). Let perpendiculars be dropped from \( \odot \) and \( \odot \) on the plane of the Equator. Let them be \( \odot N \), \( \odot D \), Join NC and DB. Then \( \odot BD = \odot CN \) = the dihedral angle between the two planes = \( \omega \); \( C \odot = R \) since in Hindu trigonometry \( H \sin 90^\circ = R \). Also \( \odot N = H \sin \odot E \) (in Hindu trigonometry) = \( H \sin \omega \); \( \odot D = H \sin \odot M = H \sin \delta \); \( \odot B = H \sin r \odot = H \sin (\lambda + a) \) where \( \lambda \) is the Hindu longitude of the Sun and \( a \) = Ayanamsas. It will be seen that \( \odot D \) is a segment of the
line of intersection of the planes PCM a plane perpendicular to the plane of the Equator, and \( \odot BD \) a plane perpendicular to the Ecliptic plane. Since \( \odot C N \odot B \) and \( \odot N N \odot D \) and \( BD \odot = CN \odot = 90^\circ \), so the two triangles are congruent

\[
\therefore \frac{\odot B}{\odot C} = \frac{\odot D}{\odot N} \text{ ie. } H \sin (\lambda + a) = \frac{R \times H \sin \delta}{H \sin \omega} \\
\therefore H \sin \delta = \frac{H \sin (\lambda + a) H \sin \omega}{R}
\]

\(NC = \odot L = \sqrt{C\odot^2 - \odot N^2} = \sqrt{R^2 - H \sin^2 \delta} = H \cos \delta. \odot L \) is called Dyujyā as explained below in note (3).

Another way of looking at the similarity of the triangles \( C \odot N \) and \( B \odot D \) is from the fact that they are formed by the intersection of parallel planes, both of which are perpendicular to the plane of the Equator. This idea will be elaborated when we explain fig. 21, wherein the so-called latitudinal triangles will be shown to be formed by the intersection of the plane of the Equator and planes of diurnal circles with the planes of the horizon and the prime vertical. In fact, the plane of a great circle and the parallel planes of the corresponding small circles form the same dihedral angle with the planes of the celestial sphere namely the planes of the meridian, horizon and prime-vertical so that right-angled triangles formed by their intersection will be all similar.

Formula II is derived from the formula \( H \sin^2 \delta + H \cos^2 \delta = R^2 \) derived from fig. 6 from which formula, formula III follows.
(3) Why $H \cos \delta$ is called Dyujyā in formula II is clear from fig. 20 wherein $p$ is the celestial pole, $(C)$ is the celestial equator and $(C)$ is the diurnal circle of a celestial body say the ☉ i.e. the Sun. Let $\odot N$ be the perpendicular dropped from ☉ on the plane of the Equator so that $\odot N = H \sin \odot A = H \sin \delta \, CN = H \cos \delta = C \odot$ = radius of the diurnal circle called Dyujyā (Dy = day) युज्याः युज्याः (Madhyamapadalopī Samāsa) formulae IV, V and VI will be dealt with in the next chapter Triprassādhyāya more elaborately but one has to understand what Charajyā is to follow the subsequent verses of this chapter so that we shall give its location and definition in the light of fig. 21. Let fig. 21 represent the celestial sphere in which SEN = Horizon, $Z =$ Zenith, $N =$ Nadir $Ez = $ prime vertical $EQR =$ celestial equator,
\[ PP' = \text{Polar axis}, \quad \text{SBS'} = \text{the diurnal circle of a celestial body } S; \quad \text{EW} = \text{the East-West line}, \quad \text{SS'} = \text{Udayāstātra} \]
or the join of the rising and setting points \( S, S' \). This \( SS' \) is evidently a diameter of the diurnal circle which is bisected by the plane of the horizon; \( \text{PEP'} \) is called the unmandala or the Equatorial horizon. \( \text{PSA} \) is the declination circle of \( S \) cutting the Equator in \( A \). \( \text{EA} \) is called the charam whose \( H \) sine is called Charajyā. The \( H \) sine of \( SB \) in the diurnal circle is called Kujyā so that as corresponding lines in the diurnal circle and the Equator stand in the ratio \( H \cos \delta : R \),

\[
\frac{\text{Kujyā}}{\text{Charajyā}} = \frac{H \cos \delta}{R}
\]
In the beginning of the Triprāṣṇādhyāya, Bhāskara says "सासे देशे खगोळवस्त्रयान्ति, तिरंचा लगोळवस्त्रयान्ति अ स्वप्तात्त्वभाषाणि क्षेत्राणयुक्तां ता। ताभ्योऽस्मतं च इव।" ie. In a place having a latitude the diurnal paths of stars and planets will be inclined to the fundamental circles of the celestial sphere namely horizon, meridian and prime vertical and so their intersection gives rise to what are called latitudinal triangles, in which the angles would be $\phi$, $90-\phi$ and $90$ where $\phi$ is the latitude. Thus in fig. 21 the projections of the triangles ESB, EDB, DSB, EDF, EBF, ESF and EQG on the meridian plane will be all triangles in which the angles will be $\phi$, $90-\phi$ and $90^\circ$ so that they are all latitudinal triangles. These are all similar to the fundamental gnomonic triangle gmn of fig. 18 wherein also the angles are $\phi$, $90-\phi$ and $90^\circ$. Here, there is one important point to be observed. We have said "their projections are all similar". In fact the corresponding spherical triangles enumerated above are all apparently similar, though they are not viewed in modern astronomy as regular spherical triangles, because all the three sides of the triangles are not arcs of great circles. It is not possible to apply Napier’s rules to these triangles for the reason mentioned above. None the less, the property of similarity of the projected right angled triangles is made use of in Hindu Astronomy to obtain the magnitudes of the sides of the projected triangles.

EW is called the prāk-pratichī sūtra, SS’ the Udāyāsta sūtra; similarly if lines through F, B, D, A parallel to EW be drawn, these sūtras will intersect the meridian planes in points which constitute the projected triangles mentioned above. Let us study these triangles which we connote by the same letters with lowered indices. Thus for example $E_1S_1B_1$ is the projected triangle of ESB where of course $E_1$ will be the centre of the celestial sphere. The lines drawn parallel to SS’ or EW through B, D etc. will be denoted as BB’, DD’, FF’ etc.
In the triangle $E_1 S_1 B_1$, $E_1 S_1$ is called Agrajya, $S_1 B_1$ Kujya, and $E_1 B_1$ Kranitija. Of the sides of ESB, ES and EB are arcs of great circles whereas SB is the arc of a small circle. In the projected triangle $E_1 S_1 B_1$, $E_1 S_1$ will be equal to the perpendicular from $S$ on $E\omega$, which will be $H \sin (ES)$ and is called Agrajya; $E_1 B_1$ will be equal to the perpendicular from $B$ on $E\omega$ which will be $H \sin BE$ and as such called Kranitija. But $S_1 B_1$, which is equal to the perpendicular from $S_1$ on $BB'$ the diameter of the diurnal circle is the $H$ sine of $SB$ in the diurnal circle and is called Kujya. It will be noted that the perpendiculars from $S$ on $E\omega$, and $BB'$ and the perpendicular from $B$ on $E\omega$ do not form a triangle by themselves but by the theorem of three perpendiculars, if $SL$ be the perpendicular from $S$ on the plane of the unmandala ie. the great circle $EBP$, and if $LM$ be perpendicular from $L$ on $E\omega$, $SM$ will be perpendicular on $E\omega$. Here the perpendicular $SL$ will be the Kujya, and $SM$ the Agrajya, whereas $LM$ is not actually the $H$ sine of $BE$ but is equal and parallel to it.

Similarly take the spherical triangle SAE. This is a regular spherical triangle because the three sides are arcs of great circles. Napier's rules can be applied to this triangle and we have the formula $\sin SA = \sin SE \sin \hat{E}$ or in Hindu form $H \sin SA = \frac{H \sin SE \times H \sin \hat{E}}{R}$ or $RH \sin \delta = Agrajya \times H \cos \phi$ ie. $Agrajya = \frac{RH \sin \delta}{H \cos \phi}$ II

Again $\sin EA = \tan SA \times \tan \hat{E}$ where $H \sin EA$ is called Charajya and $\tan \hat{E} = \cot \phi$ so that $Charajya = \tan \delta \tan \phi$ in modern form, and the Hindu form is $R \tan \delta \tan \phi$. 
From \( \frac{H \cos \delta}{R} = \frac{Kujyā}{Charajyā} \) so that

\[ Kujyā = \frac{H \cos \delta}{R} \times R \tan \delta \tan \phi = H \sin \delta \tan \phi \] III

In Triprasnađhyāya, the elements of all the eight latitudinal triangles are found by using their similarity with the fundamental gnomic triangle. The important elements that will enter into computation are (a) Charajyā (b) Kujyā (c) Agrațyā (d) Taddhriti ie. S, F, the projection of SF on the plane of the meridian (e) Sama-S'anku = E, F, = H sin EF (f) Krāntijyā = E, B, = H sin EB = H sin δ (g) Lambajyā = H sin QS = H cos ZQ = H cos φ (h) Akshajya = H sin ZQ = H sin φ (i) B, D, = Ud-Vritha-S'anku = H cos ZB (j) Dinārđha-S'anku = H cos Zq. The following points will be noted.

(i) In the triangle E, Q G, the projected triangle of EQG on the meridian plane, noting that E, is the centre of the celestial sphere E, Q = R, QG, = H cos φ and E, G, = H sin ZQ = H sin φ.

(ii) \( S_{s1} = H \sin SB + H \sin fB \) (both the H sines pertaining to the diurnal circle.

(iii) Sama S'anku is the H cosine of the Zenith-distance when the Sun or celestial body is on the prime-vertical.

(iv) BD is an arc of the great circle ZB, so that the Unmandala S'anku is the H cosine of ZB.

(v) Dinārđha - S'anku = H cosine ZQ.

(vi) These S'ankus are the H sines of altitudes or H cosines of Zenith-distances and they are in the planes of the respective great circles.
(vii) Lambāmsa-chāpa is the arc of the colatitude QS where S is the South point so that Lambajyā is the H sine of QS or H cosine of ZQ. This Lambajyā will be seen to be the diameter of a small circle parallel to the Equator and passing through Z.


This chara can be had by the so-called chara-segments of the locality using a process similar to that of finding the H sines of the smaller table of nine H sines using \( \frac{\lambda}{3} \) where \( \lambda \) is the Sāyana longitude of the Sun (i.e. The Hindu longitude plus the arc of ayanāmsas is called the Sāyana longitude or the modern longitude measured from \( \nu \) along the ecliptic to the Sun). Find the charas of 30°, 60° and 90° of the ecliptic measured from \( \nu \). Subtract the first from the second, the second from the third. Thus we have C30°, C60°–C30°, C90°–C60°, (where Cθ° signifies the chara of θ°) which are called chara-Khandas or chara-segments. The equinoctial shadow multiplied by 10, 3, 3½ gives the approximate values of the chara-segments in Vinādis (a sidereal day is divided into 60 nādis and each nādi consists of 60 Vinādis). The chara-segments thus measured in Vinādis are rather approximate. If further exactitude is required, better take the arc in units each of which rises in \( \frac{1}{6} \)th of a Vinādi (This \( \frac{1}{6} \)th part of a Vinādi is known as a prāna i.e. the duration of the interval between two inhales of a healthy person reckoned as 4° of time).

Comm. The charas of 30°, 60° and 90° of modern longitude are the values of the arc EA, (Ref. Fig. 21) when S has longitudes 30°, 60° and 90°. We have the formula

\[ \text{Charajyā} = R \tan \delta \tan \phi \]
Taking the equinoctial shadow equal to 1 Angula means \( \tan \phi = \frac{1}{12} \). As charajyā is proportional to \( \tan \phi \), for any equinoctial shadow of \( s \) angulas, \( \tan \phi \) being equal to \( \frac{s}{12} \) the charajya got above is to be multiplied by \( s \) only to obtain the charajyā in any place where the equinoctial shadow is \( s \) angulas. Putting the modern longitudes equal to 30° & 60°, if the corresponding declinations be \( \delta_1 \), \( \delta_2 \) \( \sin \delta_1 = \sin 30 \sin \omega \), \( \sin \delta_2 = \sin 60 \sin \omega \). Taking \( \omega = 24° \) and applying logarithmic tables
\[
\log \sin \delta_1 = 9.6990 + 9.6093 = 9.3083 \text{ so that } \delta_1 = 11°-44' \\
\log \sin \delta_2 = 9.9375 + 9.6093 = 9.5468 \text{ so that } \delta_2 = 20°-38'
\]

Now from the formula for charajyā cited above viz. \( \text{H sine (chara)} = R \tan \delta \tan \phi \) or \( \text{sine (chara)} = \tan \phi \tan \delta \), putting \( \tan \phi = \frac{1}{12} \) and applying tables, using the values of \( \delta \) got above,

charajya for 30° = \( \frac{\tan 11°-44'}{12} \)

and charajyā for 60° = \( \frac{\tan 20°-38'}{12} \) so that

\[
\log (\text{sine chara}) = 9.3175-1.0792 \text{ for 30° and for 60°} \\
\log (\text{sine chara}) = 9.5758-1.0792
\]

\( \therefore \) Chara for 30° or \( C (30°) = 59' \) and \( C (60°) = 1°-48' \)

Converting these arcs into their rising times at the rate of 6' per Vinadi, we have \( C (30°) = 10 \), and \( C (60°) = 18 \)

Noting \( \delta_3 = \omega \), \( \sin (\text{chara}) \) for 90° = \( \frac{\tan 24°}{12} \) so that

\[
\log \sin (C 90°) = 9.6436 - 1.0792 = 8.5694 \text{ so that } \\
C (90°) = 2°-8' = 21\frac{1}{3} \text{ Vinādis.}
\]

Thus \( (C 30°) = 10, C (60°) - C (30°) = 18 - 10 = 8 \)
\( C (90°) - C (60°) = 21\frac{1}{3} - 18 = 3\frac{1}{3} \) so that the chara segments are respectively 10, 8, 3\( \frac{1}{3} \) as given by Bhāskara.
For a given place of equinoctial shadow equal to $s''$, we have to multiply 10, 8, 3$\frac{1}{2}$, by $s$, which will be the chara-segments for the place.

The meaning of the first half of the verse 49 is as follows.—Let the equinoctial shadow for a place be 3 Angulas. Then the chara-segments for that place are 30, 24, 10. These are three in number for a longitude $\lambda$ of 90°. If the longitude be 44° (say) then proceed as we have done to find $\sin 24^\circ$, using the method of Bhogya Khanda Sphutikarana with respect to the table of $H$ sines namely 21, 20, 19, 17, 15, 12, 9, 5, 2. Proceeding as directed the Sphuta Bhogya Khanda for 14° is $\left(\frac{3 \times 14}{30} = \frac{7}{5}; 30 - \frac{7}{5} = 28\frac{2}{3}\right)$

\[
\frac{14 \times 28\frac{2}{3}}{30} = \frac{2002}{150} = 13\frac{1}{3}; \quad 30 + 13\frac{1}{3} =
\]

Proceeding according to the modern formula we have

Charajya = $\tan \delta \tan \phi$ where $\phi = \frac{3}{12}$, and $\sin \delta = \sin 44^\circ \sin 24^\circ$

$\log \sin \delta = 9.8418 + 9.6093 = 9.4511; \delta = 16^\circ-25$

$\log \tan \delta = 9.4693$\quad \therefore \quad \log \sin (\text{chara}) = 9.4693 + \log \tan \phi$

$= 9.4693 + \log \frac{3}{12} = 9.4693 - .6021 = 8.8672$

\therefore \quad \text{Chara} = 4^\circ-14' = \frac{254}{6} = 42\frac{2}{3}$ Vinadis whereas we have got by the Hindu method 43$\frac{1}{3}$ which is near the truth.

Verse 52. To find the durations of day and night.

Fifteen ghatis increased or decreased by the Charanadis, according as the Sun is in the northern hemisphere or southern, gives half the day of the locality and the difference of 30 nādis and the above half-day gives half the duration of night.

Comm. Ref. fig. 21. Let $S$ be the Sun rising in the hemisphere when his declination is north. Then
from the figure $\hat{APQ}$ is the rising hour-angle $= \hat{APE} + \hat{EPQ} = \hat{APE} + 90^\circ$ where $90^\circ$ correspond to 15 nādis. So we have to add $\hat{APE}$ expressed in nādis equal to the rising time of $AE$. Let us find the duration of the day for the place where $s = 3''$ and when $\lambda$ of $s = 44^\circ$; the latitude of the place will be $14^\circ$ (from tables) when $\lambda 44^\circ$, we have found above that 42–20 Vīnadīs is the chara expressed in time. Hence half the day $= 15–42–20$ or duration of day $31–25$; duration of night $= 23–35$.

Note (1) In modern astronomy we have the formula $\cos h = - \tan \phi \tan \delta$ where $h$ is the rising hour angle. Putting $h = H + 90^\circ$ where $H$ stands for the arc $EA$ of fig. 21 $\cos (90 + H) = - \sin h = - \tan \phi \tan \delta$ so that $\sin H = \tan \phi \tan \delta$ ie. $H \sin (\text{Chara}) = R \tan \phi \tan \delta$ which accords with the formula found before. The word chara used for $EA$ means etymologically रविस्वायर्यवेशन दिन-विकार: i.e. the variation in 15 ghatis of the equinoctial half-day on account of the Sun’s variation in declination,

(2) If $\phi = 0$, Chara $= 0$ so that duration of half-day is 15 ghatis ie. on the terrestrial equator, whatever be the Sun’s declination, the length of the day will be always 12 hours.

(3) Let $\delta = 0$ so that chara $= 0$ ie. whatever be the latitude (provided $\phi > 90-\delta$ as we shall see shortly). the day and night will be each of 12 hrs.

(4) Let $\delta$ be negative, so that the arc connoting chaan i.e. $EA$ will be above the horizon, and consequently the duration of half-day will be less than 6 hrs. by the time that is taken for $EA$ to rise. This can be seen otherwise also as $\cos h = - \tan \phi \tan \delta = + \ve$ so that $h < 90^\circ$ which means half-day is less than 6 hrs.

(5) We could also treat the case when $\phi$ is negative, but as this case is not in the purview of Hindu Astronomers who had only India in their mind and as such were concerned primarily with positive latitudes. If, however, we consider a negative latitude, when $\delta$ is $+ \ve$, $\cos h$ will be positive and if $\delta$ is $- \ve$, $\cos h$ will be negative. This means that when the Sun is in northern latitudes, the southern latitudes will have their day less than 12 hours and when the Sun is in southern latitudes, their day will be greater than 12 hrs.
(6) Let $\phi + \delta = 90^\circ$. (Ref. fig. 22) i.e. imagine the Sun to rise at N, the north point so that $RN + NP = \delta + \phi = 90^\circ$. Then the Sun's diurnal path will be entirely above the horizon, which means that what is called 'perpetual day' begins for that place on that day and lasts as long as $\delta > 90 - \phi$. For the same place, let $\delta = -(90 - \phi) = QS$. In this case the Sun sets at S, and what is called perpetual night begins and lasts till the southern declination of the Sun is greater than $90 - \phi$. The duration of perpetual day can be found as follows which applies to the perpetual night as well.

$$\delta = 90 - \phi$$

(Ref. fig. 23). Let A be the point at which the declination is $90 - \phi$; let S be the summer solstice and let B be the point where again the declination is equal to $90 - \phi$. So long as the Sun traces the arc $AB = 2AS =$
(90 − ρA), there will be perpetual day. But \( \sin \delta = \sin \lambda \sin \omega \) so that \( \sin \lambda \)

\[
\frac{\sin \delta}{\sin \omega}.
\]

Putting \( \delta = 90 - \phi \), \( \sin \lambda = \frac{\cos \phi}{\sin \omega} \);

\[
\sin \lambda = \sin \gamma A = \cos \alpha S \quad \therefore \quad \cos \alpha S = \frac{\cos \phi}{\sin \omega}.
\]

\[
\therefore \quad 2 \alpha S = 2 \cos^{-1}\left(\frac{\cos \phi}{\sin \omega}\right). \quad \text{Supposing } \alpha S \text{ expressed in degrees and assuming the Sun goes along the ecliptic with uniform motion, since he takes 365}\frac{1}{2} \text{ days to trace 360°, to trace 2 } \alpha S, \text{ he takes } \frac{2 \alpha S}{360} \times 365\frac{1}{2} \text{ days} = \frac{365\frac{1}{2}}{180} \times \cos^{-1}\left(\frac{\cos \phi}{\sin \omega}\right) \text{ which is the length of the perpetual day.}
\]

Verse 52. The correction known as Chara.

The daily motion of the planet being multiplied by the chara expressed in asus and divided by the asus in a day viz. 21659 and the result being subtracted from or added to the planetary position at Sunrise according as the Sun is in the northern or southern hemisphere. The result is to be added to or subtracted from the planetary position at Sunset.

Comm. The mean planets computed hold good at the Sun-rise at Lanka i.e. at zero latitude; they have to be converted to hold good at the local Sun-rise. In other words in fig. 21, the mean planet computed is B which is on the Lanka horizon, whereas we have to get S, the same planet on the local horizon. The position of S is earlier than that of B by the time interval indicated by the arc SB which is measured by EA the chara because the position B on the Lanka horizon is later than the position S on the local horizon. If the mean planet moves in a day of 21659 asus by the arc denoting its daily mean motion, by how much does it move in the time of chara expressed in asus? The result is

\[
\text{Chara in Asus } \times \text{ mean daily motion of } 21659
\]
This result is to be subtracted from the position of B to get the position of S. If the position B' which is the setting position at Lanka, and if we have to get S' the local setting position, in as much S' is later than B' by the same arc, we have to add the above result to the position B' to get S'. Here the asus in a day is given to be 21659 and not $60 \times 60 \times 6 = 21600$ because the mean daily motion is during the course of the Sāvana day i.e. the true solar day and not during the course of a sidereal day of 21600 asus. The Sāvana day exceeds the sidereal by the time taken by the arc moved by the Sun during that day which is very approximately 59'. Since a minute of arc of the Equator rises in an asu i.e. in 4", so 59' of the Sun's motion is covered in 59 asus which is to be added to 21600 asus of the sidereal day to get the Sāvana day approximately. It will be noted that this correction of chara in the planetary positions is due to latitude of the place and if the latitude is zero, it need not be done. Also if $\delta = 0$, it need not be done, for, then, the Sun or the celestial body whose $\delta = 0$ will be rising at E itself in which case the chara E A will be zero.

Verses 54, 55. The H sines of 30°, 60° and 90°, being squared and decreased by the squares of their respective declinations, the square-root of the differences being taken, and the result being multiplied by the radius, is to be divided by the respective H cosines of the declination. The first of the three results, the difference of the second and the first and the difference of the third and the second will give us the rising times of what are called the Sāvana basis of Mesha, Vrishabha and Mithuna; their reverses will then give the rising times of the next three; then the original ones those of the next three and again then reverses those of the last three.

Comm. There are twelve Rasis in the Zodiac of equal interval. Measuring from $r$, and taking aros of the
ecliptic successively each of 30°, we have what are called Sāyana Rāsis or Rāsis taking the Ayanāmsa into consideration or in other words measuring from \( r \). On the other hand successive arcs each of 30° measured from the zero point of the Hindu Zodiac, constitute what are called the Nirayana Rāsis. The words Meṣa, Vṛiṣabha etc. signifying the shapes of the constellations apply strictly to the Nirayana Rāsis. But nonetheless, by the convention what is called Upachāra, we name the Sāyana Rāsis also by the same names. In as much as the zero-points of the Hindu Zodiacs is ahead of the modern zero-point by an arc which is the arc of Ayanamsa or accumulated precession, the words Sāyana and Nirayana came into vogue. Once upon a time approximately in Varāha’s time or rather 499 A.D, the two zero-points coincided and then the Sāyana and Nirayana Rāsis were the same. Gradually on account of the phenomenon of precession \( r \) preceded, and today the distance between \( r \) and the Hindu zero-point is about 20°-30°. Of late, there has been a big controversy as to what exactly the arc is and the Calendar reform committee has adopted a value far more than what could be justified. The present author opines, that we have no right to set aside a statement of no less an astronomer than Varāhamihira who stated explicitly ‘अत्राविवारात् दिक्षिणमुत्सरसयं र्वेषविणङ्गायम्, नूनं कर्तव्यचारिस् बैन्द्रेकथे विशेषतः, सामतं सहितं कर्तव्यकार्य समाधिनायात्मुत्तल उत्सामाय: विस्तितः प्रकृतपरीक्षणवैज्ञानिकः’ i.e. True it is that once upon a time, the Sun began his southern journey when he was mid-way the constellation of Asleṣa and his northern when he was at the beginning of Dhanistiha, because it was stated in ancient texts. (The allusion is to Vedāṅga Jyotīṣa where it was stated as such); but now the southern journey of the Sun begins from the point marking \( \frac{3}{4} \) of the constellation of punarvasu which is the beginning point of the Karkata Rāsi and his norther from the beginning of Makara i.e. from the point marking \( \frac{1}{4} \)th of the constellation of Uttarāṣadha; there has been a change from what was
stated in the ancient texts; let people verify this by actual observation". Accepting an Ayanāmśa which goes against the statement of this great astronomer, who said that he observed and called upon others to observe, is really unwarranted, especially when the adopted Ayanāmśa of the Calendar reform committee goes on the basis of surmises and consensus. We shall deal with this topic in further detail in an appendix to this work, because it is a really important issue and has been wrongly solved. The importance of knowing the exact value of the arc is clear when we observe that from the correctly computed planetary positions of modern astronomy this ayanāmśa is being subtracted to give the positions measured from the Zero-point of the Hindu Zodiac. An error in the Ayanāmśa therefore vitiates the positions obtained by the above method.

Coming to the point, the problem on hand is to obtain the rising times of the Sāyana Rāśis at a place of zero latitude i.e. what are called Lankōdaya times of Sāyana Rāśis (Ref. fig. 21). Let $rS = 30^\circ$ so that $rA$ the equatorial arc gives the rising time of $rS$. We could have the magnitude of this arc by the formula derived from Napier's rules namely \[ \cos \omega = \tan \alpha \tan \delta \] I.

But as in Hindu trigonometry the tangent functions of angles are not used, Bhāskara gave the following formula

\[ \sin \alpha = \frac{\sqrt{\sin^2 \lambda - \sin^2 \delta}}{\cos \delta} \] II. This formula could be derived easily from formula I and the formulae \[ \cos \lambda = \cos \alpha \cos \delta \] III and \[ \sin \delta = \sin \lambda \sin \omega \] IV; for multiplying the right-hand sides of I and III we have \[ \cos \omega \cos \lambda = \frac{\sin \alpha \cos \delta}{\tan \lambda} \] i.e. \[ \sin \lambda \cos \omega = \cos \delta \sin \alpha \]

Thus \[ \sin \alpha = \frac{\sin \lambda \cos \omega}{\cos \delta} \] V. But from IV \[ \sin \omega = \]
\[
\frac{\sin \delta}{\sin \lambda} \text{ so that } \cos \omega = \sqrt{1 - \frac{\sin^2 \delta}{\sin^2 \lambda}} = \frac{1}{\sin \lambda} \sqrt{\sin^2 \lambda - \sin^2 \delta}.
\]
Substituting in \( V \) for \( \cos \omega \), we have
\[
H \sin \alpha = R \frac{\sqrt{H \sin^2 \lambda - H \sin^2 \delta}}{H \cos \delta} \sin \alpha = \sin \lambda \times \frac{1}{\sin \lambda} \frac{\sqrt{\sin^2 \lambda - \sin^2 \delta}}{\cos \delta} \cos\delta - \sin^2 \cos \delta
\]
\( H \cos \delta \) as stated.

Fig. 24

But let us see how this formula was derived by the Hindu astronomers. (Ref. fig. 24). Let \( rS \) be an arc of the ecliptic equal to \( \lambda \). Let \( SM \) be dropped perpendicular from \( S \) on the plane of the Equator so that \( SM = H \sin \delta \).
From M drop a perpendicular ML on \( r = \) so that SL will be perpendicular from S on \( r = \) by the theorem of three perpendiculars. \( SL = H \sin \lambda. \) Also ML will be equal to the perpendicular from S on the diameter of the diurnal circle of S parallel to \( r = \) so that \( ML = SN = H \) sine of the arc in the diurnal circle corresponding to \( Kr \)

\[
\therefore \quad SW^2 = SL^2 - LN^2 = H \sin^2 \lambda - H \sin^3 \delta
\]

\[
\therefore \quad SN = \frac{\sqrt{H \sin^2 \lambda - H \sin^3 \delta}}{H \cos \delta}
\]

\[
\therefore \quad \text{The length of the perpendicular from K on } r = = \quad R \times \frac{\sqrt{H \sin^2 \lambda - H \sin^3 \delta}}{H \cos \delta} \quad \text{since corresponding lines of the diurnal circle and the equator stand in the ratio of } H \cos \delta : R \quad \text{(Vide fig. 20).} \quad \text{But this } \perp \text{ is } H \sin \alpha = \quad R \times \frac{\sqrt{H \sin^2 \gamma - H \sin^3 \delta}}{H \cos \delta}.
\]

If \( \alpha_1, \alpha_2, \alpha_3 \) be the Right ascensions of the points on the ecliptic whose modern longitudes are 30°, 60° and 90°, expressed in asus, then \( \alpha_3 - \alpha_1, \alpha_3 - \alpha_2 \) will give the rising times of the arcs of the ecliptic which stand for Sāyana Meṣa, Sāyana Vriṣabha and Sāyana Mithuna. The rising times of the next three Rasis will be the same in reverse order since Karkata is symmetric with Mithuna with respect to the Equator and similarly Simha and Kanya symmetric with Vriṣabha and Meṣa. The next three are again symmetric with Meṣa, Vriṣabha and Mithuna and the last three with Mithuna, Vriṣabha and Meṣa.

**Verse 56.** The \( H \) cosines of the ends of the Rasis Karkata etc., being multiplied by the radius, and divided by the \( H \) cosines of their respective declinations, and the arcs of those \( H \) cosines being taken, subtract as before the preceding from the succeeding. Then we have the rising times of the Rasis beginning with Karkataka.
Comm. This is clear from fig. 24. If $S$ be the end of Vriṣabha, $S'$ in the figure denotes the end of Karkata them from the right-angled triangle SAP,

$$\sin S'\Lambda = \sin S\Lambda = \sin \hat{APS} \times \sin PS = \sin \Lambda'K \times \cos SK$$

$$\therefore \sin \Lambda'K = \frac{\sin S'\Lambda}{\cos SK}.$$ But $\sin S'\Lambda = \cos S'$ and $\cos SK = \cos \delta$ where $\delta$ is the declination at $S$.

$\Lambda'K$ converted into time gives the rising time of $AS'$ ie. Karkata.

$$\therefore \text{the rising time of Karkata} = \frac{\cos S'}{\cos \delta}$$

In the Hindu form, it will be

$$Karkata-\text{Rāsi}-Udayakāla = \frac{Karkata-\text{anta}-\text{Kotijā}}{Karkata-\text{anta}-\text{Dyujyā}} \times R$$

as stated.

In modern terms $\sin \Lambda'K = \cos rK = \frac{\cos rS}{\cos \delta}$ from the formula $\cos \lambda = \cos \alpha \cos \delta$. Thus, virtually the formula is a statement of the formula $\cos \lambda = \cos \alpha \cos \delta$. Also the formula $\sin AS' = \sin \hat{P} \sin PS$ is parallel to the formula $\sin \delta = \sin \lambda \sin \omega$ which we proved already from Hindu methods.

Verse 57. Still an alternative method.

The $H$ sines of Mēṣa etc. being multiplied by $H \cos \omega$ divided by their respective $H \cos \delta$'s and the arcs thereof being subtracted as before the preceding from the succeeding we have the rising times of Mesha etc.

Comm. From figure 24,

$$SN = SL \cos LSN$$

and $\frac{S\Lambda}{\cos SK} = \text{perpendicular from } K \text{ on}$
\[
\frac{SL \cos \omega}{\cos \delta} = H \sin rK. \quad SL = H \sin rS
\]
\[
\therefore \quad H \sin rK = \frac{H \sin rS \times H \cos \omega}{H \cos \delta}
\]

We proved the above in a modern way. The Hindu concept is derived from the similarity of SML and ACM', where C is the centre of the sphere and M' is the foot of the perpendicular from A on the plane of the equator.

\[
\therefore \quad \frac{LM}{CM'} = \frac{SL}{CA} \quad \therefore \quad LM = \frac{H \cos \omega \times H \sin rS}{R}
\]

Since LM = SN.

LM divided by \(H \cos \delta\) and multiplied by R gives \(H \sin rK\).

\[
\therefore \quad H \sin rK = \frac{H \cos \omega \times H \sin rS}{R} \times \frac{R}{H \cos \delta} = \frac{H \cos rS \times H \cos \omega}{H \cos \delta}
\]

i.e. \(H \sin \alpha = \frac{H \sin \lambda \times H \cos \omega}{H}\)

Here \(H \cos \omega\) is called Trig̽ha-dyu-maurvi because it is the \(H\) cosine of the declination of \(\lambda\) when \(\lambda = 90^\circ\).

Verses 58, 59. The magnitudes of the rising times.

Those rising times are 1670, 1793, 1987; these in the same and reverse orders diminished or increased by their respective Chara segments which are also in the same and reverse orders give the rising times of the Sayana Rasis beginning from Meṣa for the locality. The Rasis from Tulā are in a reverse direction i.e. as the Meṣa is projecting upwards above the horizon, Tulā will be projecting below the horizon so that, the time taken by Meṣa to rise is exactly the time taken by Tula to set.

Comm. We shall compute the rising times of Sayana Rasis for Lanka first i.e. for zero latitude using modern methods from the formula \(\tan \alpha = \cos \omega \tan \lambda\)
\[ \log \tan \alpha = \log \cos \omega + \log \tan \lambda; \] Put \( \lambda_1 = 30, \text{ and } \lambda_2 = 60 \) and take \( \omega = 24^\circ \); Let the corresponding \( \alpha \)'s be \( \alpha_1, \alpha_2 \)

\[ \log \tan \alpha_1 = \log \cos 24^\circ + \log \tan 30^\circ \quad (1) \]
\[ \log \tan \alpha_2 = \log \cos 24^\circ + \log \tan 60^\circ \quad (2) \]
\[ \log \tan \alpha_1 = 9.9607 + 9.7614 = 9.7221 \]
\[ \therefore \quad \alpha_1 = 27^\circ - 48' \]
\[ \log \tan \alpha_2 = 9.9607 + 10.2386 = 10.1993 \]
\[ \therefore \quad \alpha_2 = 57^\circ - 42' \]

At the rate of 1 asu for 1', \( \alpha_1 = 1668 \text{ asus } \alpha_2 = 3462 \); \( \alpha_2 - \alpha_1 = 1794 \) and since \( \alpha_3 = \) the right ascension of 90° Longitude = 90°, \( \alpha_3 = 5400 \) so that \( \alpha_3 - \alpha_3 = 1938. \) These are given by Bhāskara as 1670, 1793, 1937, the first exceeding by 2 asus, the second and third each less by one asu and the total according with the total. The rising times of Karkata etc. will be 1937, 1793, 1670, 1670, 1793, 1937, 1937, 1793, 1670 respectively.

Let us then find the rising times of these Śāyāna Rasis at a locality say of latitude 13°. Refer to fig. 21. Let \( rS \) represent Meṣa so that the rising times of \( rE \) is equal to that of \( rS. \) But \( rE = rA - AE. \) We have seen \( rA = 1670 \) using Bhāskara's value. \( \sin EA = \tan 13^\circ \tan \delta_1 \) where \( \delta_1 \) is the declination of \( S \) where \( rS = 30^\circ. \)
\( \sin \delta_1 = \sin 30^\circ \sin 24^\circ. \)

\[ \log \sin \delta_1 = 9.6990 + 9.6093 = 9.3083 \]
\[ \therefore \quad \delta_1 = 11^\circ - 44' \]

\[ \therefore \quad \log \sin EA = \log \tan 13^\circ + \log \tan 11^\circ - 44 = 
9.3634 + 9.3175 = 8.6809 \]
\[ \therefore \quad EA = 2^\circ - 45' \quad \therefore \quad rE = 1670 - 165 = 1505 \text{ asus.} \]

Similarly for \( \lambda = 60^\circ, \) putting \( \alpha_2, \delta_2 \) in the place of \( \alpha_1, \delta_1 \) and proceeding as before \( rE = rA_1 - A_1E; \) \( rA_1 = 3463; \) \( \sin EA_1 = \tan 13^\circ \tan \delta_2 \sin \delta_3 = \sin 60^\circ \sin 24^\circ. \)
\[ \log \sin \delta_2 = 9.6093 + 9.9375 = 9.5468 \]
\[ \therefore \delta_2 = 20^\circ - 30' \]
\[ \log \sin EA_1 = 9.3634 + 9.5758 = 8.9392 \]
\[ \therefore EA_1 = 4^\circ - 59' = 299' \]
\[ rE = 3463 - 299 = 3164 \text{ asus} \]
\[ \therefore \text{Rising time of Sayana Vri\textbf{\textsc{s}}abha for the locality} = 3164 - 1505 = 1659 \text{ asus}. \]
\[ rA_1 = 5400, \sin EA_2 = \tan 13 \tan \delta_2 = \tan 13 \tan 24^\circ. \]
\[ \therefore \log \sin EA_2 = 9.3634 + 9.6486 = 9.0120 \]
\[ \therefore EA_2 = 5^\circ - 54' = 354 \text{ asus} \]
\[ rE = 5400 - 354 = 5046 \]
\[ \therefore \text{Rising time of Mithuna is } 5046 - 3164 = 1882 \text{ asus}. \]

Before we proceed to find the rising times of Karkataka, Simha and Kanyak, we shall cast our previous procedure into the Hindu form. In fig. 21, let \( rS \) be the Sayana Mesha. The rising time of \( rS \) is measured by \( rE \), because when \( r \) is at E, Me\( \text{s}a \) is just about to rise and when \( r \) is in the position indicated, the extremity of Me\( \text{s}a \) namely S is rising. So it means that as \( rS \) of the ecliptic has risen, a portion \( rE \) of the Equator has risen. As time is measured by the arc of the equator which rises with a uniform speed, we measure the rising time of \( rS \) by the arc \( rE \); but \( rE = rA - AE \). \( rA \) is the Equatorial rising time of \( rS \), because when A is at E, S will be at B i.e. A and S will then be on the equatorial horizon EB simultaneously. Hence \( rA = 1670 \) as proved before and stated by Bh\( \text{\textsc{s}}\text{\textsc{k}}\text{\textsc{a}}\)\( \text{\textsc{r}}\). \( EA \) is the chara for 30°. The chara for one angula or inch (inch is here used technically, and does not mean what it means in ordinary parlour) as has been stated by Bh\( \text{\textsc{k}}\text{\textsc{s}}\text{\textsc{a}}\text{\textsc{r}}\)\( \text{\textsc{a}}\) and proved by us is 10 Vinadis. (Vide page 181)
But we have taken $13^\circ$ as our latitude, so that $\tan \phi = .2309$. Since $\frac{g}{12} = \tan \phi = .2309 \therefore s = 2.7708$; let us take this as $2.8''$ so that the chars 10, 8, 3$\frac{1}{5}$ found for one inch are to be multiplied by 2.8 to give the local chars. They are in Vinādis, 28, 22.4, 9$\frac{1}{5}$ or in asus 168, 134.4, 56; for convenience let us take 134.4 as 134, so that the chars are 168, 134, 56. Thus the rising time of Sāyana Meṣa at this locality is $1670 - 168 = 1502$ asus i.e. 250 Vinādis $= 4-10$ Nādis. Then let $rS$ now represent $60^\circ$, instead of subtracting $EA$ from $rA$ to get the combined rising time of Meṣa and Vṛiṣabha, the Hindu practice is to subtract the chara pertaining to Vṛiṣabha from the equatorial rising of Vrishabha i.e. $rE = 1793 - 134 = 1659$ as got before. Here it must be noted that $EA^\circ$ is the chara not pertaining to Vṛiṣabha alone but to Meṣa and Vṛiṣabha put together. That is why for ease, the chara to Meṣa, the increase in chara for Vṛiṣabha, and the increase in chara for Mithuna as well as their individual equatorial rising times are given. The increments in the chars are called chara-khandas just as the increments in the $H$ sines are called Jyā-khandas (khandas means segments). Similarly the rising time of Sāyana Mithuna is equal to $1937 - 56 = 1881$ asus $= 313.5$ Vinadis $= 5-14$ Nadis. Now with respect to Karkataka, its equatorial rising time is 1793, for, from fig. 25, the equator at the equatorial place being prime Vertical, if $rM$ be Meṣa, its time of rising is given by $rE$ where $E$ is the foot of the declination circle of $M$. Similarly if $MV$ represents Vṛiṣabha, when $V$ comes to the horizon, $N$ the foot of the declination circle comes to the horizon. Thus the rising time of any arc of the ecliptic at an equatorial place is given by the corresponding arc of the equator, which is intercepted between the declination circles of the ends of the arc. So from fig. 26 if $r ed$ be the equator, $rED$ = the ecliptic, $A$ the Ayana or Summer solstice, $P$ the pole $rE$, $ED$, $DA$ etc. the Sāyana Rasis Meṣa etc. a, b, c etc. the feet of the
declination circles of A, B, C etc., since PAE is secondary both to the ecliptic and equator (i.e. perpendicular circle) spherical triangles PAD, PAB are congruent; PBC, PDE are congruent and PC = is congruent with PE r. Hence ab = ad i.e. rising times of Karkataka and Mithuna are equal; bc = de i.e. those of Simha and Vriśabha are
equal and similarly those of Meṣa and Kanya. It will be noted that the equatorial risings alone are equal in the above cases but not at any other place, for in a place with some latitude when a point like B (fig. 26) comes to the horizon, the foot of its declination circle namely b will not be on the horizon and there arises the chara in between, which has to be taken into account, and be subtracted from or added to the equatorial rising time as the case may be.

Now regarding the chara-khanda of Karkata it is again 56; why it should be so is not proved by Bhāskara but merely stated, nor any commentator took the pains to prove. It can be proved as follows. Let in fig. 26, \( \delta_1, \delta_2, \delta_3 \) be the declinations at the ends of Vṛisabha, Mithuna and Karkata respectively. We know \( \delta_1 = \delta_2 \).

The chara-segments for Mithuna and Karkata i.e. when Mithuna and Karkata are rising are to be proved to be equal, here 56 asus. Their expressions are \( \tan \phi \tan \delta_2 - \tan \phi \tan \delta_1 \) and \( \tan \phi \tan \delta_3 - \tan \phi \tan \delta_2 \) i.e. \( \tan \phi (\tan \delta_3 - \tan \delta_1) \) and \( \tan \phi (\tan \delta_3 - \tan \delta_2) \). Since \( \delta_1 = \delta_2 \) we perceive that they are equal but of opposite signs. So Bhāskara says rightly "अवचियमानत्रात चन्द्रम" i.e. because of negative sign, the chara-segment of Karkata while being subtracted will be rendered positive. Hence the rising times of Karkata, Simha and Kanya will be respectively 1937+56, 1793+134, 1670+168 asus or 1993, 1927, 1838 asus or 332, 321, 306 Vinadis or 5-32, 5-21 and 5-6 nadis. Thus, in as much as the rising times of Meṣa to Kanya are 1670-168, 1793-134, 1937-56, 1937+56, 1793+134, 1670+168 their total is 30 nadis as should be expected because the equator bisects the ecliptic between \( r \) and \( = \) and the equatorial interval between \( r \) and \( = \) is 30 nadis. It will be noted that while the equatorial rising times of Meṣa to Kanya are symmetrical as 1670, 1793, 1937, 1937, 1793, 1670, their rising times at any other place are not like that but

We have now to comment upon the statement “तुलादिलोकोमथा च नितोमस्तथा:” which means that the rising times from Tula to Mina are in the reverse order i.e. the rising time of Tula equals that of Kanya; that of Vrischika equals that of Simha and so on the rising time of Mina equalling that of Meṣa. Thus the rising times of Tula to Mina being in the reverse order are 1670+168, 1793+134, 1937+55, 1937—55, 1793—134, 1670—168 for the aforesaid locality. Why it should be so can be easily seen from the fact that Kanya and Tula are symmetric with respect to the line \( r = \) which bisects the ecliptic (Ref. fig. 27). Or again we can see this in another way; the chara-segments are successively \((\tan \phi \tan \delta_1 - \tan \phi \tan 0), (\tan \phi \tan \delta_2 - \tan \phi \tan \delta_1), (\tan \phi \tan \omega - \tan \phi \tan \delta_2), (\tan \phi \tan \delta_3 - \tan \phi \tan \omega), \tan \phi \tan \delta_1 - \tan \phi \tan \delta_2 (\tan \phi \tan 0 - \tan \phi \tan \delta_1), (\tan \phi \tan \delta_1 - \tan \phi \tan 0), (\tan \phi \tan \delta_3 - \tan \phi \tan \delta_1), (\tan \phi \tan \omega - \tan \phi \tan \delta_3), (\tan \phi \tan \delta_2 - \tan \phi \tan \omega) (\tan \phi \tan \delta_1 - \tan \phi \tan \delta_2), (\tan \phi \tan 0 - \tan \phi \tan \delta_1)\) where \(\delta_1 = \) declination of 30\(^o\), and \(\delta_2\) that of 60\(^o\). These are there as found before 56, 134, 168,—168,—134,—56 upto Kanya. But the remaining, though apparently are 56, 134, 168, —168,—134,—56 must be taken with a reverse sign because the rising point of the ecliptic will be to the south of the east point send \(\delta\) will be negative from 180\(^o\) to 360\(^o\) longitude. Hence the chara Segments are 56, 134, 168, —168, —134,—56,—56,—134,—168, 168, 134, 56 so that from Tula onwards they are in the reverse order as Bhāskara.
has stated. Bhāskara’s statement yet has another meaning. The ecliptic from 180° to 360° being in the reverse as can be seen in fig. 27, the time by which a particular Rāsi rises, is equal to the time by which its seventh Rāsi or diametrically opposite Rāsi sets. It is important to note that the rising time of a particular Rāsi is not equal to its setting time as can be gauged from the rising times, for, then the rising times of Meṣa and Tulā must be equal which is not the case. The word चिन्होमक्ष्या has this meaning as well, rising and setting being reverse directions.

Incidentally there is what is called a handed down i.e. a traditional statement which mentions “मीमाम्सी चन्द्रपल्ल; साँध बल्वारिगोघटी etc.” This clearly pertains to the local rising times of the Sāyana Rāsis at a place of latitude 17°-45' i.e. in between Rajamundry and Vizianagaram, as in this latitude the chara Segments in asus will be 230, 184, 73 which give rise to such rising times.

We next find what are called Nirayana Swādayas i.e. the rising times for the locality of the Nirayana Rāsis i.e. the Rāsis from the zero-point of the Hindu zodiac. We shall obtain these magnitudes for Rajahmundry whose latitude is 17°-2'. We shall however take it as 17°; also we shall take the Ayanāmsas to be 21° at present following Varāhamihira. This is one of the contexts where the knowledge of Ayanāmsas is essential. The method is the same as followed before; only, we have to find the declinations, right ascensions, charas and therefrom the rising times of arcs of magnitude 21°, 51°, 81°......351°. Subtracting the preceding rising times from the following we have successively the required rising times. We shall give them under the following table.
<table>
<thead>
<tr>
<th>Longitude</th>
<th>21°</th>
<th>51°</th>
<th>81</th>
<th>111</th>
<th>141</th>
<th>171</th>
<th>201</th>
</tr>
</thead>
<tbody>
<tr>
<td>Declination</td>
<td>8-13</td>
<td>18-13</td>
<td>23-12</td>
<td>21-51</td>
<td>14-32</td>
<td>3-37</td>
<td>8-13</td>
</tr>
<tr>
<td>Right ascension</td>
<td>19-24</td>
<td>48-14</td>
<td>80-12</td>
<td>112-43</td>
<td>143-24</td>
<td>171-44</td>
<td>199-21</td>
</tr>
<tr>
<td>Chara (ascentional difference)</td>
<td>2-30</td>
<td>5-41</td>
<td>7-32</td>
<td>7-3</td>
<td>4-33</td>
<td>1-6</td>
<td>2-30</td>
</tr>
<tr>
<td>Rising time in Vinādis</td>
<td>169</td>
<td>429</td>
<td>727</td>
<td>1057</td>
<td>1389</td>
<td>1706</td>
<td>2019</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Longitude</th>
<th>231</th>
<th>261</th>
<th>291</th>
<th>321</th>
<th>351</th>
<th>21°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Declination</td>
<td>18-3</td>
<td>23-12</td>
<td>21-51</td>
<td>14-32</td>
<td>3-37</td>
<td></td>
</tr>
<tr>
<td>Right ascension</td>
<td>228-33</td>
<td>260-12</td>
<td>292-42</td>
<td>323-24</td>
<td>351-44</td>
<td></td>
</tr>
<tr>
<td>Chara</td>
<td>5-41</td>
<td>7-3</td>
<td>4-43</td>
<td>4-43?</td>
<td>1-6</td>
<td></td>
</tr>
<tr>
<td>Rising time in Vinādis</td>
<td>2342</td>
<td>2677</td>
<td>2998</td>
<td>3279</td>
<td>3528</td>
<td></td>
</tr>
</tbody>
</table>
In modern terms, the rising times of Rasis could be found by solving the spherical triangle \( rEA \) (fig. 28) using the Inner side inner angle formula namely

\[
\cos x \times \cos \omega = \sin x \cot \lambda + \sin \omega \tan
\]

\[
\sin x \cot \lambda - \cos x \cos \omega = - \sin \omega \tan
\]

putting \( \sin x = t \), this reduces to

\[
t \cot \lambda - \sqrt{1 - t^2} = - \sin \omega \tan
\]

or \((t \cot \lambda + \sin \omega \tan \phi)^2 = (1 - t^2) \cos \)

\[
: t^2 (\cos^2 \omega + \cot^2 \lambda) + \sin \omega \tan \phi \cot \lambda = \cos^2 \omega = 0
\]

\[
t = -\sin \omega \tan \ , \cos \lambda \pm \ , \cot ^2 \cos ^2
\]

Putting successively \( \lambda = 30^\circ, 60^\circ...360^\circ \) and ignoring the negative sign of the radical we have the sines of rising times of arcs of the ecliptic of 30°, 60° etc. Subtracting
the preceding from the succeeding, we have successively the rising times. Thus the rising time of Mesha is
\[
\frac{\sqrt{3} \sin^3 \omega \tan^3 \phi + \cos^3 \omega (\cos^3 \omega + 3 - \sqrt{3} \sin \omega \tan \phi)}{3 + 1}
\]

That of Meṣa and Vriṣabha put together, the rising time is
\[
\frac{\sqrt{\sin^3 \omega \tan^3 \phi + 3 \cos^3 \omega (\cos^3 \omega + \frac{1}{3}) - \sin \omega \tan \phi}}{\sqrt{3} (\cos^3 \omega + \frac{1}{3})}
\]

The combined rising time of the 3 Rasis Mesha, Vriṣabha and Mithuna is from I using tables \(\cos^{-1}(\tan \omega \tan \phi) = 5046\) asus which exactly accords with what we have found previously namely 1505 + 1659 + 1882 = 5046. Also putting in \(\lambda = 180^\circ\), we have \(\sin x = 0\) or \(x = 180^\circ = 10800\) asus; subtracting 5046, we have the combined times of rising of Karkataka, Simha abd Kavya to be 5754 asus as we have had. Putting again \(\lambda = 270\), we have \(\cos^{-1}(\tan \omega \tan \phi) = 2\pi - 5046\) which signifies that the sum of the rising times of the last three rasis is the same as that of the first three which again means that the sum of the rising times of Tula to Dhanus is equal to the sum of the rising times of Karkataka, Simha and Kanya establishing Bhaskara's statement "कि:

Verse 60. Computations of Lagna, Udayantara and the like from the rising times of big arcs of the ecliptic like Rasis will be approximate, whereas one desirous of greater approximation has to find the same from the rising times of smaller arcs likes Horas and द्रक्कानंस, so as to be more correct.

Comm. Rasis divided into halves are called horas and if divided into one-third parts are called द्रक्कानंस. The meaning of the verse is that if after having found the rising time of a particular Rāsi say Meṣa, we say that one-third of that rising time is that of one-third of that Rāsi, we will be making only an appro-
ximate statement just like saying that 'Since 12 Rasis
erise in the course of a sidereal day, so each rasi rises in
1800 asus' which is far from truth being based on a crude
rule of three. So, Bhāskara says, Acharyas like Aryabhata
insisted on finding the rising times of Dṛkkānas, in as
much, as while computing the Lagna or the point of inter-
section of the Ecliptic with the horizon, we will be nearer
the truth by using the rising times of smaller arcs like
Dṛkkānas than broadly using those of Rasis.

Verse 61. Bhujāntara correction.

The equation of centre of the Sun multiplied by the
equatorial rising time of the Rāsi occupied by the Sun,
and divided by 1800, and then again multiplied by the
ture daily motion of a planet and divided by the number
of asus in a day, is the correction in the planet positive
or negative according as the equation of centre of the Sun
is positive or negative.

Comm. This Bhujāntara correction arising out of
the Sun's equation of centre is prescribed even for the Sun,
as well as for the other planets. The planets are originally
computed for the rising time of the Mean Sun, whereas
we are interested to know the positions at the True Sun-
rise. The position of the Sun also is originally computed
for his mean rise and is therefore to be rectified to get his
position at his true rise. So even the Sun is not exempt
from the correction. It will be noted here that as the
equation of centre pertains to the eccentricity of the Sun's
orbit, this correction of Bhujāntara is a correction for the
so-called modern 'Equation of time due to eccentricity'.
In other words, the Sun's equation of centre converted
into time is exactly what is called the Equation of time
due to eccentricity.

The formula prescribed for the correction is as follows.
Suppose the Sun is in a particular Rāsi which rises at the
equator in x asus. Let the equation of centre of the Sun be
E minutes of arc. Then the equatorial rising time of $E$ is \( \frac{E \times x}{1800} \) asus because there are 1800' in a Rasi. We have to provide a correction in the planetary position including that of the Sun for this time. If the planet goes $Y'$ during 21659 asus of a day, what arc is covered by the planet or the Sun in $\frac{E \times x}{1800}$ asus? The result is $\frac{E \times x}{1800} \times \frac{Y}{216}$ minutes of arc. This correction is positive if the equation of centre of the Sun is positive, for, we want the planetary position for a latter time than Mean Sunrise, since the positive equation of centre advances the True Sun over the Mean. This correction will be appreciable only in the case of the Moon having a rapid motion.

Verses 62, 63. The correction known as Udayāntara.

The difference in minutes of arc in the longitude of the Śāyana mean Sun and the asus in his Right ascension, multiplied by the daily motion of the planet and divided by 21659 is the result to be added to or subtracted from the planet's longitude according as the asus in the Sun's Right ascension are greater or less than the minutes of arc of his longitude. This is what is called Udayāntara correction in the planetary position.

Comm. (1) We have seen that the Bhujāntara is a correction in the mean planetary position due to the Equation of time in Eccentricity.

(2) This Udayāntara is a correction in the same due to the Equation of time in obliquity.

(3) Some have misconstrued that this Udayāntara correction is the Equation of time in the obliquity itself, whereas it is a correction to be effected in the planetary position due to the Equation of time due to obliquity.
(4) The maximum equation of centre in the Sun has a magnitude \(\frac{13}{3} \times \frac{1}{2\pi} \times \frac{41}{3} \times \frac{7}{44} = \) 

Converting this into time at the rate of 15° per hour (since in one hour diurnal rotation of the earth is equal to 15°) we have \(\frac{13}{6} \times 4\text{ minutes} = \frac{26}{3} = 8' - 40''\). The maximum equation of centre according to modern astronomy is \(2e\) expressed in radius where \(e = \frac{1}{60}(0.0167339) = 1\) 

radius = \(2 \times \frac{1}{60} \times \frac{180 \times 7}{22}\) degrees = \(\frac{21}{11} = 1° - 54'\) - 

As such the max. Equation of time due to eccentricity is \(\frac{21}{11} \times 4' = \frac{84}{11} = 7' - 38''\). The small difference in the two values arises out of the difference in the max. equations of centre. Any way it is clear that, in as much as the Bhujāntara correction is necessitated on account of the equation of centre in the Sun, to obtain the planetary positions computed for the mean Sunrise at the time of True Sunrise, the Bhujāntara correction is a correction in the planetary position, on account of the equation of time due to eccentricity. The max. correction to be effected in even the quick moving Moon amounts to \(\frac{8 \times 790}{24 \times 60} = \frac{79}{18} = \)

(5) We have said in the translation of the verse, 'The asus in the Right ascension of the mean Sun', where what exactly is stated by Bhāskara is, the time of rising of the small arc covered by the Sun in the particular Sāyana Rasi in which the Mean Sun is, (सूक्ष्मसव:) together with the rising times of the previous Sāyana Rasis covered by the Sun. The meaning is therefore the rising time of an arc of the ecliptic equal to the Sāyana longitude of the ecliptic which is measured by his Right ascension at the
rate of 15° per hour or 6° per nadi or 10 Vina
dis per degree or 60 asus per 60 minutes of arc or as many asus as there are minutes of arc in the right ascension of the mean Sun. So, what is stated by Bhāskara is the difference of the minutes of arc in the mean longitude of the Sun and the minutes of arc in his right ascension i.e. \((l - \alpha)\) expressed in minutes. We know that the total equation of time arising out of unequal motion in the true longitude of the Sun i.e. \(\odot\) in comparison with the equal motion in his right ascension i.e. \(\alpha\) is measured by \(\odot - \alpha\) which may be expressed as \((\odot - l) + (l - \alpha)\) where \(l\) is his mean longitude, \(\odot - l\) = Equation of centre and so the time expressed by \(\odot - l\) is the equation of time due to obliquity. The difference \(l - \alpha\) arises out of the obliquity of the ecliptic and so the time expressed by \(l - \alpha\) is the equation of time due to obliquity. We have the modern formula

\[
\cos \omega = \frac{\tan \alpha}{\tan l} \text{ so that } \frac{1 - \cos \omega}{1 + \cos \omega} = \frac{\tan \alpha - \tan l}{\tan \alpha + \tan l} = \frac{\sin \alpha - l}{\sin \alpha + l}
\]

\(\therefore\) Sin \((\alpha - l) = \tan^3 \omega/2 \sin (\alpha + l)\). As \(\alpha\) is very nearly equal to \(l\), we could write, when expressed in radius \(\alpha - l = \tan^2 \omega/2 \sin 2l\). Thus the maximum difference between \(\alpha\) and \(l\) arises when \(2l = 90°\) i.e. \(l = 45\) degrees i.e. at the mid-point of the first quadrant; the minimum difference is when \(2l = 270\) i.e. \(l = 135\) i.e. at the middle point of the 2nd quadrant. Also the numerical magnitudes of the max. as well as the minimum value are each \(\tan^3 \omega/2\) i.e. they are equal. Since this is expressed in radius, converting into time the numerical value of the max. and minimum equation of time due to obliquity is 9.87'. Thus we can write \(l - \alpha = 9.87' \sin 2l\). Again where \(l = 225\), \(2l = 450\) so that \(l - \alpha\) will have a positive max.; and again when \(l = 315\), \(2l = 630\) so that \(l - \alpha\) will have a negative maximum value. Also when \(l\) has values 0, 90, 180, 270 it is zero. Thus the equation of time due to obliquity is zero at \(r\), i.e. the vernal equinox; +ve in the first quadrant.
increasing from zero to 45 degrees and then decreasing from 45° to 90°; assuming the value zero at 90°, then negative in the second quadrant negatively increasing from zero to a maximum as \( l \) increases from 90° to 135°, and then negatively decreasing from the maximum value to zero at the end of the second quadrant; again behaving in the 3rd quadrant as in the first and in the fourth as in the second.

(6) It was stated by Mr. Mazumdar in his introduction to the Siddhānta Śekhara of Śripati published by the Calcutta University as well as by pandit Babuaji Misra, the editor thereof that this Udayāntara correction was first mentioned by Śripati, and it meant equation of time due to obliquity. In fact Śripati states (Verse I ch. eleven)

"अत्यधिकमं शृणिता रक्षितवाचिववीर्यांभरणं बिहितं, फलःकामुक्तेऽण, 
वाहोऽकाशं रहितास्ववर्धेपरं तेन, यातास्वरो युगुयोऽऽपरोऽणरणम्"

\( l \) expressed in minutes \( - \frac{H \sin^{-1} \frac{H \sin \gamma + H \cos \omega}{H \cos \delta}} \) expressed in asus = what are called elapsed asus and are +ve, +ve, +ve and -ve in the successive quadrants. We saw before that \( H \sin \alpha = \frac{H \sin \gamma + H \cos \omega}{H \cos \delta} \) so that Śripati meant \( l - \alpha \), the former expressed in minutes of arc and the latter in asus, or what is the same \( (l - \alpha) \) both expressed in minutes or asus. These give the gain of \( l \) over \( \alpha \). Immediately after this verse Śripati goes to a different topic, and never mentions (as understood from the printed text) any further details as to what is to be done with these Yātāsus. Since Bhāskara says explicitly that for these Yātāsus, the planets are to be corrected, we may surmise that there should have been in Śripati’s text also another verse detailing the usage of those asus.

(7) We shall now attend to what Bhāskara gives by way of explanation of the verses in question, The Ahār-