SEMESTER: - II   **CLASS: - F.Y.B.COM.**

DEPARTMENT: - MATHEMATICS AND STATISTICS

SUBJECT: - MATHEMATICAL AND STATISTICAL TECHNIQUES

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UNIT-I: - A) Functions and applications
B) Derivatives and applications

Q.1 The demand function for wheat on a given day in a certain market is given by $8p + 7q = 296$, where p = price in Rs per kg and q = quantity demanded in thousands of tons. If the corresponding supply function is $4p - 3q = -112$, what will be the equilibrium price and corresponding quantity?

Q.2 If the total revenue function of a company is $R = 49x - 2x^2$ and the total cost function is $C = 70 + 10x$, find the minimum number of units which can be produced and sold to avoid loss.

Q.3 **Find the derivatives of the following functions:**

1	$y = (e^x + 2)(2x^2 + x + 4)$	5	$y = x - e^x + 29$
2	$y = (x + e^x)(\log x - 10)$	6	$y = 6x^5 + \log 90 + 2(4^x) + e^x$
3	$y = 4x^7 - \log x + \sqrt{x}$	7	$y = (\log x + x)(5x^5 + 55)$
4	$y = \frac{e^x + 2^x}{x^2 + 1}$	8	$y = x^5(2x + 4)$

Q.4 If the total revenue is $R = 10D - D^2$ where D = demand., find (i) demand function, (ii) marginal revenue function, (iii) AR when MR = 0

Q.5 The demand function and the supply function for a commodity are given by $D = 400 - p^2$ and $S = 100 + 2p^2$ respectively. Find the rate of change in demand with respect to price at the equilibrium price.

Q.6 If the demand function is given by $D = 15 - 4p + p^2$, find the price elasticity of demand at p = 1.

Q.7 The demand function is given by $p = 18 + D - D^2$. Find the total revenue and the marginal revenue function.

Q.8 The total cost of manufacturing x items of a product is given by $C = 2x^2 + 3x + 10$. Find the total cost, average cost and marginal cost 10 items are manufactured..

Q.9 Find the output at which the profit is maximum and the maximum profit, if the total cost and the total revenue functions are given by $C = 500 + 20x$ and $R = 260 - 3x^2$ respectively.

Q.10 What is the elasticity of demand when MR = 0?

UNIT-II: - Interest and Annuities

Q.1 A person borrowed Rs.100000 from 2 different lenders. The first lender charged 12% p.a. & the second charged 20% p.a. He paid in all Rs.27200 as simple interest. How much he borrowed from each lender?

Q.2 A principal amounts to Rs.11800 after 4 years & to Rs.14040 after 7 years. Find the sum & the rate of simple interest.

Q.3 The simple interest at 20% p.a. on a certain sum of money for 4 years is Rs.25600. Find the compound interest on the same sum at the same rate & for the same period.

Q.4 Rasika invested Rs.20000 for 5 years at 9% p.a. simple interest. To gain the same simple interest in 4 years at the same rate, how much she should invest?

Q.5 Amit keeps a fixed deposit of Rs.25000/- in a bank for 3 years. If the rate of interest is 10% per annum compounded annually, find the total amount he will receive at the time of maturity after 3 years.

Q.6 The difference between the simple interest & the compound interest on a sum for 4 years at 10% p.a. be Rs.10255.60. Find the principal amount.

Q.7 Bhavin repaid the total amount of Rs.366025/- to Ketan against the loan taken by him before 4 years. If the rate of interest was 12.5% p.a. compound interest, find the loan amount.

Q.8 Find the number of years for which an annuity of Rs.10000 is paid at the end of each year, if its accumulated amount works out to be Rs.53680 with interest compounded at 20% p.a. *(Oct-17)*

Q.9 Find the amount at the end of 1 year of an annuity of Rs.5000/-payable at each quarter with rate of interest 12% p.a.

Q.10 A loan of Rs.80000 is to be returned in 4 equal monthly installments at 12% p.a.
      (a) Find the EMI using reducing balance method.
      (b) Calculate for each month the break- up of EMI into interest payment & principal repayment.

Q.11 A machine costing Rs.58000 is purchased for Rs.10000 down payment & the balance in 3 equal installments at the end of each year for the next 3 years, with compound interest at 9% p.a. Find the amount of each installment.

Unit – III- Correlation and Regression Analysis

1. Write a short note on Scatter diagram.
2. Explain in brief how one can identify the different types of correlation using Scatter diagram.
3. Explain briefly the idea of correlation. Define the coefficient of correlation. State the limiting values of r and comment on the same.
4. Define Spearman's rank correlation coefficient, explaining the symbols used. Discuss the limiting values of it.
   How to calculate the rank correlation coefficient when some values are repeated in the sets?
5. What is regression? Why are there in general, two regression lines? When do they coincide? Explain the use of regression equations.
6. Distinguish between correlation and regression. Explain the relation between the correlation coefficient and the regression coefficients.

7. Calculate product moment correlation coefficient for the following data-

Marks in English	10	2	5	7	9	4	8
Marks in Economics	8	4	4	8	5	3	7

8. The following data gives the experience (X) in years of eight machine operators and their performance ratings (Y). Calculate Karl Pearson's coefficient of correlation between them.

X	16	13	17	4	3	11	7	14
Y	88	87	89	72	70	82	78	84

9. From the following data calculate the coefficient of correlation-

No. of pairs of observations	= 12
Sum of X values	= 35
Sum of Y values	= 60
Sum of squares of X values	= 148
Sum of squares of Y values	= 450
Sum of products of X and Y	= 105

10. Calculate Spearman's rank correlation coefficient for the following data-

X	105	112	107	115	160	152	148	132
Y	120	127	135	123	140	142	138	110

11. Calculate Spearman's rank correlation coefficient for the following data-

X	15	18	20	25	25	32	46	46
Y	18	20	24	24	29	30	20	48

12. Calculate Spearman's rank correlation coefficient for the following data representing marks in two tests for a group of seven students-

Marks in Test I	52	34	47	65	43	34	54
Marks in Test II	65	59	65	68	82	60	57

13. The value of Spearman's rank correlation coefficient between a set of n values of variables X and Y was found to be 0.75. If the sum of squares of the differences in ranks was 30, find n.

14. Spearman's coefficient of rank correlation is given to be -0.25. The sum of squares of differences between is 105. Find the number of pairs in the data.

15. From the following data, obtain the yield when the rainfall is 30 inches. The correlation coefficient between rainfall and yield is 0.8

	Rainfall (inches)	Yield (per acre)
Mean	27	40
Standard Deviation	3	6

16. Find the two regression equations given the following data
$\bar{x} = 15$, $\bar{y} = 20$,
Regression coefficient of y on x = 0.3 ,
Regression coefficient of x on y = 0.7.
Find the coefficient of correlation.

17. Find the regression of y on x given the following data.
n = 10, $\bar{x} = 30$, $\bar{y} = 50$, $\sigma_x = 4$, $\sigma_y = 5$, $r_{xy} = 0.7$
Estimate y when x = 37.

18. Find the two regression equations from the following data.
$\bar{x} = 23$, $\bar{y} = 35$, $\sigma_x = 2$, $\sigma_y = 3$, $r_{xy} = 0.6$
Estimate (i) y when x = 20
(ii) x when y = 38

19. Given the following regression equations find $\bar{x}$, $\bar{y}$ and r.
$100y - 45x - 1400 = 0$ and $4y - 5x + 200 = 0$

20. The regression of y on x for certain bivariate data was found to be $10y = 3x + 155$ and that of x on y was $10x = 7y + 10$. Find $\bar{x}$, $\bar{y}$ and r.

21. Regression equations for certain bivariate data are found to be $45y = 14x + 490$ and $18y = 35x - 2744$. Find $\bar{x}$, $\bar{y}$ and r. It is known that $\sigma_y = 7$, find the value of σ_x.

Unit – IV- Time series and Index numbers

1. What is a time series? Describe the various components of a time series.
2. Describe secular trend component of time series.
3. What are seasonal variations? Explain briefly with example.
4. What are the merits and demerits of the method of moving averages?
5. Find 3 yearly moving averages for the following time series giving Exports of a company.

Year	2000	2001	2002	2003	2004	2005	2006	2007
Exports	46	53	72	57	62	78	60	85

Also plot the original time series and the moving averages on a graph paper.

6. Find trend values using 4 yearly moving averages for the following data.

Year	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
y	242	250	253	249	254	256	250	257	262	268	260

7. Fit a straight line trend to the following data representing imports in million Rs. Of a certain company. Also find an estimate for the year 2008.

Year	2000	2001	2002	2003	2004	2005	2006
Imports	48	50	58	52	45	41	49

8. Use the method of least squares to find straight line trend for the following time series of production in thousand units during 1995 – 2002. Also estimate production for the year 2003.

Year	1995	1996	1997	1998	1999	2000	2001	2002
Production	80	90	92	83	94	99	92	102

9. Calculate seasonal indices for the following data.

Expenditure (in lakhs of Rs.)			
Year	I	II	III	IV
2003	55	53	57	51
2004	56	55	60	53
2005	57	56	61	54

10. What is an Index number? State its uses and limitations.
11. Why index numbers are called as Economic barometers? Explain.
12. Describe the aggregative expenditure method and the family budget method of construction of cost of living index number.
13. Calculate for the following data the index number as given below:
    (i) Unweighted aggregative method   (ii) Weighted aggregative method
    (iii) Unweighted average of price relatives   (iv) Weighted average of price relatives

Commodity	Weight	Price in Rs.	
		Base Year	Current Year
A	130	550	1345
B	450	630	1250
C	75	150	335
D	225	450	778
E	120	225	886

14. From the following data calculate : I_L, I_P, I_F, I_{DB}, I_{ME}

Commodity	Base Year		Current Year	
	Price	Quantity	Price	Quantity
Rice	4	15	5	20
Pulses	8	20	12	30
Sugar	6	25	8	20
Oil	14	10	21	15

15. Calculate the cost of living index number for the following data.

Group	Weights W	Index Number I
Food	48	160
Fuel and lighting	7	120
Clothing	10	140
House Rent	10	100
Miscellaneous	15	80

Unit – V- Elementary Probability Distributions

1. Write the probability mass function (p.m.f.) of Binomial distribution and state its properties.
2. Write the probability mass function (p.m.f.) of Poisson distribution and state its properties.
3. State the properties of Normal distribution.
4. 30% of the students in the class are girls. Find the probability that a randomly selected group of 5 students include 3 girls.
5. The probability that a student is a swimmer is 4/5. Out of 5 students selected, find the probability that, (i) 4 are swimmers (ii) 1 or less are swimmers.
6. If mean and variance of a Binomial distribution are 4 and 2 respectively, find the probability that number of successes is (i) only 3 (ii) at least 7
7. If mean and standard deviation of Binomial distribution are 8 and 2 respectively, find probability that no. of successes is (i) only 2   (ii) 3 or 4.
8. A student calculates mean as 5 and variance as 9 for a Binomial distribution.
   Is his calculation correct? Justify.
9. The average number of phone calls per minute in a call centre is 4. Find the probability that during a specific minute, the number of calls is (i) only 1 (ii) at least 2
   (Given e $^{-4}$ = 0.0183)
10. A random variable X follows Binomial distribution with n = 4.
    If 4 P(X = 2) = 9 P(X = 3) , find P(X = 0)
11. Akash receives, on an average, 5 messages per day. Find the probability that on a specific day,

he will receive (i) only 2   (ii) at least 4   (iii) less than 3 messages.
(Given e $^{-5}$ = 0.0067)

12. For a Poisson distribution with standard deviation of 2 units, find the probability
(i) P(X = 1)   (ii) P (X < 3)   (iii) X ≥ 3)
(Given: e $^{-2}$ = 0.1353, e $^{-4}$ = 0.0183)

13. The income distribution of a group of 10000 persons was found to be normal with mean Rs. 7500/- p.m. and standard deviation Rs. 500. What percentage of this group had income
(i) exceeding Rs. 6600/-   (ii) exceeding Rs. 8320/- ?
(Given area under standard normal curve between ordinates of ± 1.64 is 0.8990)

14. The weekly wages of 8000 workers are normally distributed with mean Rs. 770/- and standard deviation Rs. 70/-. Find the no. of workers whose wages are below Rs 700/-.
(Given : area between z = 0 and z = 1 is 0.3413)

15. If the heights of 1000 soldiers in a regiment are distributed normally with a mean of 172 cms and a standard deviation 5 cms., how many soldiers have heights greater than 180 cms?
(The area of a standard normal variate z between z = -1.6 and z = 1.6 is 0.8904)